

# CLASS: XI: MATHEMATICS

## TRIGONOMETRY FORMULAE

### PRACTICE QUESTIONS ON COMPOUND ANGLES

#### FORMULA USED

1.  $\sin(A + B) = \sin A \cos B + \cos A \sin B$
2.  $\sin(A - B) = \sin A \cos B - \cos A \sin B$
3.  $\cos(A + B) = \cos A \cos B - \sin A \sin B$
4.  $\cos(A - B) = \cos A \cos B + \sin A \sin B$
5.  $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
6.  $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
7.  $\cot(A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$
8.  $\cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

#### **Some Useful Results:**

1.  $\sin(A + B) \cdot \sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$
2.  $\cos(A + B) \cdot \cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

1. Evaluate: (i)  $\cos 105^\circ$  (ii)  $\tan 75^\circ + \cot 75^\circ$
2. Find the value of (i)  $\cos 70^\circ \cos 10^\circ + \sin 70^\circ \sin 10^\circ$  (ii)  $\cos 130^\circ \cos 40^\circ + \sin 130^\circ \sin 40^\circ$
3. Find the value of  $\sin 780^\circ \sin 480^\circ + \cos 240^\circ \cos 300^\circ$
4. Show that:  $\tan 70^\circ = 2 \tan 50^\circ + \tan 20^\circ$
5. Show that  $\tan 75^\circ - \tan 30^\circ - \tan 75^\circ \tan 30^\circ = 1$
6. Show that  $2 \tan 70^\circ = \tan 80^\circ - \tan 10^\circ$ .
7. Prove that  $\tan 3A \tan 2A \tan A = \tan 3A - \tan 2A - \tan A$ .
8. Prove that:  $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$
9. Prove that  $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$
10. Prove that:  $\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \tan 37^\circ$
11. Prove that:  $\tan 9^\circ = \frac{\cos 36^\circ - \sin 36^\circ}{\cos 36^\circ + \sin 36^\circ}$
12. Show that:  $\frac{\sin(A - B)}{\sin(A + B)} = \frac{\tan A - \tan B}{\tan A + \tan B}$

13. Show that:  $\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} = 0$
14. Prove that:  $\frac{1 + \sin A - \cos A}{1 + \sin A + \cos A} = \tan \frac{A}{2}$
15. Prove that:  $\sin^2\left(\frac{\pi}{8} + \frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{A}{2}\right) = \frac{1}{\sqrt{2}} \sin A$
16. Show that:  $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$
17. If  $\cos \alpha = -\frac{12}{13}$ ,  $\cot \beta = \frac{24}{7}$ ,  $\alpha$  lies in II quadrant,  $\beta$  lies in III quadrant. Find (i)  $\sin(\alpha + \beta)$   
(ii)  $\cos(\alpha + \beta)$  (iii)  $\tan(\alpha + \beta)$
18. If  $\tan x = \frac{3}{4}$ ,  $\pi < x < \frac{3\pi}{2}$ , find the value of  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$
19. Show that  $\sin^2(A+B) - \sin^2(A-B) = \sin 2A \sin 2B$
20. Prove that  $\sin^2 A = \cos^2(A-B) + \cos^2 B - 2 \cos(A-B) \cos A \cos B$
21. If A, B, C and D are angles of a cyclic quadrilateral, prove that  $\cos A + \cos B + \cos C + \cos D = 0$
22. If  $3 \tan \theta \tan \phi = 1$ , prove that  $2 \cos(\theta + \phi) = \cos(\theta - \phi)$ .
23. If  $\cot \alpha \cdot \cot \beta = 2$ , show that  $\frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} = 2$
24. Show that  $\cos^2 \frac{\theta - \phi}{2} - \sin^2 \frac{\theta + \phi}{2} = \cos \theta \cdot \cos \phi$
25. If  $\tan A = \frac{m}{m-1}$  and  $\tan B = \frac{1}{2m-1}$ , prove that  $A - B = \frac{\pi}{4}$ .
26. If  $\tan \beta = \frac{n \sin \alpha \cdot \cos \alpha}{1 - n \sin^2 \alpha}$ , prove that  $\tan(\alpha - \beta) = (1-n) \tan \alpha$ .
27. If  $\sin x + \sin y = a$  and  $\cos x + \cos y = b$ , show that  $\cos(x-y) = \frac{1}{2}(a^2 + b^2 - 2)$ .
28. If  $\tan \theta + \tan \phi = a$  and  $\cot \theta + \cot \phi = b$ , prove that  $\cot(\theta + \phi) = \frac{1}{a} - \frac{1}{b}$ .
29. If  $\sin(\alpha + \beta) = 1$  and  $\sin(\alpha - \beta) = \frac{1}{2}$ , where  $0 \leq \alpha, \beta \leq \frac{\pi}{2}$ , find the values of  $\tan(\alpha + 2\beta)$  and  $\tan(2\alpha + \beta)$ .
30. If  $\tan \frac{\alpha}{2}$  and  $\tan \frac{\beta}{2}$  are the roots of the equation  $8x^2 - 26x + 15 = 0$ , then find the value of  $\cos(\alpha + \beta)$ .
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# CLASS: XI: MATHEMATICS

