MATRICES

CHAPTER – 3: MATRICES

MARKS WEIGHTAGE – 03 marks

QUICK REVISION (Important Concepts & Formulae)

☞ **Matrix**

A *matrix* is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix. We denote matrices by capital letters.

☞ ***Order of a matrix***

A matrix having m rows and n columns is called a matrix of *order* $m \times n$ or simply $m \times n$ matrix (read as an m by n matrix).

In general, an $m \times n$ matrix has the following rectangular array:

$$\begin{bmatrix} a_{11} & a_{12} & ..... & a_{1n} \\ a_{21} & a_{22} & ..... & a_{2n} \\ . & . & . & . \\ . & . & . & . \\ a_{m1} & a_{m2} & ..... & a_{mn} \end{bmatrix}$$

or $A = [a_{ij}]_{m \times n}$, $1 \le i \le m$, $1 \le j \le n$ $i, j \in N$

Thus the ith row consists of the elements $a_{i1}, a_{i2}, a_{i3}, ..., a_{in}$, while the j^{th} column consists of the elements $a_{1j}, a_{2j}, a_{3j}, ..., a_{mj}$,

In general a_{ij}, is an element lying in the i^{th} row and j^{th} column. We can also call it as the $(i, j)^{th}$ element of A. The number of elements in an $m \times n$ matrix will be equal to mn.

We can also represent any point (x, y) in a plane by a matrix (column or row) as $\begin{bmatrix} x \\ y \end{bmatrix} or \begin{bmatrix} x, y \end{bmatrix}$

☞ **Types of Matrices**

(i) **Column matrix**

A matrix is said to be a *column matrix* if it has only one column. In general, $A = [a_{ij}]_{m \times 1}$ is a column matrix of order $m \times 1$.

(ii) **Row matrix**

A matrix is said to be a *row matrix* if it has only one row. In general, $B = [b_{ij}]_{1 \times n}$ is a row matrix of order $1 \times n$.

(iii) **Square matrix**

A matrix in which the number of rows are equal to the number of columns, is said to be a *square matrix*. Thus an $m \times n$ matrix is said to be a square matrix if $m = n$ and is known as a square matrix of order 'n'.

- ☞ In general, $A = [a_{ij}]_{m \times m}$ is a square matrix of order m.
- ☞ If $A = [a_{ij}]$ is a square matrix of order n, then elements (entries) $a_{11}, a_{22}, ..., a_{nn}$ are said to constitute the *diagonal*, of the matrix A.

(iv) **Diagonal matrix**

A square matrix $B = [b_{ij}]_{m \times m}$ is said to be a *diagonal matrix* if all its non diagonal elements are zero, that is a matrix $B = [b_{ij}]_{m \times m}$ is said to be a diagonal matrix if $b_{ij} = 0$, when $i \neq j$.

(v) **Scalar matrix**

A diagonal matrix is said to be a *scalar matrix* if its diagonal elements are equal, that is, a square matrix B = $[b_{ij}]_{n \times n}$ is said to be a scalar matrix if $b_{ij} = 0$, when $i \neq j$
$b_{ij} = k$, when $i = j$, for some constant k.

(vi) **Identity matrix**
A square matrix in which elements in the diagonal are all 1 and rest are all zero is called an *identity matrix*. In other words, the square matrix A = [aij] $n \times n$ is an identity matrix, if $a_{ij} = \begin{cases} 1 \, if \; i = j \\ 0 \, if \; i \neq j \end{cases}$

We denote the identity matrix of order n by I_n. When order is clear from the context, we simply write it as I.

Observe that a scalar matrix is an identity matrix when $k = 1$. But every identity matrix is clearly a scalar matrix.

(vii) **Zero matrix**
A matrix is said to be *zero matrix* or *null matrix* if all its elements are zero. We denote zero matrix by O.

☞ ***Equality of matrices***
Two matrices A = $[a_{ij}]$ and B = $[b_{ij}]$ are said to be equal if
(i) they are of the same order
(ii) each element of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ for all i and j.

☞ **Operations on Matrices**
☞ ***Addition of matrices***
The sum of two matrices is a matrix obtained by adding the corresponding elements of the given matrices. Furthermore, the two matrices have to be of the same order.
Thus, if A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$ is a 2 × 3 matrix and B = $\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$ is another 2×3 matrix. Then, we define
A + B = $\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \end{bmatrix}$.
In general, if A = $[a_{ij}]$ and B = $[b_{ij}]$ are two matrices of the same order, say $m \times n$.
Then, the sum of the two matrices A and B is *defined* as a matrix C = $[c_{ij}]_{m \times n}$, where $c_{ij} = a_{ij} + b_{ij}$, for all possible values of i and j.

If A and B are not of the same order, then A + B is not defined.

☞ ***Multiplication of a matrix by a scalar***
If A = $[a_{ij}]_{m \times n}$ is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by the scalar k.
In other words, kA = $k\,[a_{ij}]_{m \times n} = [k\,(a_{ij})]_{m \times n}$, that is, (i, j)th element of kA is ka_{ij} for all possible values of i and j.

☞ **Negative of a matrix** The negative of a matrix is denoted by –A. We define –A = (– 1) A.

☞ **Difference of matrices** If A = $[a_{ij}]$, B = $[b_{ij}]$ are two matrices of the same order, say $m \times n$, then difference A – B is defined as a matrix D = $[d_{ij}]$, where $d_{ij} = a_{ij} - b_{ij}$, for all value of i and j. In other words, D = A – B = A + (–1) B, that is sum of the matrix A and the matrix – B.

☞ ***Properties of matrix addition***

(i) **Commutative Law** If A = $[a_{ij}]$, B = $[b_{ij}]$ are matrices of the same order, say $m \times n$, then A + B = B + A.

(ii) **Associative Law** For any three matrices A = $[a_{ij}]$, B = $[b_{ij}]$, C = $[c_{ij}]$ of the same order, say $m \times n$, (A + B) + C = A + (B + C).

(iii) **Existence of additive identity** Let A = $[a_{ij}]$ be an $m \times n$ matrix and O be an $m \times n$ zero matrix, then A + O = O + A = A. In other words, O is the additive identity for matrix addition.

