DETERMINANTS

CHAPTER – 4: DETERMINANTS

MARKS WEIGHTAGE – 10 marks

QUICK REVISION (Important Concepts & Formulae)

☞ ***Determinant***

If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A is written as |A| = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ = det (A) or Δ

(i) For matrix A, |A| is read as determinant of A and not modulus of A.

(ii) Only square matrices have determinants.

☞ ***Determinant of a matrix of order one***

Let A = [a] be the matrix of order 1, then determinant of A is defined to be equal to a

☞ ***Determinant of a matrix of order two***

Let A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a matrix of order 2 × 2, then the determinant of A is defined as:

det (A) = |A| = Δ = $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ = $a_{11}a_{22} - a_{21}a_{12}$

☞ ***Determinant of a matrix of order* 3 × 3**

Determinant of a matrix of order three can be determined by expressing it in terms of second order determinants. This is known as expansion of a determinant along a row (or a column). There are six ways of expanding a determinant of order 3 corresponding to each of three rows (R_1, R_2 and R_3) and three columns (C_1, C_2 and C_3) giving the same value as shown below.

Consider the determinant of square matrix A = $[a_{ij}]_{3 \times 3}$

i.e., $$| A | = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

☞ **Expansion along first Row (R_1)**

Step 1

Multiply first element a_{11} of R_1 by $(-1)^{(1+1)}$ $[(-1)^{\text{sum of suffixes in } a11}]$ and with the second order determinant obtained by deleting the elements of first row (R_1) and first column (C_1) of | A | as a_{11} lies in R_1 and C_1,

i.e., $(-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

Step 2

Multiply 2nd element a_{12} of R_1 by $(-1)^{1+2}$ $[(-1)^{\text{sum of suffixes in } a12}]$ and the second order determinant obtained by deleting elements of first row (R_1) and 2nd column (C_2) of | A | as a_{12} lies in R_1 and C_2,

i.e., $(-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

Step 3

Multiply third element a13 of R1 by $(-1)^{1+3}$ $[(-1)^{\text{sum of suffixes in } a13}]$ and the second order determinant obtained by deleting elements of first row (R_1) and third column (C_3) of | A | as a_{13} lies in R_1 and C_3, i.e., $(-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Step 4

Now the expansion of determinant of A, that is, | A | written as sum of all three terms obtained in steps 1, 2 and 3 above is given by

$$|A| = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

or $|A| = a_{11} (a_{22}a_{33} - a_{32} a_{23}) - a_{12} (a_{21}a_{33} - a_{31}a_{23}) + a_{13} (a_{21}a_{32} - a_{31}a_{22})$

- **Expansion along second row (R_2)**

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Expanding along R_2, we get

$$|A| = (-1)^{2+1} a_{21} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{2+2} a_{22} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{2+3} a_{23} \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

- **Expansion along first Column (C_1)**

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

By expanding along C_1, we get

$$|A| = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{2+1} a_{21} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{3+1} a_{31} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

- For easier calculations, we shall expand the determinant along that row or column which contains maximum number of zeros.

- While expanding, instead of multiplying by $(-1)^{i+j}$, we can multiply by +1 or –1 according as $(i + j)$ is even or odd.

- If A = kB where A and B are square matrices of order n, then $|A| = k^n |B|$, where n = 1, 2, 3

Properties of Determinants

- **Property 1**

The value of the determinant remains unchanged if its rows and columns are interchanged.
if A is a square matrix, then det (A) = det (A′), where A′ = transpose of A.

  - If $R_i = i^{th}$ row and $C_i = i^{th}$ column, then for interchange of row and columns, we will symbolically write $C_i \leftrightarrow R_i$

- **Property 2**

If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.

- We can denote the interchange of rows by $R_i \leftrightarrow R_j$ and interchange of columns by $C_i \leftrightarrow C_j$.

☞ **Property 3**

If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then value of determinant is zero.

☞ **Property 4**

If each element of a row (or a column) of a determinant is multiplied by a constant k, then its value gets multiplied by k.

- By this property, we can take out any common factor from any one row or any one column of a given determinant.
- If corresponding elements of any two rows (or columns) of a determinant are proportional (in the same ratio), then its value is zero.

☞ **Property 5**

If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.

☞ **Property 6**

If, to each element of any row or column of a determinant, the equimultiples of corresponding elements of other row (or column) are added, then value of determinant remains the same, i.e., the value of determinant remain same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + k\,C_j$.

☞ If Δ_1 is the determinant obtained by applying $R_i \to kR_i$ or $C_i \to kC_i$ to the determinant Δ, then $\Delta_1 = k\Delta$.

☞ If more than one operation like $R_i \to R_i + kR_j$ is done in one step, care should be taken to see that a row that is affected in one operation should not be used in another operation. A similar remark applies to column operations.

☞ ***Area of triangle***

Area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3), is given by the expression

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

☞ Since area is a positive quantity, we always take the absolute value of the determinant in (1).
☞ If area is given, use both positive and negative values of the determinant for calculation.
☞ The area of the triangle formed by three collinear points is zero.

☞ **Minors and Cofactors**

Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} lies. Minor of an element $a=$ is denoted by M_{ij}.

