

KVS Junior Mathematics Olympiad (JMO) – 2001

M.M. 100

Time : 3 hours

Note : (i) Please check that there are two printed pages and ten question in all.

(ii) Attempt all questions. All questions carry equal marks.

1. Fill in the blanks :

- (a) If $x + y = 1$, $x^3 + y^3 = 4$, then $x^2 + y^2 = \dots\dots\dots$
- (b) After 15 litres of petrol was added to the fuel tank of a car, the tank was 75% full. If the capacity of the tank is 28 litres, then the number of litres in the tank before adding the petrol was
- (c) The perimeter of a rectangle is 56 metres. The ratio of its length to width is 4:3. The length of the diagonal in metres is
- (d) If April 23 falls on Tuesday, then March 23 of the same year was a
- (e) The sum of the digits of the number $2^{2000}5^{2004}$ is

2. (a) Arrange the following in ascending order :

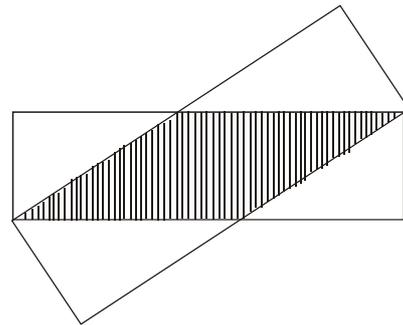
$$2^{5555}, 3^{3333}, 6^{2222}$$

- (b) Two rectangles, each measuring 3 cm x 7 cm, are placed as in the adjoining figure :

Find the area of the overlapping portion (shaded) in cm^2 .

3. (a) Solve :

$$\frac{\log_{10}(35 - x^3)}{\log_{10}(5 - x)} = 3$$



(b) Simplify :

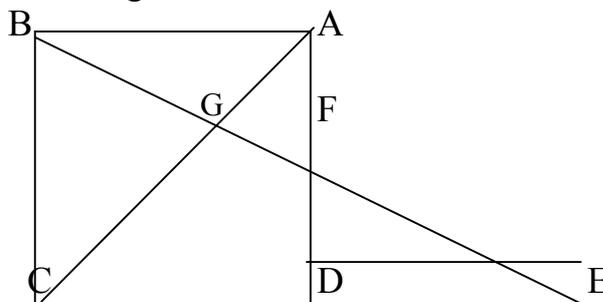
$$\frac{a-b}{a+b} + \frac{b-c}{b+c} + \frac{c-a}{c+a} + \frac{(a-b)(b-c)(c-a)}{(a+b)(b+c)(c+a)}$$

4. (a) Factorize :

$$(x-y)^3 + (y-z)^3 + (z-x)^3$$

- (b) If $x^2 - x - 1 = 0$, then find the value of $x^3 - 2x + 1$

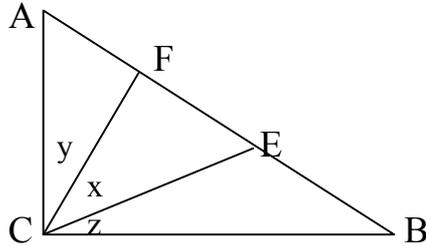
5. ABCD is a square. A line through B intersects CD produced at E, the side AD at F and the diagonal AC at G.



If $BG = 3$, and $GF = 1$, then find the length of FE ,

6. (a) Find all integers n such that $(n^2 - n - 1)^{n+2} = 1$

- (b) If $x = \frac{4ab}{a+b}$, find the value of $\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}$
7. (a) Find all the positive perfect cubes that divide 9^9 .
 (b) Find the integer closest to $100(12 - \sqrt{143})$
8. In a triangle ABC, $\angle BCA = 90^\circ$. Points E and F lie on the hypotenuse AB such that $AE = AC$ and $BF = BC$. Find $\angle ECF$.



9. An ant crawls 1 centimetre north, 2 centimetres west, 3 centimetres south, 4 centimetres east, 5 centimetres north and so on, at 1 centimetre per second. Each segment is 1 centimetre longer than the preceding one, and at the end of a segment, the ant makes a left turn. In which direction is the ant moving 1 minute after the start?
10. Find the lengths of the sides of a triangle with 20, 28 and 35 as the lengths of its altitudes.