(iv) **The existence of additive inverse** Let A = $[a_{ij}]_{m \times n}$ be any matrix, then we have another matrix as – A = $[-a_{ij}]_{m \times n}$ such that A + (– A) = (– A) + A= O. So – A is the additive inverse of A or negative of A.

☞ ***Properties of scalar multiplication of a matrix***

If A = $[a_{ij}]$ and B = $[b_{ij}]$ be two matrices of the same order, say $m \times n$, and k and l are scalars, then (i) k(A +B) = k A + kB, (ii) (k + l)A = k A + l A

☞ ***Multiplication of matrices***

The *product* of two matrices A and B is *defined* if the number of columns of A is equal to the number of rows of B. Let A = $[a_{ij}]$ be an $m \times n$ matrix and B = $[b_{jk}]$ be an $n \times p$ matrix. Then the product of the matrices A and B is the matrix C of order $m \times p$.

To get the $(i, k)^{th}$ element c_{ik} of the matrix C, we take the ith row of A and k^{th} column of B, multiply them elementwise and take the sum of all these products. In other words, if A = $[a_{ij}]_{m \times n}$, B = $[b_{jk}]_{n \times p}$, then the ith row of A is $[a_{i1}\ a_{i2}\ ...\ a_{in}]$ and the kth column of B is $\begin{bmatrix} b_{1k} \\ b_{2k} \\ . \\ . \\ b_{nk} \end{bmatrix}$ then

$c_{ik} = a_{i1}\, b_{1k} + a_{i2}\, b_{2k} + a_{i3}\, b_{3k} + ... + a_{in}\, b_{nk}$ = The matrix C = $[c_{ik}]_{m \times p}$ is the product of A and B.

If AB is defined, then BA need not be defined. In the above example, AB is defined but BA is not defined because B has 3 column while A has only 2 (and not 3) rows. If A, B are, respectively $m \times n$, $k \times l$ matrices, then both AB and BA are defined **if and only if** $n = k$ and $l = m$. In particular, if both A and B are square matrices of the same order, then both AB and BA are defined.

☞ **Non-commutativity of multiplication of matrices**

Now, we shall see by an example that even if AB and BA are both defined, it is not necessary that AB = BA.

☞ **Zero matrix as the product of two non zero matrices**

We know that, for real numbers a, b if $ab = 0$, then either $a = 0$ or $b = 0$.

If the product of two matrices is a zero matrix, it is not necessary that one of the matrices is a zero matrix.

☞ ***Properties of multiplication of matrices***

The multiplication of matrices possesses the following properties:

☞ **The associative law** For any three matrices A, B and C. We have (AB) C = A (BC), whenever both sides of the equality are defined.

☞ **The distributive law** For three matrices A, B and C.
A (B+C) = AB + AC
(A+B) C = AC + BC, whenever both sides of equality are defined.

☞ **The existence of multiplicative identity** For every square matrix A, there exist an identity matrix of same order such that IA = AI = A.

☞ **Transpose of a Matrix**
If A = $[a_{ij}]$ be an $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called the *transpose* of A. Transpose of the matrix A is denoted by A′ or (A^T). In other words, if A = $[a_{ij}]_{m \times n}$, then A′ = $[a_{ji}]_{n \times m}$.

☞ ***Properties of transpose of the matrices***
For any matrices A and B of suitable orders, we have
(i) (A′)′ = A, (ii) (kA)′ = kA′ (where k is any constant)
(iii) (A + B)′ = A′ + B′ (iv) (A B)′ = B′ A′

☞ **Symmetric and Skew Symmetric Matrices**
A square matrix A = $[a_{ij}]$ is said to be *symmetric* if A′ = A, that is, $[a_{ij}] = [a_{ji}]$ for all possible values of i and j.

☞ A square matrix A = $[a_{ij}]$ is said to be *skew symmetric* matrix if A′ = – A, that is *aji* = – *aij* for all possible values of i and j. Now, if we put $i = j$, we have $a_{ii} = – a_{ii}$. Therefore $2a_{ii} = 0$ or $a_{ii} = 0$ for all i's.
This means that all the diagonal elements of a skew symmetric matrix are zero.

☞ **Theorem 1** For any square matrix A with real number entries, A + A′ is a symmetric matrix and A – A′ is a skew symmetric matrix.

☞ **Theorem 2** Any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

☞ **Elementary Operation (Transformation) of a Matrix**
There are six operations (transformations) on a matrix, three of which are due to rows and three due to columns, which are known as *elementary operations* or *transformations*.

☞ *(i) The interchange of any two rows or two columns.* Symbolically the interchange of ith and jth rows is denoted by $R_i \leftrightarrow R_j$ and interchange of ith and jth column is denoted by $C_i \leftrightarrow C_j$.

☞ (ii) *The multiplication of the elements of any row or column by a non zero number.* Symbolically, the multiplication of each element of the ith row by k, where $k \neq 0$ is denoted by $R_i \rightarrow k R_i$.
The corresponding column operation is denoted by $C_i \rightarrow kC_i$

☞ (iii) *The addition to the elements of any row or column, the corresponding elements of any other row or column multiplied by any non zero number.*
Symbolically, the addition to the elements of i^{th} row, the corresponding elements of j^{th} row multiplied by k is denoted by $R_i \rightarrow R_i + kR_j$.
The corresponding column operation is denoted by $C_i \rightarrow C_i + kC_j$.

☞ **Invertible Matrices**
If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the *inverse* matrix of A and it is denoted by A^{-1}. In that case A is said to be invertible.

- ☞ A rectangular matrix does not possess inverse matrix, since for products BA and AB to be defined and to be equal, it is necessary that matrices A and B should be square matrices of the same order.

- ☞ If B is the inverse of A, then A is also the inverse of B.

- ☞ **Theorem 3** (Uniqueness of inverse) Inverse of a square matrix, if it exists, is unique.

- ☞ **Theorem 4** If A and B are invertible matrices of the same order, then $(AB)^{-1} = B^{-1}A^{-1}$.