☞ Minor of an element of a determinant of order $n(n \geq 2)$ is a determinant of order $n - 1$.

☞ Cofactor of an element a_{ij}, denoted by A_{ij} is defined by $Aij = (-1)^{i+j} M_{ij}$, where M_{ij} is minor of a_{ij}.

☞ If elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero.

☞ **Adjoint and Inverse of a Matrix**

The adjoint of a square matrix $A = [a_{ij}]_{n \times n}$ is defined as the transpose of the matrix $[A_{ij}]_{n \times n}$, where A_{ij} is the cofactor of the element a_{ij}. Adjoint of the matrix A is denoted by *adj* A.

For a square matrix of order 2, given by $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

The *adj* A can also be obtained by interchanging a_{11} and a_{22} and by changing signs of a_{12} and a_{21}, i.e.,

$$adj\ \text{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

Change sign Interchange

☞ **Theorem 1** If A be any given square matrix of order *n*, then A(*adj* A) = (*adj* A) A = |A| I, where I is the identity matrix of order *n*

☞ A square matrix A is said to be singular if |A| = 0.

☞ A square matrix A is said to be non-singular if |A| $\neq$ 0

☞ **Theorem 2** If A and B are nonsingular matrices of the same order, then AB and BA are also nonsingular matrices of the same order.

☞ **Theorem 3** The determinant of the product of matrices is equal to product of their respective determinants, that is, |AB| = |A| |B|, where A and B are square matrices of the same order

☞ If A is *a* square matrix of order *n*, then $|adj(\text{A})| = |\text{A}|^{n-1}$.

☞ **Theorem 4** A square matrix A is invertible if and only if A is nonsingular matrix.
Let A is a non-singular matrix then we have |A| $\neq$ 0
$\Rightarrow$ A is invertible and $A^{-1} = \frac{1}{|A|} adjA$

☞ **Applications of Determinants and Matrices**
Application of determinants and matrices for solving the system of linear equations in two or three variables and for checking the consistency of the system of linear equations:

☞ **Consistent system** A system of equations is said to be *consistent* if its solution (one or more) exists.

☞ **Inconsistent system** A system of equations is said to be *inconsistent* if its solution does not exist.

☞ ***Solution of system of linear equations using inverse of a matrix***
Consider the system of equations

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

$$Let\ \ A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}\ \ and\ \ B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Then, the system of equations can be written as, AX = B, i.e.,

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

☞ **Case I**

If A is a nonsingular matrix, then its inverse exists. Now AX = B

or A^{-1} (AX) = A^{-1} B (premultiplying by A^{-1})

or (A^{-1}A) X = A^{-1} B (by associative property)

or I X = A^{-1} B

or X = A^{-1} B

This matrix equation provides unique solution for the given system of equations as inverse of a matrix is unique. This method of solving system of equations is known as Matrix Method.

☞ **Case II**

If A is a singular matrix, then |A| = 0. In this case, we calculate (*adj* A) B.

If (*adj* A) B ≠ O, (O being zero matrix), then solution does not exist and the system of equations is called inconsistent.

If (*adj* A) B = O, then system may be either consistent or inconsistent according as the system have either infinitely many solutions or no solution.

CHAPTER – 4: DETERMINANTS

MARKS WEIGHTAGE – 10 marks

NCERT Important Questions & Answers

1. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$**, then find the value of *x*.**

Ans:

Given that $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

On expanding both determinants, we get

$x \times x - 18 \times 2 = 6 \times 6 - 18 \times 2 \Rightarrow x^2 - 36 = 36 - 36$

$\Rightarrow x^2 - 36 = 0 \Rightarrow x^2 = 36$

$\Rightarrow x = \pm 6$

2. Prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

Ans:

Applying operations $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$ to the given determinant Δ, we have

$$\Delta = \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 3a & 7a+3b \end{vmatrix}$$

Now applying $R_3 \to R_3 - 3R_2$, we get

$$\Delta = \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 0 & a \end{vmatrix}$$

Expanding along C_1, we obtain

$$\Delta = a\begin{vmatrix} a & 2a+b \\ 0 & a \end{vmatrix} + 0 + 0 = a(a^2 - 0) = a(a^2) = a^3$$

3. Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

Ans:

$$Let\ \ \Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2 - R_3$ to Δ, we get

$$\Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Expanding along R_1, we obtain

$$\Delta = 0\begin{vmatrix} c+a & b \\ c & a+b \end{vmatrix} - (-2c)\begin{vmatrix} b & b \\ c & a+b \end{vmatrix} + (-2b)\begin{vmatrix} b & c+a \\ c & c \end{vmatrix}$$

$= 2c(ab + b^2 - bc) - 2b(bc - c^2 - ac)$
$= 2abc + 2cb^2 - 2bc^2 - 2b^2c + 2bc^2 + 2abc$
$= 4abc$

4. If *x, y, z* are different and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ **then show that 1 + *xyz* = 0**

Ans:

We have $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$

Now, we know that If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.