## TRIGONOMETRY FORMULAE

### PRACTICE QUESTIONS ON TRANSFORMATION OF FORMULAE

#### FORMULA USED

##### Product to Sum or Difference Formulae

1.  $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$
2.  $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$
3.  $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$
4.  $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

##### Sum or Difference to Product Formulae

5.  $\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$
6.  $\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$
7.  $\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$
8.  $\cos C - \cos D = 2 \sin \frac{C+D}{2} \sin \frac{D-C}{2}$

1. Find the value of

(i)  $2 \sin 15^\circ \cos 75^\circ$

(ii)  $2 \cos 45^\circ \sin 15^\circ$

(iii)  $2 \sin 75^\circ \sin 15^\circ$

2. Find the value of  $2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13}$ .

3. Find the value of  $\sin \frac{5\pi}{12} \sin \frac{\pi}{12}$ .

4. Prove that:  $\sin 20^\circ \sin 40^\circ \sin 80^\circ = \frac{\sqrt{3}}{8}$ .

5. Prove that:  $\tan(45^\circ + \theta) - \tan(45^\circ - \theta) = 2 \tan 2\theta$

6. Prove that:  $\tan 20^\circ \tan 40^\circ \tan 80^\circ = \tan 60^\circ$

7. Prove that:  $\sec\left(\frac{\pi}{4} + \theta\right) \cdot \sec\left(\frac{\pi}{4} - \theta\right) = 2 \sec 2\theta$

8. Prove that  $\frac{2 \sin(\alpha - \gamma) \cos \gamma - \sin(\alpha - 2\gamma)}{2 \sin(\beta - \gamma) \cos \gamma - \sin(\beta - 2\gamma)} = \frac{\sin \alpha}{\sin \beta}$ .