KVS Junior Mathematics Olympiad (JMO) – 2002

M.M. 100

Time : 3 hours

Note : (i) Please check that there are two printed pages and 10 questions in the question paper.

(ii) Attempt all questions.

1. Fill in the blanks

- (a) Yash is carrying 100 hundred – rupee notes, 50 fifty –rupee notes, 20 twenty – rupee notes, 10 ten –rupee notes and 5 five-rupee notes. The total amount of money he is carrying in Rupee, is
- (b) In a school, the ratio of boys to girls is 4:3 and the ratio of girls to teachers is 8 :1. The ratio of students to teachers is
- (c) The value of $\left(0.5 + \frac{1}{0.5}\right)^2$ is
- (d) $(123456)^2 + 123456 + 123457$ is the square of
- (e) The area of square is 25 square centimeters. In perimeter, in centimeters, is
2. (a) How many four digit numbers can be formed using the digits 1,2 only so that each of these digits is used at least once ?
 (b) Find the greatest number of four digits which when increased by 1 is exactly divisible by 2, 3, 4, 5, 6 and 7.
3. (a) If $f(x) = ax^7 + bx^5 + cx^3 - 6$, and $f(-9) = 3$, find $f(9)$.
 (b) Find the value of

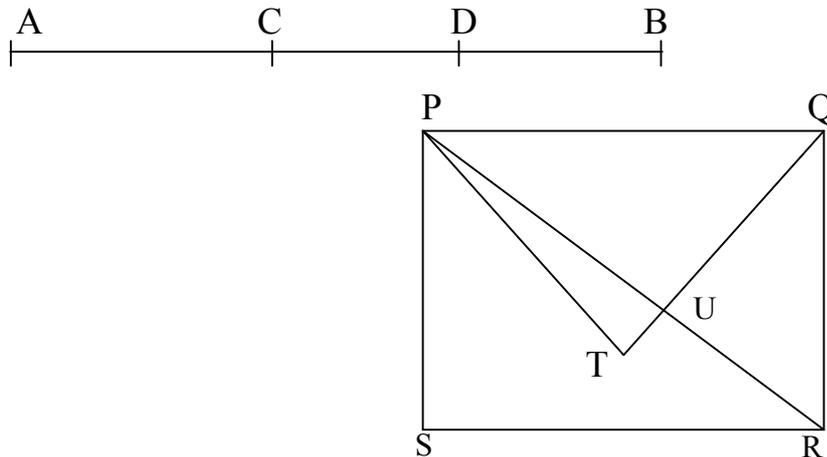
$$\frac{(2002)^3 - (1002)^3 - (1000)^3}{3 \times (1002) \times (1000)}$$

4.(a) If $x > 0$ and $x^4 + \frac{1}{x^4} = 47$, find the value of $x^3 + \frac{1}{x^3}$

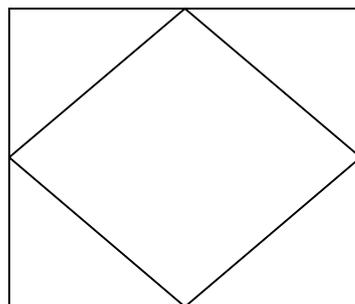
(b) If $8^{2x} = 16^{1-2x}$, find the value of 3^{7x} .

5. A train, after traveling 70 km from a station A towards a station B, develops a fault in the engine at C, and covers the remaining journey to B at $\frac{3}{4}$ of its earlier speed and arrives at B 1 hour and 20 minutes late. If the fault had developed 35 km further on at D, it would have arrived 20 minutes sooner. Find the speed of the train and the distance from A to B.

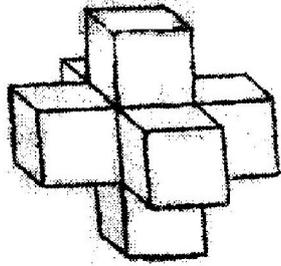
6. The adjoining diagram shows a square PQRS with each side of length 10 cm. Triangle PQT is equilateral. Find the area of the triangle UQR.