$\therefore \Delta = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}$

$= (-1)^2 \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ (Using $C_3 \leftrightarrow C_2$ and then $C_1 \leftrightarrow C_2$)

$= (1+xyz) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$

$= (1+xyz) \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$ (Using $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$)

Taking out common factor $(y - x)$ from R_2 and $(z - x)$ from R_3, we get

$$\Delta = (1+xyz)(y-x)(z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix}$$

$= (1 + xyz)(y - x)(z - x)(z - y)$ (on expanding along C_1)

Since $\Delta = 0$ and x, y, z are all different, i.e., $x - y \neq 0$, $y - z \neq 0$, $z - x \neq 0$, we get
$1 + xyz = 0$

5. Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc+bc+ca+ab$

Ans:

$LHS = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

Taking out factors a,b,c common from R_1, R_2 and R_3, we get

$$\Delta = abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we have

$$\Delta = abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

Now applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, we get

$$\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)[1(1-0)]$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc+bc+ca+ab = \text{RHS}$$

6. Using the property of determinants and without expanding, prove that

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

Ans:

$$LHS = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$= \begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$$ (interchange row and column)

$$= \begin{vmatrix} b+c & c+a & -2c \\ q+r & r+p & -2r \\ y+z & z+x & -2z \end{vmatrix}$$ [using $C_3 \to C_3 - (C_1 + C_2)$]

$$= (-2)\begin{vmatrix} b+c & c+a & c \\ q+r & r+p & r \\ y+z & z+x & z \end{vmatrix}$$ (taking '–2' common from C_3)

$$= (-2)\begin{vmatrix} b & a & c \\ q & p & r \\ y & x & z \end{vmatrix}$$ (using $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$)

$$= 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$ (using $C_1 \leftrightarrow C_2$)

$$= 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = RHS$$ (interchange row and column)

7. Using the property of determinants and without expanding, prove that

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

Ans:

$$LHS = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$ [taking out factors a from R_1, b from R_2 and c from R_3]

$$= (abc)(abc)\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$ (taking out factors a from C_1, b from C_2 and c from C_3)

$$= a^2b^2c^2\begin{vmatrix} 0 & 0 & 2 \\ 0 & -2 & 2 \\ 1 & 1 & -1 \end{vmatrix}$$ (using $R_1 \to R_1 + R_2$ and $R_2 \to R_2 - R_3$)

Expanding corresponding to first row R_1, we get

$$= a^2b^2c^2 \times 2\begin{vmatrix} 0 & -2 \\ 1 & 1 \end{vmatrix}$$

$$= a^2b^2c^2 \times 2(0+2) = 4a^2b^2c^2 = RHS$$

8. Using the property of determinants and without expanding, prove that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

Ans:

$$LHS = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$ *and* $R_2 \to R_2 - R_3$, we get

$$= \begin{vmatrix} 0 & a-c & a^2-c^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 0 & a-c & (a-c)(a+c) \\ 0 & b-c & (b-c)(b+c) \\ 1 & c & c^2 \end{vmatrix}$$

Taking common factors $(a - c)$ and $(b - c)$ from R_1 and R_2 respectively, we get

$$= (a-c)(b-c)\begin{vmatrix} 0 & 1 & (a+c) \\ 0 & 1 & (b+c) \\ 1 & c & c^2 \end{vmatrix}$$

Now, expanding corresponding to C_1, we get

$= (a - c) (b - c) (b + c - a - c) = (a - b) (b - c) (c - a) = \text{RHS}$

9. Using the property of determinants and without expanding, prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

Ans:

$$LHS = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$, we get

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ (a-b)(a^2+ab+b^2) & (b-c)(b^2+bc+c^2) & c^3 \end{vmatrix}$$

Taking common $(a - b)$ from C_1 and $(b - c)$ from C_2, we get

$$= (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ (a^2+ab+b^2) & (b^2+bc+c^2) & c^3 \end{vmatrix}$$

Now, expanding along R_1, we get

$= (a - b) (b - c) [1 \times (b^2 + bc + c^2) - 1 \times (a^2 + ab + b^2)]$

$= (a - b) (b - c) [b^2 + bc + c^2 - a^2 - ab - b^2]$

$= (a - b) (b - c) (bc - ab + c^2 - a^2)$

$= (a - b) (b - c) [b(c - a) + (c - a) (c + a)]$

$= (a - b) (b - c) (c - a) (a + b + c)= \text{RHS}.$

10. Using the property of determinants and without expanding, prove that

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

Ans:

$$LHS = \begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$$

Applying $R_1 \to xR_1, R_2 \to yR_2$ and $R_3 \to zR_3$, we have

$$= \frac{1}{xyz}\begin{vmatrix} x^2 & x^3 & xyz \\ y^2 & y^3 & xyz \\ z^2 & z^3 & xyz \end{vmatrix}$$

$$= \frac{xyz}{xyz}\begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$$ (take out xyz common from C_3)

$$= \begin{vmatrix} x^2 & x^3 & 1 \\ y^2 - x^2 & y^3 - x^3 & 0 \\ z^2 - x^2 & z^3 - x^3 & 0 \end{vmatrix}$$ (using $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$)

Expanding corresponding to C_3, we get

$$= 1\begin{vmatrix} y^2 - x^2 & y^3 - x^3 \\ z^2 - x^2 & z^3 - x^3 \end{vmatrix}$$