9. Prove that  $\cos(120^\circ + \alpha) \cdot \cos(120^\circ - \alpha) = \frac{2 \cos 2\alpha - 1}{4}$

10. Prove that  $\tan 20^\circ \tan 40^\circ \tan 60^\circ \tan 80^\circ = 3$ .
11. Prove that:  $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$ .
12. Prove that  $\cos^2 A + \cos^2 B - 2 \cos A \cos B \cos(A + B) = \sin^2(A + B)$
13. Prove that  $\sin^2 A + \sin^2(A - B) - 2 \sin A \cos B \sin(A - B) = \sin^2 B$
14. Prove that  $\tan(A + 30^\circ) + \cot(A - 30^\circ) = \frac{1}{\sin 2A - \sin 60^\circ}$
15. Prove that  $4 \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{4}$
16. Prove that  $4 \cos A \cos(60^\circ - A) \cos(60^\circ + A) = \cos 3A$
17. Prove that  $\sin A \sin(B - C) + \sin B \sin(C - A) + \sin C \sin(A - B) = 0$
18. Prove that  $\sin(60^\circ + A) \sin(420^\circ - A) = \frac{1 + 2 \cos 2A}{4}$
19. Prove that:  $(\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 = 4 \cos^2 \left( \frac{\alpha - \beta}{2} \right)$
20. If  $\cos A + \cos B = \frac{1}{3}$  and  $\sin A + \sin B = \frac{1}{4}$ , then prove that  $\tan \frac{A + B}{2} = \frac{3}{4}$
21. Find the value of (i)  $\sin 75^\circ + \sin 15^\circ$  (ii)  $\cos 75^\circ + \sin 15^\circ$  (iii)  $\cos \frac{4\pi}{5} + \cos \frac{\pi}{5}$   
(iv)  $\sin 50^\circ - \cos 80^\circ$
22. Find the value of  $\frac{\cos 20^\circ - \cos 70^\circ}{\sin 70^\circ - \sin 20^\circ}$ .
23. Find the value of  $\frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ - \cos 15^\circ}$ .
24. Show that:  $\sin \left( \frac{\pi}{4} + \theta \right) + \sin \left( \frac{\pi}{4} - \theta \right) = \sqrt{2} \cos \theta$
25. Show that  $\cos \left( \frac{2\pi}{3} + \theta \right) + \cos \left( \frac{2\pi}{3} - \theta \right) = \sqrt{3} \cos \theta$
26. Prove that  $\cos 15^\circ - \sin 15^\circ = \frac{1}{\sqrt{2}}$
27. Prove that  $\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4 \cos 2\theta \cos 4\theta$
28. Prove that  $\frac{\cos \theta + \cos 2\theta + \cos 3\theta + \cos 4\theta}{\sin \theta + \sin 2\theta + \sin 3\theta + \sin 4\theta} = \cot \frac{5\theta}{2}$
29. Prove that  $\frac{\cos 3\theta + 2 \cos 5\theta + \cos 7\theta}{\cos \theta + 2 \cos 3\theta + \cos 5\theta} = \cos 2\theta - \sin 2\theta \tan 3\theta$

30. If  $x \cos \theta = y \cos \left( \theta + \frac{2\pi}{3} \right) = z \cos \left( \theta + \frac{4\pi}{3} \right)$ , then show that  $xy + yz + zx = 0$ .

31. Prove that  $\cos \alpha + \cos \beta + \cos \gamma + \cos(\alpha + \beta + \gamma) = 4 \cos \frac{\alpha + \beta}{2} \cos \frac{\beta + \gamma}{2} \cos \frac{\gamma + \alpha}{2}$ .

32. Prove that  $\sin \alpha + \sin \beta + \sin \gamma - \sin(\alpha + \beta + \gamma) = 4 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta + \gamma}{2} \sin \frac{\gamma + \alpha}{2}$ .

33. Prove that  $\frac{\cos 2A \cos 3A - \cos 2A \cos 7A + \cos A \cos 10A}{\sin 4A \sin 3A - \sin 2A \sin 5A + \sin 4A \sin 7A} = \cot 6A \cot 5A$

34. Prove that  $\frac{\cos \left( \frac{\pi}{4} + \theta \right) - \cos \left( \frac{\pi}{4} - \theta \right)}{\sin \left( \frac{2\pi}{3} + \theta \right) - \sin \left( \frac{2\pi}{3} - \theta \right)} = \sqrt{2}$

35. Prove that  $\frac{\sin(4A - 2B) + \sin(4B - 2A)}{\cos(4A - 2B) + \cos(4B - 2A)} = \tan(A + B)$

36. If  $A + B + C = \pi$ , prove that  $\frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B} = 2$

37. Prove that  $\sin(y + z - x) + \sin(z + x - y) + \sin(x + y - z) - \sin(x + y + z) = 4 \sin x \sin y \sin z$

38. If  $b \sin \beta = a \sin(2\alpha + \beta)$ , prove that  $(b + a) \cot(\alpha + \beta) = (b - a) \cot \alpha$

39. If  $\sin A + \sin B = a$  and  $\cos A + \cos B = b$ , then prove that (i)  $\tan \frac{A+B}{2}$  and (ii)  $\tan \frac{A-B}{2}$ .

