# CLASS: XI: MATHEMATICS

## PERMUTATIONS AND COMBINATIONS

## PRACTICE QUESTIONS ON FACTORIAL AND FUNDAMENTAL PRINCIPLES OF COUNTING

***FORMULA USED***

Factorial notation: $n!$ *or* $\lfloor n$

**1.** $n! = n(n-1)n-2)(n-3).....3.2.1$

**2.** $0! = 1$

When n is negative or a fraction, n! is not defined.

Prove the following for $n \in N$,

**1.** $(2n)! = 2^n.n!.[1.3.5.....(2n-1)]$

**2.** $(n+1)[n!n + (n-1)!(2n-1) + (n-2)!(n-1)!] = (n+2)!$

**3.** $\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} = \frac{(n+1)!}{r!(n-r+1)!}$

**4.** $\frac{n!}{r!} = n(n-1)(n-2).....(r+1)$

**5.** $(n-r+1).\frac{n!}{(n-r+1)!} = \frac{n!}{(n-r)!}$

**6.** $33!$ is divisible by $2^{15}$

**7.** $\frac{(2n!)}{[(n-1)!]^2} = \frac{n(n+1)(n+2)....(2n-1)(2n)}{(n-1)!}$

**8.** $(n!+1)$ is not divisible by any natural number between 2 and n.

**9.** $(n!)^2 \le n^n.n! < (2n)!$

Find n, if

**10.** $\frac{n!}{(n-2)!} = 930, n \ge 2$

**11.** $\frac{n!}{(n-5)!} = 20.\frac{n!}{(n-3)!}, n \ge 5$

**12.** $(n+2)! = 60.(n-1)!$

**13.** $(n+2)! = 2550.n!$

**14.** $\frac{1}{9!} + \frac{1}{10!} = \frac{n}{11!}$

**15.** $\frac{n!}{(n-3)!} = 990$

**16.** $\frac{(n+1)!}{(n-1)!}=6$

**17.** $\frac{(n+2)!}{(2n-1)!}.\frac{(2n+1)!}{(n+3)!}=\frac{72}{7}$

**18.** $\frac{(2n)!}{5!(2n-3)!}:\frac{n!}{4!(n-2)!}=52:5$

**19.** $\frac{n!}{2(n-2)!}:\frac{n!}{4!(n-4)!}=2:1, n\geq 4$

**20.** How many 3 letter words(with or without meaning) can be formed out of the letters of the word LOGARITHMS, if repetition of letter is not allowed?

**21.** Aditi wants to arrange 4 English, 3 Maths and 2 Physics books on a shelf. If the books on the same subject are different, determine the number of all possible arrangement.

**22.** A flag is in the form of three blocks each to be coloured differently. If there are eight different colours to choose from, how many such flags are possible?

**23.** In how many ways two books of different languages can be selected from 10 Hindi, 5 English and 7 Sanskrit books?

**24.** Four persons enter a bus and they find seven seats vacant. In how many ways can they be seated?

**25.** How many 5-digit numbers are there with all distinct digits?

**26.** How many digit odd numbers can be formed by using the digits 1,2,3,4,5,6 when (i) repetition of digits is not allowed (ii) the repetition of digits is allowed?

**27.** How many words are there with or without meaning of three distinct alphabets?

**28.** There are four routes between Delhi and Mumbai. In how many ways can a person go from Delhi to Mumbai and return if for returning (i) any route is taken (ii) the same route is taken (iii) the same route is not taken.

**29.** How many 4-digits odd numbers can be formed with the help of the digits 1,2,3, 4 and 5 if (i) no digit is repeated (iii) digits are repeated?

**30.** How many odd numbers less than 10,000 can be formed using the digits 0, 2, 3, 5 allowing repetition of digits?

**31.** How many 4-digits numbers can be formed using the digits 0,1,2,3,4,5, no digit being repeated?

**32.** How many 3-digit numbers are there such that 5 is at units place?

**33.** How many numbers are there between 100 and 1000 such that at least one of the digits is 6?

**34.** How many then three digit numbers are there which have exactly one of the digits as 6?

**35.** For a set of six true or false questions, no student has written all answers and no two students have given the same sequence of answers. What is the maximum number of students in the class for this job to be possible?