- ☞ ***Inverse of a matrix by elementary operations***
  If A is a matrix such that A^{-1} exists, then to find A^{-1} using elementary row operations, write A = IA and apply a sequence of row operation on A = IA till we get, I = BA. The matrix B will be the inverse of A. Similarly, if we wish to find A–1 using column operations, then, write A = AI and apply a sequence of column operations on A = AI till we get, I = AB.

  In case, after applying one or more elementary row (column) operations on A = IA (A = AI), if we obtain all zeros in one or more rows of the matrix A on L.H.S., then A^{-1} does not exist.

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MARKS WEIGHTAGE – 03 marks

NCERT Important Questions & Answers

1. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Ans:

Since, a matrix containing 18 elements can have any one of the following orders :

$$1 \times 18,\ 18 \times 1,\ 2 \times 9,\ 9 \times 2,\ 3 \times 6, 6 \times 3$$

Similarly, a matrix containing 5 elements can have order 1×5 or 5×1.

2. Construct a 3 × 4 matrix, whose elements are given by:

(i) $a_{ij} = \frac{1}{2}|-3i + j|$ (ii) $a_{ij} = 2i - j$

Ans:

(i) The order of given matrix is 3 × 4, so the required matrix is

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times4}, \text{ where } a_{ij} = \frac{1}{2}|-3i + j|$$

Putting the values in place of i and j, we will find all the elements of matrix A.

$$a_{11} = \frac{1}{2}|-3+1| = 1,\ a_{12} = \frac{1}{2}|-3+2| = \frac{1}{2},\ a_{13} = \frac{1}{2}|-3+3| = 0$$

$$a_{14} = \frac{1}{2}|-3+4| = \frac{1}{2},\ a_{21} = \frac{1}{2}|-6+1| = \frac{5}{2},\ a_{22} = \frac{1}{2}|-6+2| = 2$$

$$a_{23} = \frac{1}{2}|-6+3| = \frac{3}{2},\ a_{24} = \frac{1}{2}|-6+4| = 1,\ a_{31} = \frac{1}{2}|-9+1| = 4$$

$$a_{32} = \frac{1}{2}|-9+2| = \frac{7}{2},\ a_{33} = \frac{1}{2}|-9+3| = 3,\ a_{34} = \frac{1}{2}|-9+4| = \frac{5}{2}$$

Hence, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}_{3\times4}$

(ii) Here, $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times4}$,where $a_{ij} = 2i - j$

$a_{11} = 2 - 1 = 1$, $a_{12} = 2 - 2 = 0$,
$a_{13} = 2 - 3 = -1$, $a_{14} = 2 - 4 = -2$,
$a_{21} = 4 - 1 = 3$, $a_{22} = 4 - 2 = 2$,
$a_{23} = 4 - 3 = 1$, $a_{24} = 4 - 4 = 0$
$a_{31} = 6 - 1 = 5$, $a_{32} = 6 - 2 = 4$,
$a_{33} = 6 - 3 = 3$ and $a_{34} = 6 - 4 = 2$

Hence, the required matrix is $A=\begin{bmatrix}1 & 0 & -1 & -2\\ 3 & 2 & 1 & 0\\ 5 & 4 & 3 & 2\end{bmatrix}_{3\times 4}$

3. Find the value of *a*, *b*, *c* and *d* from the equation: $\begin{bmatrix}a-b & 2a+c\\ 2a-b & 3c+d\end{bmatrix}=\begin{bmatrix}-1 & 5\\ 0 & 13\end{bmatrix}$

Ans:

Given that $\begin{bmatrix}a-b & 2a+c\\ 2a-b & 3c+d\end{bmatrix}=\begin{bmatrix}-1 & 5\\ 0 & 13\end{bmatrix}$

By definition of equality of matrix as the given matrices are equal, their corresponding elements are equal. Comparing the corresponding elements, we get

$a - b = -1$ …(i)

$2a - b = 0$ …(ii)

$2a + c = 5$ …(iii)

and $3c + d = 13$ …(iv)

Subtracting Eq.(i) from Eq.(ii), we get $a = 1$

Putting $a = 1$ in Eq. (i) and Eq. (iii), we get

$1 - b = -1$ and $2 + c = 5$

$\Rightarrow b = 2$ and $c = 3$

Substituting $c = 3$ in Eq. (iv), we obtain

$3 \times 3 + d = 13 \Rightarrow d = 13 - 9 = 4$

Hence, $a = 1, b = 2, c = 3$ and $d = 4$.

4. Find X and Y, if X + Y = $\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}$ **and X – Y =** $\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$**.**

Ans:

$$(X+Y)+(X-Y)=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}+\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}8 & 8\\ 0 & 8\end{bmatrix}\Rightarrow X=\frac{1}{2}\begin{bmatrix}8 & 8\\ 0 & 8\end{bmatrix}$$

$$\Rightarrow X=\begin{bmatrix}4 & 4\\ 0 & 4\end{bmatrix}$$

$$Now,(X+Y)-(X-Y)=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}-\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$$

$$\Rightarrow 2Y=\begin{bmatrix}2 & -4\\ 0 & 10\end{bmatrix}\Rightarrow X=\frac{1}{2}\begin{bmatrix}2 & -4\\ 0 & 10\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}1 & -2\\ 0 & 5\end{bmatrix}$$

5. Find the values of *x* and *y* from the following equation:

$$2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$$

Ans:

$$2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2x & 10\\ 14 & 2y-6\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

or $2x + 3 = 7$ and $2y - 4 = 14$

or $2x = 7 - 3$ and $2y = 18$

or $x = \frac{4}{2}$ and $y = \frac{18}{2}$

i.e. $x = 2$ and $y = 9$.

6. Find AB, if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$**.**

Ans:

We have $AB = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Thus, if the product of two matrices is a zero matrix, it is not necessary that one of the matrices is a zero matrix.

7. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$**, then show that $A^3 - 23A - 40\,I = O$**

Ans:

$$A^2 = A.A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$So, A^3 = A.A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix} = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

$$Now, A^3 - 23A - 40I = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} + \begin{bmatrix} -23 & -46 & -69 \\ -69 & 46 & -23 \\ -92 & -46 & -23 \end{bmatrix} + \begin{bmatrix} -40 & 0 & 0 \\ 0 & -40 & 0 \\ 0 & 0 & -40 \end{bmatrix}$$

$$= \begin{bmatrix} 63-23-40 & 46-46+0 & 69-69+0 \\ 69-69+0 & -6+46-40 & 23-23+0 \\ 92-92+0 & 46-46+0 & 63-23-40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

8. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$**, find the values of *x* and *y*.**

Ans:

$$x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

By definition of equality of matrix as the given matrices are equal, their corresponding elements are equal. Comparing the corresponding elements, we get

$$2x - y = 10 \quad \ldots(i)$$
$$\text{and } 3x + y = 5 \quad \ldots(ii)$$

Adding Eqs. (i) and (ii), we get

$$5x = 15 \Rightarrow x = 3$$

Substituting $x = 3$ in Eq. (i), we get

$$2 \times 3 - y = 10 \Rightarrow y = 6 - 10 = -4$$

9. Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$**, find the values of *x*, *y*, *z* and *w*.**

Ans:

By definition of equality of matrix as the given matrices are equal, their corresponding elements are equal. Comparing the corresponding elements, we get

$$3x = x + 4 \Rightarrow 2x = 4 \Rightarrow x = 2$$

$$\text{and } 3y = 6 + x + y \Rightarrow 2y = 6 + x \Rightarrow y = \frac{6+x}{2}$$

Putting the value of x, we get

$$y = \frac{6+2}{2} = \frac{8}{2} = 4$$

Now, $3z = -1 + z + w$, $2z = -1 + w$

$$z = \frac{-1+w}{2} \quad \ldots(i)$$

Now, $3w = 2w + 3 \Rightarrow w = 3$

Putting the value of w in Eq. (i), we get

$$z = \frac{-1+3}{2} = \frac{2}{2} = 1$$

Hence, the values of x, y, z and w are 2, 4, 1 and 3.

10. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$**, show that F(*x*) F(*y*) = F(*x* + *y*).**

Ans:

$$LHS = F(x)F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\sin y \cos x - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y) = RHS$$

11. Find A^2 – 5A + 6I, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Ans:

$$A^2 = A.A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\therefore A^2 - 5A + 6I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10+6 & -1-0+0 & 2-5+0 \\ 9-10+0 & -2-5+6 & 5-15+0 \\ 0-5+0 & -1+5+0 & -2+0+6 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

12. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$

Ans:

$$A^2 = A.A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$$\therefore A^3 - 6A^2 + 7A + 2I = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21-30+7+2 & 0-0+0+0 & 34-48+14+0 \\ 12-12+0+0 & 8-24+14+2 & 23-30+7+0 \\ 34-48+14+0 & 0-0+0+0 & 55-78+21+2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

13. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find *k* so that $A^2 = kA - 2I$

Ans:

Given than $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

By definition of equality of matrix as the given matrices are equal, their corresponding elements are equal. Comparing the corresponding elements, we get

$3k - 2 = 1 \Rightarrow k = 1$
$-2k = -2 \Rightarrow k = 1$
$4k = 4 \Rightarrow k = 1$
$-4 = -2k - 2 \Rightarrow k = 1$

Hence, $k = 1$

14. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ **and I is the identity matrix of order 2, show that**

I + A = (I – A)$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Ans:

Let $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$ *where* $x = \tan\frac{\alpha}{2}$

Now, $\cos\alpha = \dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \dfrac{1-x^2}{1+x^2}$ *and* $\cos\alpha = \dfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \dfrac{2x}{1+x^2}$

$$RHS = (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}\right)\begin{bmatrix} \frac{1-x^2}{1+x^2} & -\frac{2x}{1+x^2} \\ \frac{2x}{1+x^2} & \frac{1-x^2}{1+x^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & x \\ -x & 1 \end{bmatrix}\begin{bmatrix} \frac{1-x^2}{1+x^2} & -\frac{2x}{1+x^2} \\ \frac{2x}{1+x^2} & \frac{1-x^2}{1+x^2} \end{bmatrix} = \begin{bmatrix} \frac{1-x^2+2x^2}{1+x^2} & \frac{-2x+x(1-x^2)}{1+x^2} \\ \frac{-x(1-x^2)+2x}{1+x^2} & \frac{2x^2+1-x^2}{1+x^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1+x^2}{1+x^2} & \frac{-2x+x-x^3}{1+x^2} \\ \frac{-x+x^3+2x}{1+x^2} & \frac{1+x^2}{1+x^2} \end{bmatrix} = \begin{bmatrix} 1 & \frac{-x-x^3}{1+x^2} \\ \frac{x^3+x}{1+x^2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{-x(1+x^2)}{1+x^2} \\ \frac{x(x^2+1)}{1+x^2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & x \\ -x & 1 \end{bmatrix}$$

$$LHS = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} = \begin{bmatrix} 1 & x \\ -x & 1 \end{bmatrix} = RHS$$

15. Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ **as the sum of a symmetric and a skew symmetric matrix.**

Ans:

$$B=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}\Rightarrow B'=\begin{bmatrix}2&-1&1\\-2&3&-2\\-4&4&-3\end{bmatrix}$$

$$Let\ \ P=\frac{1}{2}(B+B')=\frac{1}{2}\begin{bmatrix}4&-3&-3\\-3&6&2\\-3&2&-6\end{bmatrix}=\begin{bmatrix}2&-3/2&-3/2\\-3/2&3&1\\-3/2&1&-3\end{bmatrix}$$

$$Now\ \ P'=\begin{bmatrix}2&-3/2&-3/2\\-3/2&3&1\\-3/2&1&-3\end{bmatrix}=P$$

$Thus\ \ P=\frac{1}{2}(B+B')$ is a symmetric matrix.

$$Also, let\ \ Q=\frac{1}{2}(B-B')=\frac{1}{2}\begin{bmatrix}0&-1&-5\\1&0&6\\5&-6&0\end{bmatrix}=\begin{bmatrix}0&-1/2&-5/2\\1/2&0&3\\5/2&-3&0\end{bmatrix}$$

$$Now\ \ Q'=\begin{bmatrix}0&1/2&5/2\\-1/2&0&-3\\-5/2&3&0\end{bmatrix}=-Q$$

$Thus\ \ Q=\frac{1}{2}(B-B')$ is a skew symmetric matrix.

$$Now, P+Q=\begin{bmatrix}2&-3/2&-3/2\\-3/2&3&1\\-3/2&1&-3\end{bmatrix}+\begin{bmatrix}0&-1/2&-5/2\\1/2&0&3\\5/2&-3&0\end{bmatrix}=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}=B$$

Thus, B is represented as the sum of a symmetric and a skew symmetric matrix.

16. Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

$$(i)\begin{bmatrix}3&5\\1&-1\end{bmatrix}\qquad (ii)\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}\qquad (iii)\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}\qquad (iv)\begin{bmatrix}1&5\\-1&2\end{bmatrix}$$

Ans:

(i)

Let $A=\begin{bmatrix}3&5\\1&-1\end{bmatrix}, then\ \ A=P+Q$

$$where,\ \ P=\frac{1}{2}(A+A')\ \ and\ \ Q=\frac{1}{2}(A-A')$$

$$Now, P=\frac{1}{2}(A+A')=\frac{1}{2}\left(\begin{bmatrix}3&5\\1&-1\end{bmatrix}+\begin{bmatrix}3&1\\5&-1\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}6&6\\6&-2\end{bmatrix}=\begin{bmatrix}3&3\\3&-1\end{bmatrix}$$

$$\therefore P'=\begin{bmatrix}3&3\\3&-1\end{bmatrix}=\begin{bmatrix}3&3\\3&-1\end{bmatrix}=P$$

$Thus\ \ P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$Now, Q=\frac{1}{2}(A-A')=\frac{1}{2}\left(\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}-\begin{bmatrix}3 & 1\\ 5 & -1\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}0 & 4\\ -4 & 0\end{bmatrix}=\begin{bmatrix}0 & 2\\ -2 & 0\end{bmatrix}$$

$$\therefore Q'=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}=-Q$$

$Thus\ \ Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q,

$$P+Q=\begin{bmatrix}3 & 3\\ 3 & -1\end{bmatrix}+\begin{bmatrix}0 & 2\\ -2 & 0\end{bmatrix}=\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}=A$$

(ii)

Let $A=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}, then\ \ A=P+Q$

$where,\ \ P=\frac{1}{2}(A+A')\ \ and\ \ Q=\frac{1}{2}(A-A')$

$$Now, P=\frac{1}{2}(A+A')=\frac{1}{2}\left(\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}+\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}12 & -4 & 4\\ -4 & 6 & -2\\ 4 & -2 & 6\end{bmatrix}=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$$

$$\therefore P'=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}=P$$

$Thus\ \ P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$Now, Q=\frac{1}{2}(A-A')=\frac{1}{2}\left(\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}-\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\therefore Q'=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=-Q$$

$Thus\ \ Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q,

$$P+Q=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}=A$$

(iii)

Let $A=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}, then\ \ A=P+Q$

$where,\ \ P=\frac{1}{2}(A+A')\ \ and\ \ Q=\frac{1}{2}(A-A')$

$$Now, P=\frac{1}{2}(A+A')=\frac{1}{2}\left(\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}+\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}6 & 1 & -5\\ 1 & -4 & -4\\ -5 & -4 & 4\end{bmatrix}=\begin{bmatrix}3 & 1/2 & -5/2\\ 1/2 & -2 & -2\\ -5/2 & -2 & 2\end{bmatrix}$$

$$\therefore P'=\begin{bmatrix}3 & 1/2 & -5/2\\ 1/2 & -2 & -2\\ -5/2 & -2 & 2\end{bmatrix}=\begin{bmatrix}3 & 1/2 & -5/2\\ 1/2 & -2 & -2\\ -5/2 & -2 & 2\end{bmatrix}=P$$

Thus $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$Now, Q=\frac{1}{2}(A-A')=\frac{1}{2}\left(\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}-\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{bmatrix}=\begin{bmatrix}0 & 5/2 & 3/2\\ -5/2 & 0 & 2\\ -3/2 & -2 & 0\end{bmatrix}$$

$$\therefore Q'=\begin{bmatrix}0 & -5/2 & -3/2\\ 5/2 & 0 & 2\\ 3/2 & -2 & 0\end{bmatrix}=-Q$$

Thus $Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing *A* as the sum of *P* and *Q*,

$$P+Q=\begin{bmatrix}3 & 1/2 & -5/2\\ 1/2 & -2 & -2\\ -5/2 & -2 & 2\end{bmatrix}+\begin{bmatrix}0 & 5/2 & 3/2\\ -5/2 & 0 & 2\\ -3/2 & -2 & 0\end{bmatrix}=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}=A$$

(iv)

Let $A=\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}, then\ \ A=P+Q$

$where,\ \ P=\frac{1}{2}(A+A')\ \ and\ \ Q=\frac{1}{2}(A-A')$

$$Now, P=\frac{1}{2}(A+A')=\frac{1}{2}\left(\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}+\begin{bmatrix}1 & -1\\ 5 & 2\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}2 & 4\\ 4 & 4\end{bmatrix}=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}$$

$$\therefore P'=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}=P$$

Thus $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$Now, Q=\frac{1}{2}(A-A')=\frac{1}{2}\left(\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1\\ 5 & 2\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}0 & 6\\ -6 & 0\end{bmatrix}=\begin{bmatrix}0 & 3\\ -3 & 0\end{bmatrix}$$

$$\therefore Q'=\begin{bmatrix}0 & -3\\ 3 & 0\end{bmatrix}=-Q$$

Thus $Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing *A* as the sum of *P* and *Q*,

$$P+Q=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}+\begin{bmatrix}0 & 3\\ -3 & 0\end{bmatrix}=\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}=A$$

17. If A = $\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ and A + A′ = I, find the value of α .

Ans:

$$A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\Rightarrow A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$Now, A+A'=I$$

$$\Rightarrow\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Comparing the corresponding elements of the above matrices, we have

$$2\cos\alpha=1\Rightarrow\cos\alpha=\frac{1}{2}=\cos\frac{\pi}{3}\Rightarrow\alpha=\frac{\pi}{3}$$

18. Obtain the inverse of the following matrix using elementary operations: $A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$

Ans:

Write A = I A, i.e., $\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$

$$\Rightarrow\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 2\\ 3 & 1 & 1\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}A \qquad \text{(applying } R_1\leftrightarrow R_2\text{)}$$

$$\Rightarrow\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 2\\ 0 & -5 & -8\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & -3 & 1\end{bmatrix}A \text{ (applying } R_3\to R_3-3R_1\text{)}$$

$$\Rightarrow\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 2\\ 0 & -5 & -8\end{bmatrix}=\begin{bmatrix}-2 & 1 & 0\\ 1 & 0 & 0\\ 0 & -3 & 1\end{bmatrix}A \qquad \text{(applying } R_1\to R_1-2R_2\text{)}$$

$$\Rightarrow\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 2\\ 0 & 0 & 2\end{bmatrix}=\begin{bmatrix}-2 & 1 & 0\\ 1 & 0 & 0\\ 5 & -3 & 1\end{bmatrix}A \text{ (applying } R_3\to R_3+5R_2\text{)}$$

$$\Rightarrow\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}-2 & 1 & 0\\ 1 & 0 & 0\\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2}\end{bmatrix}A \qquad \text{(applying } R_3\to\frac{1}{2}R_3\text{)}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A \quad \text{(applying } R_1 \to R_1 + R_3\text{)}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A \qquad \text{(applying } R_2 \to R_2 - 2R_3\text{)}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$$

19. Using elementary transformations, find the inverse of $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$, if it exists.

Ans:

Let $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{-1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left(\text{Using } R_1 \to \frac{1}{6} R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{-1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A \quad \left(\text{Using } R_2 \to R_2 + 2R_1\right)$$

Now, in the above equation, we can see all the elements are zero in the second row of the matrix on the LHS. Therefore, A^{-1} does not exist.

Note *Suppose A = IA, after applying the elementary transformation, if any row or column of a matrix on LHS is zero, then A^{-1} does not exist.*

20. Show that the matrix $B' AB$ is symmetric or skew-symmetric according as A is symmetric or skew-symmetric.

Ans:

We suppose that A is a symmetric matrix, then $A' = A$

Consider $(B' AB)' = \{B' (AB)\}' = (AB)' (B')'$ $\quad [\because (AB)' = B' A']$

$= B' A' (B)$ $\quad [\because (B')' = B]$

$= B' (A' B) = B' (AB)$ $\quad [\because A' = A]$

$\Rightarrow (B' AB)' = B' AB$

which shows that $B' AB$ is a symmetric matrix.

Now, we suppose that A is a skew-symmetric matrix.

Then, $A' = -A$

Consider $(B' AB)' = [B' (AB)]' = (AB)' (B')'$ $\quad [\because (AB)' = B' A'$ and $(A')' = A]$

$= (B'A')B = B'(-A)B = -B'AB$ $[\because A' = -A]$

$\Rightarrow (B'AB)' = -B'AB$

which shows that $B'AB$ is a skew-symmetric matrix.

21. If A and B are symmetric matrices, prove that $AB - BA$ is a skew-symmetric matrix.

Ans:

Here, A and $B$are symmetric matrices, then $A' = A$ and $B' = B$

Now, $(AB - BA)' = (AB)' - (BA)'$ $(\because (A - B)' = A' - B'$ and $(AB)' = B'A')$

$= B'A' - A'B' = BA - AB$ $\quad (\because B' = B$and $A' = A)$

$= -(AB - BA)$

$\Rightarrow (AB - BA)' = -(AB - BA)$

Thus, $(AB - BA)$ is a skew-symmetric matrix.

22. Using elementary transformations, find the inverse of $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$, if it exists.

Ans:

$Let \ A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$. We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad (\text{Using } R_1 \to R_1 + R_2 - R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & -5 \\ 0 & -5 & -10 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & 2 \\ -3 & -3 & 4 \end{bmatrix} A \qquad (\text{Using } R_2 \to R_2 - 2R_1 \text{ and } R_3 \to R_3 - 3R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 0 & -5 & -10 \\ 0 & 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -3 & -3 & 4 \\ -2 & -1 & 2 \end{bmatrix} A \qquad (\text{Using } R_2 \leftrightarrow R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ \frac{3}{5} & \frac{3}{5} & \frac{-4}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix} A \qquad (\text{Using } R_2 \to -\frac{1}{5}R_2 \text{ and } R3 \to -\frac{1}{5}R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-3}{5} & \frac{1}{5} & \frac{3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix} A \qquad (\text{Using } R_2 \to R_2 - 2R_3 \text{ and } R_1 \to R1 - 4R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix} A \qquad \text{(Using } R_1 \to R_1 - R_2\text{)}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix}$$

23. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, where n is any positive integer.

Ans:

We are required to prove that for all $n \in N$

$$P(n) = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$$

Let n = 1, then $P(1) = \begin{bmatrix} 1+2(1) & -4(1) \\ 1 & 1-2(1) \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ ....(i)

which is true for n = 1.

Let the result be true for $n = k$.

$$P(k) = A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \text{ ....}(ii)$$

Let $n = k + 1$

$$P(k+1) = A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix} = \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}$$

$$Now, LHS = A^{k+1} = A^k A^1 = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (1+2k).3+(-4k).1 & (1+2k).(-4)+(-4k)(-1) \\ k.3+(1-2k).1 & k.(-4)+(1-2k)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 3+2k & -4-4k \\ k+1 & -1-2k \end{bmatrix} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & -1-2(k+1) \end{bmatrix}$$

Therefore, the result is true for $n = k + 1$whenever it is true for $n = k$. So, by principle of mathematical induction, it is true for all $n \in N$.