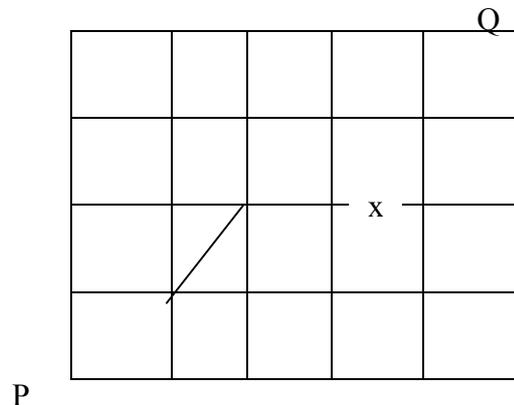
A square of side – length 64 cm is given. A second square is obtained by connecting the mid points of the sides of the first square (as shown in the diagram). If the process of forming smaller inner squares by connecting the mid points of the sides of the previous squares is continued, what will be the side-length of the eleventh square, counting the original square as the first square ?



7. Seven cubes of the same size are glued together face to face as shown in the adjoining diagram. What is the surface area, in square centimeters, of the solid if its volume is 448 cubic centimeters ?



8. Anil, Bhavna, Chintoo, Dolly and Eashwar play a game in which each is either a FOX or a RABBIT. FOXES' statements are always false and RABBITS' statements are always true.
 Anil says that Bhavna is a RABBIT.
 Chintoo says that Dolly is a FOX.
 Eashwar says that Anil is not a FOX.
 Bhavna says that Chintoo is not a RABBIT.
 Dolly says that Eashwar and Anil are different kinds of animals.
 How many FOXES are there ? (Justify your answer).
10. The accompanying diagram is a road-plan of a city. All the roads go east-west or north-south, with the exception of one shown. Due to repairs one road is impassable at the point X, of all the possible routes from P to Q, there are several shortest routes. How many such shortest routes are there ?



KVS Junior Mathematics Olympiad (JMO) – 2003

M.M. 100

Time : 3 hours

Note : (i) *Please check that there are two printed pages and 10 questions in the question paper.*

(ii) *All questions carry equal marks.*

1. Fill in the blanks

- (a) The digits of the number 2978 are arranged first in descending order and then in ascending order. The difference between the resulting two numbers is
- (b) Yash is riding his bicycle at a constant speed of 12 kilometers per hour. The number of metres he travels each minute is

- (c) The square root of $35 \times 65 \times 91$ is
- (d) The number 81 is 15% of
- (e) A train leaves New Delhi at 9.45 am and reaches Agra at 12.58 pm. The time taken in the journey, in minutes, is
2. (a) Find the largest prime factor of 203203.
- (b) Find the last two (tens' and units') digits of $(2003)^{2003}$.
3. (a) Find the number of perfect cubes between 1 and 1000009 which are exactly divisible by 9.

(b) If $x = 5 + 2\sqrt{6}$, find the value of

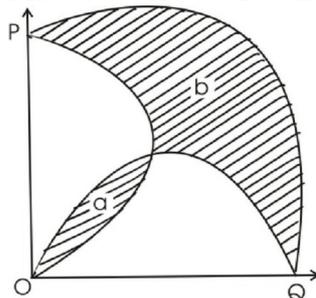
(i) $\sqrt{x} + \frac{1}{\sqrt{x}}$ (ii) $x^3 + \frac{1}{x^3}$

4. (a) Solve :

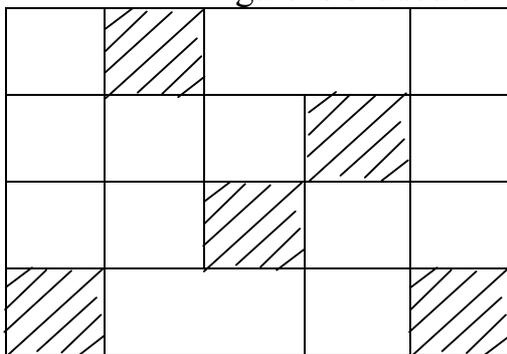
$$\frac{x^2 - 1}{x^2 - 4} - \frac{x^2 - 5}{x^2 - 8} = \frac{x^2 - 2}{x^2 - 5} - \frac{x^2 - 6}{x^2 - 9}$$

(b) Find the remainder when $x^{81} + x^{49} + x^{25} + x^9 + x$ is divided by $x^3 - x$.