# CLASS: XI: MATHEMATICS

## PERMUTATIONS AND COMBINATIONS

PRACTICE QUESTIONS ON PERMUTATIONS

***FORMULA USED***

The different arrangement which can be made out of a given number of things by taking some or all at a time, are called permutations.

Let $1 \le r \le n$, then the number of all permutations of n dissimilar things taken at a time is given by ${}^nP_r$ *or* $P(n,r)$

$${}^nP_r = n(n-1)(n-2).....(n-r+1) = \frac{n!}{(n-r)!}$$

Properties: ${}^nP_n = n!, \quad {}^nP_{n-1} = n!, \quad {}^nP_0 = 1$

**Circular Permutation:**

The number of circular permutation of n different objects is (n – 1)!

The number of ways in which n persons can be seated round a table is (n – 1)!

The number of ways in which n different beads can be arranged to form a necklace is $\frac{1}{2}(n-1)!$

Prove the following for $n \in N$,

**1.** ${}^nP_r = \frac{n!}{(n-r)!}$

**2.** ${}^nP_n = {}^nP_{n-1}$

**3.** ${}^nP_r = n.{}^{n-1}P_{r-1}$

**4.** ${}^nP_r = {}^{n-1}P_r + r.{}^{n-1}P_{r-1}$

**5.** ${}^nP_n = 2.{}^nP_{n-2}$

Find n, if,

**6.** ${}^nP_4 = 20\,{}^nP_2$

**7.** ${}^{2n}P_3 = 100.{}^nP_2$

**8.** $16.{}^nP_3 = 13.{}^{n+1}P_3$

**9.** ${}^nP_5 = 20.{}^nP_3$

**10.** $30.{}^nP_6 = {}^{n+2}P_7$

**11.** ${}^nP_5 : {}^{n-1}P_4 = 6:1$

**12.** ${}^nP_4 : {}^{n-1}P_3 = 9:1$

**13.** ${}^{n-1}P_3 : {}^{n+1}P_3 = 5:12$

**14.** ${}^{2n-1}P_n : {}^{2n+1}P_{n-1} = 22:7$

Find r, if

**15.** ${}^{5}P_4 = {}^{6}P_{r-1}$

**16.** ${}^{15}P_r = 2730$

**17.** ${}^{10}P_r = 2\,{}^{9}P_r$

**18.** $4.{}^{6}P_r = {}^{6}P_{r+1}$

**19.** ${}^{20}P_r = 13.{}^{20}P_{r-1}$

**20.** ${}^{5}P_r = 2.{}^{6}P_{r-1}$

**21.** ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800:1$

**22.** How many three-digit even numbers can be formed from the digits 1,2,3,4,5,6 if the digits can be repeated?

**23.** How many 5 digit telephone number can be made using the digits 0 to 9, if each number starts with 67 and no digit appears more than once?

**24.** In how many ways can a party of 4 men and 4 women be seated at a circular table so that no two women are adjacent?

**25.** How many 4-digit numbers can be formed with the digits 1,2,3,4,5,6 when the repetition of the digits is allowed?

**26.** How many numbers can be formed with the digits 1,2,3,4,3,2,1 so that the odd digits always occupy the odd places?

**27.** How many different signals can be made from 4 red, 2 white and 3 green flags by arranging all of them vertically on a flag staff?

**28.** There are how many types of calendar for the month of February?

**29.** In how many ways can 4 letters be posted in 3 letter boxes?

**30.** A boy has 6 pockets. In how many ways can he put 5 coins in his pockets?

**31.** In how many ways can three prizes be distributed among 4 boys when (i) no one gets more than one prize (ii) a boy can get any number of prizes.

**32.** How many different permutations each containing the letter of the word STATESMAN can be formed?

**33.** Find the number of ways in which the letter of the word MACHINE can be arranged such that the vowels may occupy only odd ppositions.

**34.** How many words can be formed from the letters of the word SUNDAY? How many of these begin with D?

**35.** In how many ways can be letters of the word DIRECTOR be arranged so that all the vowels are never together?

**36.** The letter of the word OUGHT are written in all possible order and these words are written out as in a dictionary. Find the rank of the word TOUGH in this dictionary.