24. For what values of x : $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$?

Ans:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

Since Matrix multiplication is associative, therefore $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}0+4+0\\0+0+x\\0+0+2x\end{bmatrix}=O$

$\Rightarrow\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}4\\x\\2x\end{bmatrix}=O$

$\Rightarrow[4+2x+2x]=O\Rightarrow[4+4x]=O$

$\Rightarrow 4+4x=0\Rightarrow 4x=-4\Rightarrow x=-1$

25. If $A=\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$, show that $A^2-5A+7I=0$.

Ans:

Given that $A=\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$

$A^2=A.A=\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}=\begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix}$

$\therefore A^2-5A+7I=\begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix}-5\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$=\begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\-5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\0 & 7\end{bmatrix}$

$=\begin{bmatrix}8-15+7 & 5-5+0\\-5+5+0 & 3-10+7\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}=O$

26. Find *x*, if $\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix}=O$?

Ans:

Given that $\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix}=O$

Since Matrix multiplication is associative, therefore $\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}x+0+2\\0+8+1\\2x+0+3\end{bmatrix}=O$

$\Rightarrow\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}x+2\\9\\2x+3\end{bmatrix}=O$

$\Rightarrow[x(x+2)+(-5).9+(-1)(2x+3)]=O$

$\Rightarrow[x^2-48]=O\Rightarrow x^2-48=0\Rightarrow x^2=48$

$\Rightarrow x=\pm\sqrt{48}=\pm 4\sqrt{3}$

27. Find the matrix X so that $X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & 3\\2 & 4 & 6\end{bmatrix}$

Ans:

Given that $X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & 3\\ 2 & 4 & 6\end{bmatrix}$

The matrix given on the RHS of the equation is a 2 × 3matrix and the one given on the LHS of the equation is as a 2 × 3 matrix. Therefore, X has to be a 2 × 2 matrix.

Now, let $X=\begin{bmatrix}a & c\\ b & d\end{bmatrix}$

$\begin{bmatrix}a+4c & 2a+5c & 3a+6c\\ b+4d & 2b+5d & 3b+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & 3\\ 2 & 4 & 6\end{bmatrix}$

Equating the corresponding elements of the two matrices, we have

$a + 4c = -7,\quad 2a + 5c = -8,\quad 3a + 6c = -9$

$b + 4d = 2,\quad 2b + 5d = 4,\quad 3b + 6d = 6$

Now, $a + 4c = -7 \Rightarrow a = -7 - 4c$

$2a + 5c = -8 \Rightarrow -14 - 8c + 5c = -8$

$\Rightarrow -3c = 6 \Rightarrow c = -2$

$\therefore\ a = -7 - 4(-2) = -7 + 8 = 1$

Now, $b + 4d = 2 \Rightarrow b = 2 - 4d$ and $2b + 5d = 4 \Rightarrow 4 - 8d + 5d = 4$

$\therefore\ -3d = 0 \Rightarrow d = 0$

$\therefore\ b = 2 - 4(0) = 2$

Thus, $a = 1, b = 2, c = -2, d = 0$

Hence, the required matrix X is $\begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$

28. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$**, then prove that** $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}, n\in N$

Ans:

We shall prove the result by using principle of mathematical induction.

We have P(n) : If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}, n\in N$

Let n = 1, then $P(1)=A^1=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

Therefore, the result is true for n = 1.

Let the result be true for $n = k$. So

$P(k)=A^k=\begin{bmatrix}\cos k\theta & \sin k\theta\\ -\sin k\theta & \cos k\theta\end{bmatrix}$

Now, we prove that the result holds for $n = k + 1$

i.e. $P(k+1)=A^{k+1}=\begin{bmatrix}\cos(k+1)\theta & \sin(k+1)\theta\\ -\sin(k+1)\theta & \cos(k+1)\theta\end{bmatrix}$

Now, $P(k+1)=A^{k+1}=A^k.A=\begin{bmatrix}\cos k\theta & \sin k\theta\\ -\sin k\theta & \cos k\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$=\begin{bmatrix}\cos\theta\cos k\theta-\sin\theta\sin k\theta & \cos\theta\sin k\theta+\sin\theta\cos k\theta\\ -\sin\theta\cos k\theta+\cos\theta\sin k\theta & -\sin\theta\sin k\theta+\cos\theta\cos k\theta\end{bmatrix}$

$=\begin{bmatrix}\cos(k\theta+\theta) & \sin(k\theta+\theta)\\ -\sin(k\theta+\theta) & \cos(k\theta+\theta)\end{bmatrix}=\begin{bmatrix}\cos(k+1)\theta & \sin(k+1)\theta\\ -\sin(k+1)\theta & \cos(k+1)\theta\end{bmatrix}$

Therefore, the result is true for $n = k + 1$. Thus by principle of mathematical induction, we have

$A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}, n\in N$

29. Let $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}, C=\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$. Find a matrix D such that CD – AB = O.

Ans:

Since A, B, C are all square matrices of order 2, and CD – AB is well defined, D must be a square matrix of order 2.

$$Let\ \ D=\begin{bmatrix}a & b\\ c & d\end{bmatrix},\ then\ \ CD-AB=0\Rightarrow\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}-\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}=O$$

$$\Rightarrow\begin{bmatrix}2a+5c & 2b+5d\\ 3a+8c & 3b+8d\end{bmatrix}-\begin{bmatrix}3 & 0\\ 43 & 22\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2a+5c-3 & 2b+5d\\ 3a+8c-43 & 3b+8d-22\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

By equality of matrices, we get

$2a + 5c – 3 = 0$ ... (1)

$3a + 8c – 43 = 0$ ... (2)

$2b + 5d = 0$ ... (3)

and $3b + 8d – 22 = 0$ ... (4)

Solving (1) and (2), we get $a = –191$, $c = 77$. Solving (3) and (4), we get $b = – 110$, $d = 44$.

Therefore $D=\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}-191 & -110\\ 77 & 44\end{bmatrix}$

30. By using elementary operations, find the inverse of the matrix $A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$

Ans:

In order to use elementary row operations we may write A = IA.