$= \left[(y^2 - x^2)(z^3 - x^3) - (z^2 - x^2)(y^3 - x^3) \right]$

$= (y + x)\,(y - x)\,(z - x)\,(z^2 + x^2 + xz) - (z + x)\,(z - x)\,(y - x)\,(y^2 + x^2 + xy)$

$= (y - x)\,(z - x)\,[(y + x)\,(z^2 + x^2 + xz) - (z + x)\,(y^2 + x^2 + xy)]$

$= (y - x)(z - x)[yz^2 + yx^2 + xyz + xz^2 + x^3 + x^2z - zy^2 - zx^2 - xyz - xy^2 - x^3 - x^2y]$

$= (y - x)(z - x)[yz^2 - zy^2 + xz^2 - xy^2]$

$= (y - x)(z - x)[yz(z - y) + x(z^2 - y^2)]$

$= (y - x)(z - x)[yz(z - y) + x(z - y)(z + y)]$

$= (y - x)\,(z - x)\,[(z - y)\,(xy + yz + zx)]$

$= (x - y)\,(y - z)\,(z - x)\,(xy + yz + zx)$ = RHS.

11. Using the property of determinants and without expanding, prove that

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$$

Ans:

$$LHS = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$= \begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix}$$ (using $C_1 \to C_1 + C_2 + C_3$)

$$= (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix}$$ [take out $(5x + 4)$ common from C_1].

$$= (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -x+4 & 0 \\ 0 & 0 & -x+4 \end{vmatrix}$$ (Using $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$)

Expanding along C_1, we get

$= (5x + 4)\,\{1(4 - x)\,(4 - x)\}$

$= (5x+4)(4-x)^2$ = RHS.

12. Using the property of determinants and without expanding, prove that

$$\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$$

Ans:

$$LHS = \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

$$= \begin{vmatrix} 3y+k & y & y \\ 3y+k & y+k & y \\ 3y+k & y & y+k \end{vmatrix}$$ (using $C_1 \to C_1 + C_2 + C_3$)

$$= (3y+k)\begin{vmatrix} 1 & y & y \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix}$$ [take out (5*x* + 4) common from C_1].

$$= (3y+k)\begin{vmatrix} 1 & y & y \\ 0 & k & 0 \\ 0 & 0 & k \end{vmatrix}$$ (Using $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$)

Expanding along C_3, we get

$= (3y+k)\left[1(k^2 - 0)\right]$

$= k^2(3y+k)$ = RHS

13. Using the property of determinants and without expanding, prove that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

Ans:

$$LHS = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$ (Using $R_1 \to R_1 + R_2 + R_3$)

Take out (*a* + *b* + *c*) common from R_1, we get

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & c-a-b \end{vmatrix}$$ (Using $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$)

Expanding along R_1, we get

$= (a + b + c)\ \{1(- b - c - a)\ (- c - a - b)\}$

$= (a + b + c)\ [- (b + c + a) \times (-)\ (c + a + b)]$

$= (a+b+c)(a+b+c)(a+b+c) = (a+b+c)^3$ = RHS

14. Using the property of determinants and without expanding, prove that

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$$

Ans:

$$LHS = \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$$

$$= \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$$ (using $C_1 \to C_1 + C_2 + C_3$)

$$= 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$$ [take out 2(x + y + z) common from C_1].

$$= 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 0 & y+z+x & 0 \\ 0 & 0 & z+x+y \end{vmatrix}$$ (Using $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$)

$$= 2(x+y+z)(x+y+z)(x+y+z)\begin{vmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along R_3, we get

$$= 2(x+y+z)(x+y+z)(x+y+z)\left[1(1-0)\right]$$

$$= 2(x+y+z)(x+y+z)(x+y+z) = 2(x+y+z)^3 = \text{RHS}$$

15. Using the property of determinants and without expanding, prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \left(1-x^3\right)^2$

Ans:

$$LHS = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 1 \end{vmatrix}$$ (using $C_1 \to C_1 + C_2 + C_3$)

$$= (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix}$$ [take out $(1+x+x^2)$ common from C_1].

$$= (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1-x & x-x^2 \\ 0 & x^2-x & 1-x^2 \end{vmatrix}$$ (Using $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$)

$$= (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1-x & x(1-x) \\ 0 & x(x-1) & 1-x^2 \end{vmatrix}$$

Take out $(1-x)$ common from R_2 and same from R_3, we get

$$= (1+x+x^2)(1-x)(1-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & x \\ 0 & -x & 1+x \end{vmatrix}$$

Expanding along C_1, we get

$$= (1+x+x^2)(1-x)(1-x)\begin{vmatrix} 1 & x \\ -x & 1+x \end{vmatrix}$$

$$= (1+x+x^2)(1-x)(1-x)(1+x+x^2)$$

$$= \left(1-x^3\right)\left(1-x^3\right) = \left(1-x^3\right)^2 = \text{RHS}$$

16. Using the property of determinants and without expanding, prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

Ans:

$$LHS = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix} \qquad (\text{Using } C_1 \to C_1 - bC_3 \ \ and \ \ C_2 \to C_2 + aC_3)$$

$$= (1+a^2+b^2)^2\begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$

$$= (1+a^2+b^2)^2\begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & 0 & 1+a^2+b^2 \end{vmatrix} \qquad (R_3 \to R_3 - bR_1 + aR_2)$$