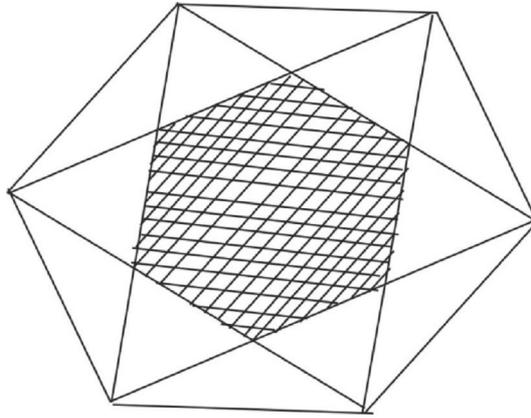
5. (a) OPQ is a quadrant of a circle and semicircles are drawn on OP and OQ. Areas a and b are shaded. Find a/b.



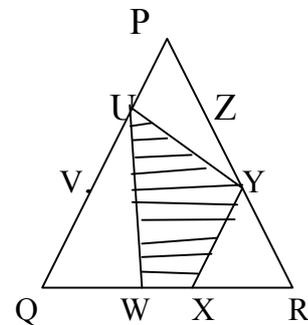
(b) Assuming all vertical lines are parallel, all angles are right angles and all the horizontal lines are equally spaced, what fraction of figure is shaded ?



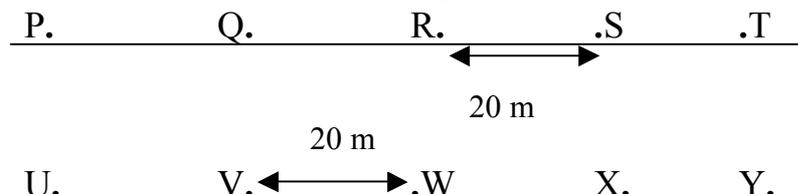
6. Alternate vertices of a regular hexagon are joined as shown. What fraction of the total area of a hexagon is shaded ? (Justify your answer)



7. In a competition consisting of 30 problems Neeta was given 12 points for each correct solution, and 7 points were subtracted from her score for each incorrect solution problems not attempted contributed 0 points to the score find the number of problems attempted correctly by Neeta.
8. A cube with each edge of lengths 4 units is painted green on all the faces. The cube is then cut into 64 unit cubes. How many of these small cubes have (i) 3 faces painted (ii) 2 faces painted (iii) one face painted (iv) no face painted.
9. Let PQR be an equilateral triangle with each side of length 3 units. Let U, V, W, X, Y and Z divide the sides into unit lengths. Find the ratio of the area U, W, X, Y and Z divide the sides into unit lengths. Find the ratio of the area U W X Y (shaded) to the area of the whole triangle PQR.



10. Five houses P, Q, R, S and T are situated on the opposite side of a street from five other houses U, V, W, X and Y as shown in the diagram :



Houses on the same side of the street are 20 metres apart A postman is trying to decide whether to deliver the letters using route PQRSTYXWVU or route PUQVRWSXTY, and finds that the total distance is the same in each case. Find the total distance in metres.

KVS Junior Mathematics Olympiad (JMO) – 2004

M.M. 100

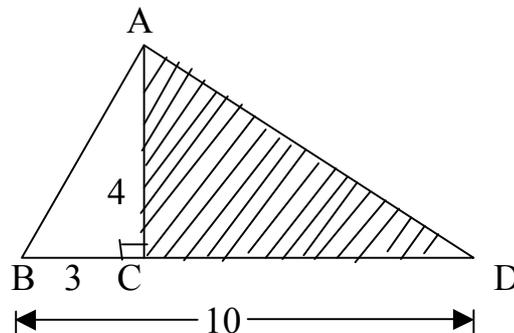
Time : 3 hours

Note : (i) Attempt all questions. Each question carries ten marks.