40. If  $\sin \theta = n \sin(\theta + 2\alpha)$ , prove that  $\tan(\theta + \alpha) = \frac{1+n}{1-n} \tan \alpha$ .

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# CLASS: XI: MATHEMATICS

## TRIGONOMETRY FORMULAE

### PRACTICE QUESTIONS ON MULTIPLES AND SUBMULTIPLES

#### FORMULA USED

##### Multiples Formulae

$$\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}, \quad \cos^2 A = \frac{1 + \cos 2A}{2} \quad \text{and} \quad \tan^2 A = \frac{1 - \cos 2A}{1 + \cos 2A}$$

$$\sin 3A = 3 \sin A - 4 \sin^3 A$$

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

##### Sub- Multiples Formulae

$$\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2} = \frac{2 \tan \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$$

$$\cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2} = 1 - 2 \sin^2 \frac{A}{2} = 2 \cos^2 \frac{A}{2} - 1 = \frac{1 - \tan^2 \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$$

$$\sin^2 \frac{A}{2} = \frac{1 - \cos A}{2}, \quad \cos^2 \frac{A}{2} = \frac{1 + \cos A}{2} \quad \text{and} \quad \tan^2 \frac{A}{2} = \frac{1 - \cos A}{1 + \cos A}$$

$$\tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$$

1. Find the value of (i)  $\sin 15^\circ$  (ii)  $\sin 7\frac{1}{2}^\circ$  (iii)  $\cos 22\frac{1}{2}^\circ$  (iv)  $\sin 22\frac{1}{2}^\circ$  (v)  $\tan 142\frac{1}{2}^\circ$

(vi)  $\cot 7\frac{1}{2}^\circ$  (vii)  $\tan 11\frac{1}{4}^\circ$

2. Find  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$  (i) if  $\sin x = \frac{1}{4}$ , x in II quadrant (ii) if  $\cos x = -\frac{1}{3}$ , x in III quadrant.

3. Evaluate:  $8 \cos^3 \frac{\pi}{9} - 6 \cos \frac{\pi}{9}$
4. If  $\sin A = \frac{3}{5}$  and A is in I quadrant, find  $\sin 2A$ ,  $\cos 2A$  and  $\tan 2A$ .
5. Prove that:  $\cos 5A = 16 \cos^5 A - 20 \cos^3 A + 5 \cos A$
6. Prove that:  $16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta = \sin 5\theta$
7. Prove that:  $\frac{1 + \sin A - \cos A}{1 + \sin A + \cos A} = \tan \frac{A}{2}$
8. Prove that:  $\frac{\cos A}{1 - \sin A} = \tan \left( 45^\circ + \frac{A}{2} \right)$
9. Prove that:  $\sqrt{2 + \sqrt{2(1 + \cos 4A)}} = 2 \cos A$
10. Prove that:  $\cos^3 x \sin^2 x = \frac{1}{16} (2 \cos x - \cos 3x - \cos 5x)$
11. If  $\tan x = \frac{1}{7}$  and  $\tan y = \frac{1}{3}$ , prove that  $\cos 2x = \sin 4y$
12. Prove that:  $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 3A}}} = 2 \cos A$
13. Prove that:  $4(\cos^3 20^\circ + \cos^3 40^\circ) = 3(\cos 20^\circ + \cos 40^\circ)$
14. Prove that:  $\tan A + 2 \tan 2A + 4 \tan 4A + 8 \cot 8A = \cot A$
15. Prove that:  $\sin A \cdot \sin(60^\circ - A) \cdot \sin(60^\circ + A) = \frac{1}{4} \sin 3A$
16. Prove that:  $\cos A \cdot \cos(60^\circ - A) \cdot \cos(60^\circ + A) = \frac{1}{4} \cos 3A$
17. Prove that:  $\cos A \cdot \cos 2A \cdot \cos 4A \dots \dots \dots \cos 2^{n-1} A = \frac{\sin(2^n A)}{2^n (\sin A)}$ .
18. If  $\tan \frac{A}{2} = \sqrt{\frac{1-x}{1+x}} \tan \frac{B}{2}$ , prove that  $\cos B = \frac{\cos A - x}{1 - x \cos A}$
19. If  $\cos A = \frac{a \cos B + b}{a + b \cos B}$ , prove that  $\tan \frac{A}{2} = \sqrt{\frac{a-b}{a+b}} \tan \frac{B}{2}$
20. If  $2 \cos A = x + \frac{1}{x}$ , prove that  $2 \cos 3A = x^3 + \frac{1}{x^3}$
21. Prove that:  $\cos^3 A + \cos^3(120^\circ + A) + \cos^3(240^\circ + A) = \frac{3}{4} \cos 3A$
22. Prove that:  $\cot A + \cot(60^\circ + A) + \cot(120^\circ + A) = 3 \cot 3A$
23. Prove that:  $\sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{5\pi}{8} + \sin^2 \frac{7\pi}{8} = \frac{3}{2}$