Expanding along R_1, we get

$$= (1+a^2+b^2)^2\left[1(1+a^2+b^2)\right]$$

$$= (1+a^2+b^2)^3 = RHS$$

17. Using the property of determinants and without expanding, prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

Ans:

$$LHS = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Taking out common factors a, b and c from R_1, R_2 and R_3 respectively, we get

$$= \begin{vmatrix} a+\frac{1}{a} & b & c \\ a & b+\frac{1}{b} & c \\ a & b & c+\frac{1}{c} \end{vmatrix}$$

$$= \begin{vmatrix} a+\frac{1}{a} & b & c \\ -\frac{1}{a} & \frac{1}{b} & 0 \\ -\frac{1}{a} & 0 & \frac{1}{c} \end{vmatrix} \quad \text{(Using } R_2 \to R_2 - R_1 \text{ and } R_3 \to R_3 - R_1\text{)}$$

Multiply and divide C_1 by a, C_2 by b and C_3 by c and then take common out from C_1, C_2 and C_3 respectively, we get

$$= abc \times \frac{1}{abc} \begin{vmatrix} a^2+1 & b^2 & c^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} a^2+1 & b^2 & c^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

Expanding along R_3, we get

$$= -1 \times (-c^2) + 1\left[1(a^2+1) + 1(b^2)\right]$$

$$= 1 + a^2 + b^2 + c^2 = RHS$$

18. Find values of k if area of triangle is 4 sq. units and vertices are
(i) (k, 0), (4, 0), (0, 2) (ii) (–2, 0), (0, 4), (0, k)
Ans:

(i) We have Area of triangle = $\frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 4$

$\Rightarrow |\, k(0 - 2) + 1(8 - 0)| = 8$
$\Rightarrow k(0 - 2) + 1(8 - 0) = \pm 8$
On taking positive sign $-2k + 8 = 8$
$\Rightarrow -2k = 0$
$\Rightarrow k = 0$
On taking negative sign $-2k + 8 = -8$
$\Rightarrow -2k = -16$
$\Rightarrow k = 8$
$\Rightarrow k = 0, 8$

(ii) We have Area of triangle = $\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} = 4$

$\Rightarrow |-2(4 - k) + 1(0 - 0)| = 8$
$\Rightarrow -2(4 - k) + 1(0 - 0) = \pm 8$
$\Rightarrow [-8 + 2k] = \pm 8$
On taking positive sign, $2k - 8 = 8 \Rightarrow 2k = 16 \Rightarrow k = 8$
On taking negative sign, $2k - 8 = -8 \Rightarrow 2k = 0 \Rightarrow k = 0$
$\Rightarrow k = 0, 8$

19. If area of triangle is 35 sq units with vertices (2, – 6), (5, 4) and (*k*, 4). Then find the value of *k*.

Ans:

We have Area of triangle $= \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = 35$

$\Rightarrow |2(4-4) + 6(5-k) + 1(20-4k)| = 70$

$\Rightarrow 2(4-4) + 6(5-k) + 1(20-4k) = \pm 70$

$\Rightarrow 30 - 6k + 20 - 4k = \pm 70$

On taking positive sign, $-10k + 50 = 70$

$\Rightarrow -10k = 20 \Rightarrow k = -2$

On taking negative sign, $-10k + 50 = -70$

$\Rightarrow -10k = -120 \Rightarrow k = 12$

$\therefore k = 12, -2$

20. Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Ans:

Given that $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Cofactors of the elements of second row

$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = -(9-16) = 7$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = (15-8) = 7$

and $A_{23} = (-1)^{2+3}\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -(10-3) = -7$

Now, expansion of Δ using cofactors of elements of second row is given by

$\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$

$= 2 \times 7 + 0 \times 7 + 1(-7) = 14 - 7 = 7$

21. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}.

Ans:

Given that A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

Now, $A^2 - 5A + 7I = O$

$A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-2 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$\therefore$ $A^2 - 5A + 7I = O$

$\because |A| = \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix} = 6+1 = 7 \neq 0$

$\therefore$ A^{-1} exists.

Now, $A.A - 5A = -7I$

Multiplying by A^{-1} on both sides, we get

$A.A\,(A^{-1}) - 5A(A^{-1}) = -7I(A^{-1})$

$\Rightarrow AI - 5I = -7A^{-1}$ (using $AA^{-1} = I$ and $IA^{-1} = A^{-1}$)

$A^{-1} = -\frac{1}{7}(A - 5I) = \frac{1}{7}(5I - A) = \frac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right)$

$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

22. For the matrix A = $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = O$.

Ans:

Given that A = $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

$A^2 = A.A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

Now, $A^2 + aA + bI = O$

$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

If two matrices are equal, then their corresponding elements are equal.