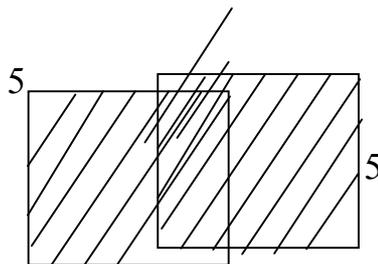
(ii) Please check that there are two printed pages and ten questions in the question paper

1. Fill in the blanks :

- (a) The number of hours from 8 p.m. Tuesday until 5 am Friday of the same week is
- (b) If $3^{x-2} = 81$, then x equals
- (c) In a school the ratio of boys to girls is 3:5 and the ratio of girls to teachers is 6:1. The ratio of boys of teachers is
- (d) If $7n + 9 > 100$ and n is an integer, the smallest possible value of n is
- (e) In the diagram, $AC = 4$, $BC = 3$, and $BD = 10$. The area of the shaded triangle is



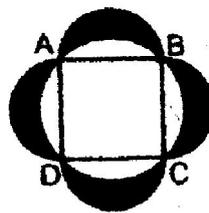
- (a) Find the number of positive integers less than or equal to 300 that are multiples of 3 or 5, but are not multiples of 10 or 15.
 - (b) The product of the digits of each of the three-digit numbers 138, 262, and 432 is 24. Write down all three-digit numbers having 24 as the product of the digits.
3. (a) Solve : $x^2 + xy + y^2 = 19$
 $x^2 - xy + y^2 = 49$
- (b) The quadratic polynomials $p(x) = a(x-3)^2 + bx + 1$ and $q(x) = 2x^2 + c(x-2) + 13$ are equal for all values of x. Find the values of a, b and c.
4. (a) Two squares, each with side length 5 cm, overlap as shown. The shape of their overlap is a square, which has an area of 4cm^2 . Find the perimeter, in centimeters of the shaded figure.



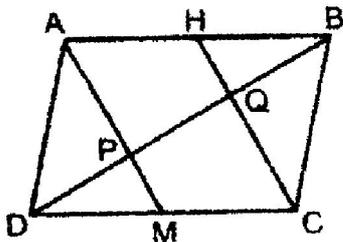
(b) A rectangle is divided into four smaller rectangles. The areas of three of these rectangles are 6, 15 and 25, as shown. Find the area of the shaded rectangle.

6	15
	25

5. (a) A square ABCD is inscribed in a circle of unit radius. Semi-circles are described on each side as a diameter. Find the area of the region bounded by the four semi-circles and the circle.

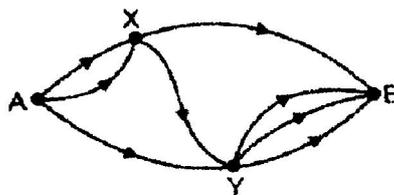


(b) In a parallelogram ABCD, H is the mid-point of AB and M is the mid-point of CD. Show that AM and CH divide the diagonal DB in three equal parts.

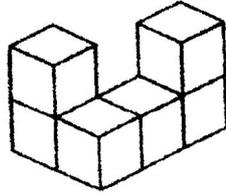


6. A two-digit number has the property that the square of its tens digit plus ten times its units digit is equal to the square of its units digit plus ten times its tens digit. Find all two digit numbers which have this property, and are prime numbers.

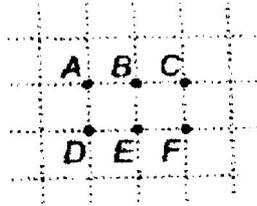
7. In the diagram, it is possible to travel only along an edge in the direction indicated by the arrow. How many different routes from A to B are there in all ?



8. The object shown in the diagram is made by gluing together the adjacent faces of six wooden cubes, each having edges of length 2 cm. Find the total surface area of the object in square centimeters.



9. Six points A, B, C, D, E and F are placed on a square grid, as shown. How many triangles that are not right-angled can be drawn by using 3 of these 6 points as vertices.



10. A distance of 200 km is to be covered by car in less than 10 hours. Yash does it in two parts. He first drives for 150 km at an average speed of 36 km/hr, without stopping. After taking rest for 30 minutes, he starts again and covers the remaining distance non-stop. His average for the entire journey (including the period of rest) exceeds that for the second part by 5km/hr. Find the speed at which he covers the second part.