$\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A,\ then\ \begin{bmatrix}1 & 2\\ 0 & -5\end{bmatrix}=\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}A$ (applying $R_2 \rightarrow R_2 – 2R_1$)

$\Rightarrow\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ \frac{2}{5} & \frac{-1}{5}\end{bmatrix}A$ (applying $R_2 \rightarrow -\frac{1}{5}R_2$)

$\Rightarrow\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{5} & \frac{2}{5}\\ \frac{2}{5} & \frac{-1}{5}\end{bmatrix}A$ (applying $R_1 \rightarrow R_1 – 2R_2$)

Thus $A^{-1}=\begin{bmatrix}\frac{1}{5} & \frac{2}{5}\\ \frac{2}{5} & \frac{-1}{5}\end{bmatrix}$

CHAPTER – 3: MATRICES

MARKS WEIGHTAGE – 03 marks

Previous Years Board Exam (Important Questions & Answers)

1. Use elementary column operation $C_2 \to C_2 - 2C_1$ in the matrix equation
$$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

Ans:

Given that $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Applying $C_2 \to C_2 - 2C_1$, we get
$$\begin{bmatrix} 4 & -6 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ 1 & -1 \end{bmatrix}$$

2. If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$ write the value of $a - 2b$.

Ans:

Give that $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$

On equating, we get

$a + 4 = 2a + 2$, $3b = b + 2$, $a - 8b = -6$

$\Rightarrow a = 2, b = 1$

Now the value of $a - 2b = 2 - (2 \times 1) = 2 - 2 = 0$

3. If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is an identity matrix.

Ans:

$7A - (I + A)^3 = 7A - \{I^3 + 3I^2A + 3I.A^2 + A^3\}$

$= 7A - \{I + 3A + 3A + A^2A\}$ $\quad [\because I^3 = I^2 = I , A^2 = A]$

$= 7A - \{I + 6A + A^2\} = 7A - \{I + 6A + A\}$

$= 7A - \{I + 7A\} = 7A - I - 7A = -I$

4. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x + y$.

Ans:

Given that $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$

Equating, we get

$x - y = -1 \ldots (i)$

$2x - y = 0 \ldots (ii)$

$z = 4, w = 5$

$(ii) - (i) \Rightarrow 2x - y - x + y = 0 + 1$

$\Rightarrow x = 1$ and $y = 2$

$\therefore x + y = 2 + 1 = 3.$

5. Solve the following matrix equation for x : $\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$

Ans:

Given that $\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} x-2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$

$\Rightarrow x - 2 = 0$

$\Rightarrow x = 2$

6. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find $(x - y)$.

Ans:

Given that $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

Equating we get $8 + y = 0$ and $2x + 1 = 5$

$\Rightarrow y = -8$ and $x = 2$

$\Rightarrow x - y = 2 + 8 = 10$

7. For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix?

Ans:

A will be skew symmetric matrix if

$A = -A'$

$\Rightarrow \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -x \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$

Equating, we get $x = 2$

8. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then write the value of k.

Ans:

Given $A^2 = kA$

$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow k = 2$

9. Find the value of *a* if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Ans:

Given that $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Equating the corresponding elements we get.

$a - b = -1 \ldots(i)$
$2a + c = 5 \ldots(ii)$
$2a - b = 0 \ldots(iii)$
$3c + d = 13 \ldots(iv)$

From (iii) $2a = b$

$\Rightarrow a = \frac{b}{2}$

Putting in (i) we get $\frac{b}{2} - b = -1$

$\Rightarrow -\frac{b}{2} = -1 \Rightarrow b = 2$

$\therefore\ a = 1$

(ii) $c = 5 - 2 \times 1 = 5 - 2 = 3$

(iv) $d = 13 - 3 \times (3) = 13 - 9 = 4$

i.e. $a = 1$, $b = 2$, $c = 3$, $d = 4$

10. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$**, then find the matrix *A*.**

Ans:

Given that $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$

11. If *A* is a square matrix such that $A^2 = A$, then write the value of $(I + A)^2 - 3A$.

Ans:

$(I + A)^2 - 3A = I^2 + A^2 + 2A - 3A$
$= I^2 + A^2 - A$
$= I^2 + A - A$ $[\because A^2 = A]$
$= I^2 = I \cdot I = I$

12. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$**, write the value of *x*.**

Ans:

Given that $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x - y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

Equating the corresponding elements we get.

$2x - y = 10$ ...(*i*)

$3x + y = 5$ ...(*ii*)

Adding (*i*) and (*ii*), we get $2x - y + 3x + y = 10 + 5$

$\Rightarrow 5x = 15 \Rightarrow x = 3$.

13. Find the value of $x + y$ from the following equation: $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Ans:

Given that $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Equating the corresponding element we get

$2x + 3 = 7$ and $2y - 4 = 14$

$\Rightarrow x = \frac{7-3}{2}$ *and* $y = \frac{14+4}{2}$

$\Rightarrow x = 2$ and $y = 9$

$\therefore x + y = 2 + 9 = 11$

14. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ ***and*** $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$**, then find $A^T - B^T$.**

Ans:

Given that $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

Now, $A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

15. If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$**, write the value of x**

Ans:

Given that $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2-6 & -6+12 \\ 5-14 & -15+28 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

Equating the corresponding elements, we get

$x = 13$

16. Simplify: $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

Ans:

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}\cos^2\theta & \sin\theta\cos\theta\\ -\sin\theta\cos\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}\cos^2\theta+\sin^2\theta & 0\\ 0 & \cos^2\theta+\sin^2\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

17. Write the values of *x* – *y* + *z* from the following equation: $\begin{bmatrix}x+y+z\\ x+z\\ y+z\end{bmatrix}=\begin{bmatrix}9\\ 5\\ 7\end{bmatrix}$

Ans:

By definition of equality of matrices, we have

$x + y + z = 9$ ... (*i*)

$x + z = 5$ .... (*ii*)

$y + z = 7$ ... (*iii*)

(*i*) – (*ii*) $\Rightarrow x + y + z - x - z = 9 - 5$

$\Rightarrow y = 4$ ... (*iv*)

(*ii*) – (*iv*) $\Rightarrow x - y + z = 5 - 4$

$\Rightarrow x - y + z = 1$

18. If $\begin{bmatrix}y+2x & 5\\ -x & 3\end{bmatrix}=\begin{bmatrix}7 & 5\\ -2 & 3\end{bmatrix}$**, then find the value of y.**

Ans:

Given that $\begin{bmatrix}y+2x & 5\\ -x & 3\end{bmatrix}=\begin{bmatrix}7 & 5\\ -2 & 3\end{bmatrix}$

By definition of equality of matrices, we have

$y + 2x = 7$

$-x = -2 \Rightarrow x = 2$

$\therefore y + 2(2) = 7 \Rightarrow y = 3$