$\Rightarrow$ $11 + 3a + b = 0$ …(i)

$8 + 2a = 0$ …(ii)

$4 + a = 0$ …(iii)

and $3 + a + b = 0$ …(iv)

Solving Eqs. (iii) and (iv), we get $4 + a = 0$

$\Rightarrow a = -4$

and $3 + a + b = 0$

$\Rightarrow 3 - 4 + b = 0 \Rightarrow b = 1$

Thus, $a = -4$ and $b = 1$

23. For the matrix A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, Show that $A^3 - 6A^2 + 5A + 11\,I = O$. Hence, find A^{-1}.

Ans:

Given that A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$$A^2 = A.A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$and \quad A^3 = A^2.A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$$\therefore A^3 - 6A^2 + 5A + 11I$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 8-24+5+11 & 7-12+5+0 & 1-6+5+0 \\ -23+18+5+0 & 27-48+10+11 & -69+84-15+0 \\ 32-42+10+0 & -13+18-5+0 & 58-84+15+11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

$$\because |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{vmatrix} = 1(6-3) - 1(3+6) + 1(-1-4) = 3 - 9 - 5 = -11 \neq 0$$

$\therefore A^{-1}$ *exist*

$Now, A^3 - 6A^2 + 5A + 11I = O$

$\Rightarrow AA(AA^{-1}) - 6A(AA^{-1}) + 5(AA^{-1}) + 11(IA^{-1}) = O$

$\Rightarrow AAI - 6AI + 5I + 11A^{-1} = O$

$\Rightarrow A^2 - 6A + 5I = -11A^{-1}$

$$\Rightarrow A^{-1} = -\frac{1}{11}(A^2 - 6A + 5I)$$

$$\Rightarrow A^{-1} = \frac{1}{11}(-A^2 + 6A - 5I)$$

$$\Rightarrow A^{-1} = \frac{1}{11}\left(\begin{bmatrix} -4 & -2 & -1 \\ 3 & -8 & 14 \\ -7 & 3 & -14 \end{bmatrix} + 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

$$\Rightarrow A^{-1} = \frac{1}{11}\left(\begin{bmatrix} -4 & -2 & -1 \\ 3 & -8 & 14 \\ -7 & 3 & -14 \end{bmatrix} + \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}\right)$$

$$\Rightarrow A^{-1} = \frac{1}{11}\begin{bmatrix} -4+6-5 & -2+6-0 & -1+6-0 \\ 3+6-0 & -8+12-5 & 14-18-0 \\ -7+12-0 & 3-6-0 & -14+18-5 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

24. Solve system of linear equations, using matrix method,

$2x + y + z = 1$

$x - 2y - z = \frac{3}{2}$

$3y - 5z = 9$

Ans:

The given system can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & -4 & -2 \\ 0 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad and \quad B = \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 2 & -4 & -2 \\ 0 & 3 & -5 \end{vmatrix} = 2(20+6) - 1(-10-0) + 1(6-0)$$

$= 52 + 10 + 6 = 68 \neq 0$

Thus, A is non-singular, Therefore, its inverse exists.

Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$.

Cofactors of A are

$A_{11} = 20 + 6 = 26$,

$A_{12} = -(-10 + 0) = 10$,

$A_{13} = 6 + 0 = 6$

$A_{21} = -(-5 - 3) = 8$,

$A_{22} = -10 - 0 = -10$,

$A_{23} = -(6 - 0) = -6$

$A_{31} = (-2 + 4) = 2$,

$A_{32} = -(-4 - 2) = 6$,

$A_{33} = -8 - 2 = -10$

$$adj(A) = \begin{bmatrix} 26 & 10 & 6 \\ 8 & -10 & -6 \\ 2 & 6 & -10 \end{bmatrix}^T = \begin{bmatrix} 26 & 8 & 2 \\ 10 & -10 & 6 \\ 6 & -6 & -10 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{68}\begin{bmatrix} 26 & 8 & 2 \\ 10 & -10 & 6 \\ 6 & -6 & -10 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{68}\begin{bmatrix} 26 & 8 & 2 \\ 10 & -10 & 6 \\ 6 & -6 & -10 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{68}\begin{bmatrix} 26+24+18 \\ 10-30+54 \\ 6-18-90 \end{bmatrix} = \frac{1}{68}\begin{bmatrix} 68 \\ 34 \\ -102 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{-3}{2} \end{bmatrix}$$

$$Hence,\ x = 1,\ y = \frac{1}{2}\ and\ z = \frac{-3}{2}$$

25. Solve system of linear equations, using matrix method,

***x* – *y* + *z* = 4**

2*x* + *y* – 3*z* = 0

***x* + *y* + *z* = 2**

Ans:

The given system can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}\ and\ B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1(1 + 3) - (-1)(2 + 3) + 1(2 - 1) = 4 + 5 + 1 = 10 \neq 0$$

Thus, A is non-singular, Therefore, its inverse exists.

Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$.