24. Prove that:  $\cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{5\pi}{8} + \cos^2 \frac{7\pi}{8} = 2$

25. Prove that:  $\cos^2 \frac{\pi}{10} + \cos^2 \frac{2\pi}{5} + \cos^2 \frac{3\pi}{5} + \cos^2 \frac{9\pi}{10} = 2$ .

26. If  $m \tan(\theta - 30^\circ) = n \tan(\theta + 120^\circ)$ , prove that  $\cos 2\theta = \frac{m+n}{2(m-n)}$ .

27. If  $\sin \alpha = \lambda \sin(\theta - \alpha)$ , prove that  $\tan\left(\alpha - \frac{\theta}{2}\right) = \frac{\lambda-1}{\lambda+1} \tan \frac{\theta}{2}$ .

28. Prove that:  $\cos^3\left(x - \frac{2\pi}{3}\right) + \cos^3 x + \cos^3\left(x + \frac{2\pi}{3}\right) = \frac{3}{4} \cos 3x$

29. If  $\theta = \frac{\pi}{2^n + 1}$ , prove that  $2^n \cos \theta \cos 2\theta \cos 4\theta \dots \cos 2^{n-1}\theta = 1$ .

30. If  $\sec(\phi + \alpha), \sec \phi$  and  $\sec(\phi - \alpha)$  are in A.P., prove that  $\cos \phi = \pm \sqrt{2 \cos^2 \frac{\alpha}{2}}$ .

31. Find the value of (i)  $\sin 18^\circ$  (ii)  $\cos 18^\circ$  (iii)  $\sin 36^\circ$  (iv)  $\cos 36^\circ$

32. Prove that:  $\sin^2 72^\circ - \sin^2 60^\circ = \frac{\sqrt{5}-1}{8}$

33. Prove that:  $\sin \frac{\pi}{5} \sin \frac{2\pi}{5} \sin \frac{3\pi}{5} \sin \frac{4\pi}{5} = \frac{5}{16}$

34. Prove that:  $\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ = \frac{1}{16}$

35. Prove that:  $\cos 36^\circ \cos 72^\circ \cos 108^\circ \cos 144^\circ = \frac{1}{16}$

36. Prove that:  $\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ = 1$

37. Prove that:  $\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{3\pi}{15} \cos \frac{4\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15} = \frac{1}{128}$

38. Prove that:  $\left(1 + \cos \frac{\pi}{10}\right) \left(1 + \cos \frac{3\pi}{10}\right) \left(1 + \cos \frac{7\pi}{10}\right) \left(1 + \cos \frac{9\pi}{10}\right) = \frac{1}{16}$

39. If  $13\alpha = \pi$ , prove that  $\cos \alpha \cos 2\alpha \cos 3\alpha \cos 4\alpha \cos 5\alpha \cos 6\alpha = \frac{1}{64}$ .