**37.** If the different permutations of the word EXAMINATION are arranged as in a dictionary, how many words are there before the first word starting with E?

**38.** In how many arrangement of the word GOLDEN will the vowels never occur together?

**39.** In how many ways can the word PENCIL be arranged so that N is always next to E?

**40.** In how many ways 8 examination papers be arranged so that the best and the worst papers are never together?

**41.** In how many ways n different books be arranged such that two particular books are never together?

**42.** How many permutations can be made out of the letters of the word TRIANGLE. How many of these will begin with T and end with E?

**43.** In how many ways 4 boys and 6 girls be seated in a line so that no two boys may sit the even places?

**44.** In how many ways 6 men and 5 women can sit in a row so that the women occupy the even places?

**45.** How many different signals can be given with 5 different flags by hoisting any number of them at a time?

**46.** A round table conference is to be held between delegates of 20 countries, in how many ways can they be seated if two particular delegates sit together?

**47.** 3 boys and 3 girls are to be seated around a table. Among them the boy X does not want any girl neighbour and the girl Y does not want any boy neighbour. How many such arrangements are possible?

**48.** Find the number of ways in which 10 different beads can be arranged to form necklace?

**49.** In how many different ways can 5 girl and 5 boys form a circle such that the boys and girls are alternate?

**50.** If 20 persons were invited to a party, in how many ways can they and the host be seated at a circular table? In how many of these ways will two particular persons be seated on either side of the host?

# CLASS: XI: MATHEMATICS

## PERMUTATIONS AND COMBINATIONS

PRACTICE QUESTIONS ON COMBINATIONS

***FORMULA USED***

Each of the different groups or selections which can be formed by taking some or all of number of objects, irrespective of their arrangement, is called a combination.
The number of all combinations of n distinct objects, taken r at a time is given by
${}^nC_r$ *or* $C(n,r)$. ${}^nC_r$ is defined only when n and r are integers such that
$n \geq r, n > 0$ *and* $r \geq 0$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

Properties:

$${}^nC_r = {}^nC_{n-r}$$

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

$${}^nC_0 = 1, \quad {}^nC_n = 1$$

If ${}^nC_p = {}^nC_q$, then p = q or + q = n.