Cofactors of A are

$A_{11} = 1 + 3 = 4$,

$A_{12} = -(2 + 3) = -5$,

$A_{13} = 2 - 1 = 1$,

$A_{21} = -(-1 - 1) = 2$,

$A_{22} = 1 - 1 = 0$,

$A_{23} = -(1 + 1) = -2$,

$A_{31} = 3 - 1 = 2$,

$A_{32} = -(-3 - 2) = 5$,

$A_{33} = 1 + 2 = 3$

$$adj(A) = \begin{bmatrix} 4 & 5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Hence, $x = 2$, $y = -1$ and $z = 1$.

26. Solve system of linear equations, using matrix method,

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3$

Ans:

The given system can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} = 2(4 + 1) - 3(-2 - 3) + 3(-1 + 6)$$

$= 10 + 15 + 15 = 40 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists. Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$

Cofactors of A are

$A_{11} = 4 + 1 = 5$,

$A_{12} = -(-2 - 3) = 5$,

$A_{13} = (-1 + 6) = 5$,

$A_{21} = -(-6 + 3) = 3$,

$A_{22} = (-4 - 9) = -13$,

$A_{23} = -(-2 - 9) = 11$,

$A_{31} = 3 + 6 = 9$,

$A_{32} = -(2 - 3) = 1$,

$A_{33} = -4 - 3 = -7$

$$adj(A) = \begin{bmatrix} 5 & 5 & 5 \\ 3 & -13 & 11 \\ 9 & 1 & -7 \end{bmatrix}^T = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Hence, $x = 1$, $y = 2$ and $z = -1$.

27. Solve system of linear equations, using matrix method,

$x - y + 2z = 7$

$3x + 4y - 5z = -5$

$2x - y + 3z = 12$

Ans:

The given system can be written as $AX = B$, where

$$A=\begin{bmatrix}1 & -1 & 2\\ 3 & 4 & -5\\ 2 & -1 & 3\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{ and } B=\begin{bmatrix}7\\ -5\\ 12\end{bmatrix}$$

$$Here, |A|=\begin{vmatrix}1 & -1 & 2\\ 3 & 4 & -5\\ 2 & -1 & 3\end{vmatrix} = 1(12-5)-(-1)(9+10)+2(-3-8)$$

$= 7 + 19 - 22 = 4 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$

Cofactors of A are

$A_{11} = 12 - 5 = 7$,

$A_{12} = -(9 + 10) = -19$,

$A_{13} = -3 - 8 = -11$,

$A_{21} = -(-3 + 2) = 1$,

$A_{22} = 3 - 4 = -1$,

$A_{23} = -(-1 + 2) = -1$,

$A_{31} = 5 - 8 = -3$,

$A_{32} = -(-5 - 6) = 11$,

$A_{33} = 4 + 3 = 7$

$$adj(A)=\begin{bmatrix}7 & -19 & -11\\ 1 & -1 & -1\\ -3 & 11 & 7\end{bmatrix}^T=\begin{bmatrix}7 & 1 & -3\\ -19 & -1 & 11\\ -11 & -1 & 7\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{4}\begin{bmatrix}7 & 1 & -3\\ -19 & -1 & 11\\ -11 & -1 & 7\end{bmatrix}$$

$$\text{Now, } X=A^{-1}B \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{4}\begin{bmatrix}7 & 1 & -3\\ -19 & -1 & 11\\ -11 & -1 & 7\end{bmatrix}\begin{bmatrix}5\\ -4\\ 3\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{4}\begin{bmatrix}49-5-36\\ -133+5+132\\ -77+5+84\end{bmatrix}=\frac{1}{4}\begin{bmatrix}8\\ 4\\ 12\end{bmatrix}=\begin{bmatrix}2\\ 1\\ 3\end{bmatrix}$$

Hence, $x = 2$, $y = 1$ and $z = 3$.

28. If A = $\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$ **find A^{-1}. Using A^{-1}, Solve system of linear equations:**

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

Ans:

The given system can be written as $AX = B$, where

$$A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{ and } B=\begin{bmatrix}11\\ -5\\ -3\end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} = 2\,(-4+4) - (-3)\,(-6+4) + 5\,(3-2)$$

$= 0 - 6 + 5 = -1 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$

Cofactors of A are

$A_{11} = -4 + 4 = 0,$

$A_{12} = -(-6 + 4) = 2,$

$A_{13} = 3 - 2 = 1,$

$A_{21} = -(6 - 5) = -1,$

$A_{22} = -4 - 5 = -9,$

$A_{23} = -(2 + 3) = -5,$

$A_{31} = (12 - 10) = 2,$

$A_{32} = -(-8 - 15) = 23,$

$A_{33} = 4 + 9 = 13$

$$adj(A) = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1$, $y = 2$ and $z = 3$.

29. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix method.

Ans:

Let the prices (per kg) of onion, wheat and rice be Rs. x, Rs. y and Rs. z, respectively then

$4x + 3y + 2z = 60$, $2x + 4y + 6z = 90$, $6x + 2y + 3z = 70$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix} = 4(12-12) - 3(6-36) + 2(4-24)$$

$= 0 + 90 - 40 = 50 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists. Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$

Cofactors of A are,

$A_{11} = 12 - 12 = 0$,
$A_{12} = -(6 - 36) = 30$,
$A_{13} = 4 - 24 = -20$,
$A_{21} = -(9 - 4) = -5$,
$A_{22} = 12 - 12 = 0$,
$A_{23} = -(8 - 18) = 10$,
$A_{31} = (18 - 8) = 10$,
$A_{32} = -(24 - 4) = -20$,
$A_{33} = 16 - 6 = 10$

$$adj(A) = \begin{bmatrix} 0 & 30 & -20 \\ -5 & 0 & 10 \\ 10 & -20 & 10 \end{bmatrix}^T = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 - 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\therefore$ $x = 5$, $y = 8$ and $z = 8$.