40. If  $15\alpha = \pi$ , prove that  $\cos 2\alpha \cos 4\alpha \cos 8\alpha \cos 14\alpha = \frac{1}{16}$ .

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# CLASS: XI: MATHEMATICS

## TRIGONOMETRY FORMULAE

### PRACTICE QUESTIONS ON SOLUTION OF TRIANGLES

#### FORMULA USED

##### Sine Formula

$$1. \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

##### Cosine Formula

$$2. a^2 = b^2 + c^2 - 2bc \cos A, b^2 = c^2 + a^2 - 2ca \cos B \text{ and } c^2 = a^2 + b^2 - 2ab \cos C$$

or

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \cos B = \frac{c^2 + a^2 - b^2}{2ca}, \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

##### Tangent Formula

$$3. \tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right) \cot \frac{C}{2}, \tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right) \cot \frac{A}{2} \text{ and}$$
$$\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right) \cot \frac{B}{2}$$

##### Projection Formulae

$$4. a = b \cos C + c \cos B$$
$$b = a \cos C + c \cos A$$
$$c = a \cos B + b \cos A$$

##### Area of triangle Formulae

$$5. \Delta = \frac{1}{2}bc \sin A, \Delta = \frac{1}{2}ca \sin B \text{ and } \Delta = \frac{1}{2}ab \sin C$$

##### Formulae for $\sin \frac{A}{2}, \sin \frac{B}{2}, \sin \frac{C}{2}$

$$6. \sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}, \sin \frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}, \sin \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

##### Formulae for $\cos \frac{A}{2}, \cos \frac{B}{2}, \cos \frac{C}{2}$

$$7. \cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}, \cos \frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}, \cos \frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

##### Formulae for $\tan \frac{A}{2}, \tan \frac{B}{2}, \tan \frac{C}{2}$

$$8. \tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}, \tan \frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}, \tan \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

1. In a  $\Delta ABC$ , if  $a = 2$ ,  $b = 3$  and  $\sin A = \frac{2}{3}$ , find  $\angle B$ .

2. In a  $\Delta ABC$ , if  $a = 18$ ,  $b = 24$  and  $c = 30$ , find  $\sin A$ ,  $\sin B$  and  $\sin C$ .

3. For any triangle  $ABC$ , prove that  $\frac{\sin(B-C)}{\sin(B+C)} = \frac{b^2 - c^2}{a^2}$ .

4. In a triangle ABC, if  $a \cos A = b \cos B$ , show that the triangle is either isosceles or right triangle.

Prove that the following:

$$5. (b+c) \cos \frac{B+C}{2} = a \cos \frac{B-C}{2}$$

$$6. \frac{c}{a+b} = \frac{1 - \tan \frac{A}{2} \tan \frac{B}{2}}{1 + \tan \frac{A}{2} \tan \frac{B}{2}}$$

$$7. \frac{c}{a-b} = \frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{\tan \frac{A}{2} - \tan \frac{B}{2}}$$

$$8. a \cos A + b \cos B + c \cos C = 2b \sin A \sin C = 2c \sin A \sin B$$

$$9. (b-c) \cot \frac{A}{2} + (c-a) \cot \frac{B}{2} + (a-b) \cot \frac{C}{2} = 0$$

$$10. \frac{a^2 \sin(B-C)}{\sin B + \sin C} + \frac{b^2 \sin(C-A)}{\sin C + \sin A} + \frac{c^2 \sin(A-B)}{\sin A + \sin B} = 0$$

$$11. \frac{b^2 - c^2}{\cos B + \cos C} + \frac{c^2 - a^2}{\cos C + \cos A} + \frac{a^2 - b^2}{\cos A + \cos B} = 0$$

$$12. \frac{a \sin(B-C)}{b^2 - c^2} = \frac{b \sin(C-A)}{c^2 - a^2} = \frac{c \sin(A-B)}{a^2 - b^2}$$

$$13. a \sin(B-C) + b \sin(C-A) + c \sin(A-B) = 0$$

$$14. \frac{1 + \cos(A-B) \cos C}{1 + \cos(A-C) \cos B} = \frac{a^2 + b^2}{a^2 + c^2}$$