Prove the following for $n \in N$,

**1.** $${}^nC_r = \frac{n!}{r!(n-r)!}$$

**2.** $$n.{}^{n-1}C_{r-1} = (n-r+1){}^nC_{r-1}$$

**3.** $$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$

**4.** $${}^{n-1}C_{r-1} + {}^{n-1}C_r = {}^nC_r$$

**5.** $${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

**6.** $${}^{2n}C_n = \frac{2^n.[1.3.5.....(2n-1)]}{n!}$$

**7.** $${}^nC_p = {}^nC_q \Rightarrow p = q \;\; or \;\; p+q = n$$

**8.** $${}^nC_r + 2.{}^nC_{r-1} + {}^nC_{r-2} = {}^{n+2}C_r$$

**9.** $$\frac{{}^nC_r}{{}^{n-1}C_{r-1}} = \frac{n}{r}$$

**10.** The product of k consecutive positive integers is divisible by k!.

Find n, if

**11.** ${}^nC_7 = {}^nC_5$

**12.** ${}^nC_{10} = {}^nC_{15}$

**13.** ${}^{n}C_{30} = {}^{n}C_{4}$

**14.** ${}^{2n}C_{3} : {}^{n}C_{3} = 11:1$

**15.** ${}^{n}C_{6} : {}^{n-3}C_{3} = 33:4$

**16.** ${}^{2n}C_{3} : {}^{n}C_{2} = 12:1$

Find r, if

**17.** ${}^{15}C_{3r} = {}^{15}C_{r+3}$

**18.** ${}^{8}C_{r} - {}^{7}C_{3} = {}^{7}C_{2}$

**19.** ${}^{18}C_{r} = {}^{18}C_{r+2}$

**20.** ${}^{15}C_{r} : {}^{15}C_{r-1} = 11:5$

Find n and r, if

**21.** ${}^{n}C_{r-1} = 36, {}^{n}C_{r} = 84, {}^{n}C_{r+1} = 126$

**22.** ${}^{n}C_{r-1} : {}^{n}C_{r} : {}^{n}C_{r+1} = 3:4:5$

**23.** ${}^{n}P_{r} = {}^{n}P_{r+1}, {}^{n}C_{r} = {}^{n}C_{r-1}$

**24.** ${}^{n+1}C_{r+1} : {}^{n}C_{r} = 11:6, {}^{n}C_{r} : {}^{n-1}C_{r-1} = 2:1$

**25.** ${}^{n}C_{r} : {}^{n}C_{r+1} : {}^{n}C_{r+2} = 1:2:3$

**26.** In how many ways can 11 players be chosen our of 15 if (i) there is no restriction (ii) a particular player is always be chosen (iii) a particular player is never chosen?

**27.** Out of 5 men and 2 women, a committee of 3 is to be formed. In how many ways can it be formed if at least one woman is to be included?

**28.** A committee of 5 is to be formed out of 6 men and 4 women. In how many ways can this be done, if (i) at least 2 women are included (ii) at most 2 women are included?

**29.** There are n points on a circle, find the number of (i) lines which can be drawn (ii) triangles which can be formed.

**30.** How many diagonals are there in a polygon of n sides?

**31.** A polygon has 35 diagonals. Find the number of its sides.

**32.** In how many ways a group of 11 boys can be divided into two groups of 6 and 5 boys each?

**33.** For the post of 5 clerks, there are 25 applicants, 2 posts are reserved for SC candidates and remaining for others, there are 7 SC candidates among the applicants. In how many ways can the selection be made?

**34.** In how many ways can 10 different books on English and 5 similar books on Hindi be placed in a row on a shelf so that two books on Hindi are not together?

**35.** In an examination, a candidate has to pass in each of the 5 subjects. In how many ways can he fail?

**36.** A question paper has two parts A and B each containing 10 questions. If a student has to choose 8 from part A and 4 from B, in how many ways can he choose the questions?

**37.** A cricket team of 11 players is to be selected from 16 players including 5 bowlers and 2 wicket keepers. In how many ways can a team be selected so as to consist of exactly 3 bowlers and one wicket keeper?

**38.** There are 15 points in a plane, of which 6 are collinear. How many (i) straight lines (ii) triangles can be formed by joining them?

**39.** A bag contains 4 red, 3 white and 2 blue balls, three balls are drawn at random, determine the number of ways of selecting balls of different colours.

**40.** A man has seven friends. In how many can he invite one or more of them to a party?

**41.** A father with 8 children wants to go to zoo as often as he can without taking the same three children more than once. How often will he go and how often will each child go?

**42.** There are two round tables one with m seats and the other with n seats around it. In how many ways can (m + n) guest be seated at them?

**43.** From 7 consonants and 4 vowels, how many different words can be formed consisting of 3 consonants and 2 vowels?

**44.** To fill 12 vacancies, there are 25 candidates of which 5 are from SC caste. If 3 of the vacancies are served for SC candidates while the rest are open to all. Find the number of ways in which the selection can be made.

**45.** From a class of 10 boys and 6 girls, 10 students are to be selected for a competition, at least including 4 boys and 4 girls. The 2 girls who won the prizes last year should be included. In how many ways the selection can be made?

**46.** A committee of 5 is to be selected from among 6 boys and 5 girls. Determine the number of ways of selections if the committee is to consist of at least one boy and one girl.

**47.** A question paper contains 12 questions divided into 3 parts, part A contains 6 questions while part B and C contains 3 questions each. A candidate is required to attempt 6 questions selecting at least one from each of parts B and C. In how many ways can the candidate select 6 questions?

**48.** A bag contains 4 red, 2 white and 3 blue balls, three balls are drawn random, determine the number of ways of selecting balls at least one black ball to be included in the draw.

**49.** A candidate is required to attempt 6 out of 10 questions, which are divided into groups each containing 5 questions, and he is not permitted to attempt more than 4 questions from each group. In how many ways can he make up his choice?

**50.** A candidate is required to attempt 7 out of 12 questions, which are divided into groups each containing 6 questions, and he is not permitted to attempt more than 5 questions from each group. In how many ways can he choose 7 questions?