$$15. 2(bc \cos A + ca \cos B + ab \cos C) = a^2 + b^2 + c^2$$

$$16. a(b \cos C - c \cos B) = b^2 - c^2$$

$$17. (b^2 - c^2) \cot A + (c^2 - a^2) \cot B + (a^2 - b^2) \cot C = 0$$

$$18. \frac{b^2 - c^2}{a^2} \sin 2A + \frac{c^2 - a^2}{b^2} \sin 2B + \frac{a^2 - b^2}{c^2} \sin 2C = 0$$

$$19. (c^2 - a^2 + b^2) \tan A = (a^2 - b^2 + c^2) \tan B = (b^2 - c^2 + a^2) \tan C$$

$$20. a^3 \cos(B-C) + b^3 \cos(C-A) + c^3 \cos(A-B) = 3abc$$

$$21. 2 \left( b \cos^2 \frac{C}{2} + c \cos^2 \frac{B}{2} \right) = a + b + c$$

$$22. 2 \left( a \sin^2 \frac{C}{2} + c \sin^2 \frac{A}{2} \right) = a + c - b$$

23. 
$$\frac{\cos A}{b \cos C + c \cos B} + \frac{\cos B}{c \cos A + a \cos C} + \frac{\cos C}{a \cos B + b \cos A} = \frac{a^2 + b^2 + c^2}{2abc}$$
24. If in a triangle ABC,  $\frac{2 \cos A}{a} + \frac{\cos B}{b} + \frac{2 \cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$ , then prove that the triangle is right angled.
25. In a triangle ABC, if  $\cos B = \frac{\sin B}{2 \sin C}$ , show that the triangle is isosceles.
26. In any triangle ABC, if  $C=60^\circ$ , prove that  $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ .
27. In a triangle ABC, if  $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$  then prove that  $\frac{\cos A}{7} = \frac{\cos B}{19} = \frac{\cos C}{25}$ .
28. In a triangle ABC, if  $B=60^\circ$ , prove that  $(a+b+c)(a-b+c) = 3ca$ .
29. The sides of a triangle are  $a = 4$ ,  $b = 6$  and  $c = 8$ , show that  $8 \cos A + 16 \cos B + 4 \cos C = 17$ .
30. In any triangle ABC, if  $A = 30^\circ$ ,  $b = 3$  and  $c = 3\sqrt{3}$ , then find  $\angle B$  and  $\angle C$ .
31. In any triangle ABC, prove that  $4\Delta(\cot A + \cot B + \cot C) = a^2 + b^2 + c^2$
32. In any triangle ABC, prove that  $a \cos A + b \cos B + c \cos C = 2a \sin B \sin C = \frac{8\Delta^2}{abc}$
33. In any triangle ABC, prove that  $\frac{a^2 - b^2}{2} \cdot \frac{\sin A \sin B}{\sin(A-B)} = \Delta$
34. In any triangle ABC, prove that  $4\Delta \cot A = b^2 + c^2 - a^2$
35. In a triangle ABC,  $a = 3$ ,  $b = 5$ ,  $c = 6$ . Calculate (i)  $\sin \frac{A}{2}$  (ii)  $\cos \frac{A}{2}$  (iii) area of triangle.
36. In a triangle ABC, if  $a = 18$ ,  $b = 24$  and  $c = 30$ , find  $\tan \frac{A}{2}$ ,  $\tan \frac{B}{2}$ ,  $\tan \frac{C}{2}$ .
37. In a triangle ABC, prove that  $\frac{(a+b+c)^2}{a^2 + b^2 + c^2} = \frac{\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}}{\cot A + \cot B + \cot C}$
38. In a triangle ABC, prove that  $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \frac{a+b+c}{a+b-c} \cot \frac{C}{2}$
39. In a triangle ABC, prove that  $(b+c-a) \left( \cot \frac{B}{2} + \cot \frac{C}{2} \right) = 2a \cot \frac{A}{2}$
40. In a triangle ABC, prove that  $(a+b+c) \left( \tan \frac{A}{2} + \tan \frac{B}{2} \right) = 2c \cot \frac{C}{2}$