Hence, price of onion per kg is Rs. 5, price of wheat per kg is Rs. 8 and that of rice per kg is Rs. 8.

30. Without expanding the determinant, prove that $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

Ans:

$$LHS = \begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix}$$

Applying $R_1 \to aR_1, R_2 \to bR_2$ *and* $R_3 \to cR_3$, we get

$$= \frac{1}{abc}\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix}$$

$$= \frac{abc}{abc}\begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$$ [Taking out factor *abc* from C_3]

$$= (-1)^2\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$ (using $C_1 \leftrightarrow C_3$ and $C_2 \leftrightarrow C_3$)

$$= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = RHS$$

31. If *a*, *b* and *c* are real numbers, and $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$**. Show that either *a* + *b* + *c* = 0 or *a* = *b* = *c*.**

Ans:

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$= \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \\ 2(a+b+c) & b+c & c+a \end{vmatrix} \quad \text{(using } C_1 \to C_1 + C_2 + C_3\text{)}$$

$$= 2(a+b+c)\begin{vmatrix} 1 & c+a & a+b \\ 1 & a+b & b+c \\ 1 & b+c & c+a \end{vmatrix} \quad \text{[take out } 2(a+b+c) \text{ common from } C_1\text{].}$$

$$= 2(a+b+c)\begin{vmatrix} 1 & c+a & a+b \\ 0 & b-c & c-a \\ 0 & b-a & c-b \end{vmatrix} \quad \text{(Using } R_2 \to R_2 - R_1 \text{ and } R_3 \to R_3 - R_1\text{)}$$

Expanding along C_1, we get

$$= 2(a+b+c)\begin{vmatrix} b-c & c-a \\ b-a & c-b \end{vmatrix}$$

$$= 2(a+b+c)\left[(b-c)(c-b)-(c-a)(b-a)\right]$$

$$= 2(a+b+c)\left[bc-b^2-c^2+bc-(bc-ac-ab+a^2)\right]$$

$$= 2(a+b+c)\left[bc-b^2-c^2+bc-bc+ac+ab-a^2\right]$$

$$= 2(a+b+c)\left[ab+bc+ac-a^2-b^2-c^2\right]$$

It is given that $\Delta = 0$,

$$2(a+b+c)\left[ab+bc+ac-a^2-b^2-c^2\right]=0$$

$$\Rightarrow \text{Either } a+b+c=0 \text{ or } ab+bc+ac-a^2-b^2-c^2=0$$

$$\Rightarrow ab+bc+ac-a^2-b^2-c^2=0$$

$$\Rightarrow 2ab+2bc+2ac-2a^2-2b^2-2c^2=0$$

$$\Rightarrow 2a^2+2b^2+2c^2-2ab-2bc-2ac=0$$

$$\Rightarrow a^2+b^2-2ab+b^2+c^2-2bc+c^2+a^2-2ac=0$$

$$\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2=0$$

$$\Rightarrow (a-b)^2=(b-c)^2=(c-a)^2=0 \quad \text{[since square of any real number is never negative]}$$

$$\Rightarrow (a-b)=(b-c)=(c-a)=0$$

$$\Rightarrow a=b, b=c, c=a$$

$$\Rightarrow a=b=c$$

32. Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

Ans:

$$LHS = \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

Taking out a from C_1, b from C_2 and c from C_3, we get

$$= abc\begin{vmatrix} a & b & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$$= abc\begin{vmatrix} 0 & c & a+c \\ 2b & b & a \\ 2b & b+c & c \end{vmatrix} \qquad [\text{Using } C_1 \to C_1 + C_2 - C_3]$$

$$= abc\begin{vmatrix} 0 & c & a+c \\ 0 & -c & a-c \\ 2b & b+c & c \end{vmatrix} \qquad [\text{Using } R_2 \to R_2 - R_3]$$

Expanding along C_1,we get

$= (abc)\ [\ (2\ b)\ \{\ c(a-c) + c\ (a+c)\ \}\ ]$

$= 2\ (ab^2c)\ (2ac) = 4\ a^2b^2c^2$ = RHS.

33. Using properties of determinants, prove that $\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$

Ans:

$$LHS = \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

$$= \begin{vmatrix} \alpha & \alpha^2 & \alpha+\beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha+\beta \\ \gamma & \gamma^2 & \alpha+\beta+\gamma \end{vmatrix} \qquad (\text{using } C_3 \to C_3 + C_1)$$

$$= (\alpha+\beta+\gamma)\begin{vmatrix} \alpha & \alpha^2 & 1 \\ \beta & \beta^2 & 1 \\ \gamma & \gamma^2 & 1 \end{vmatrix} \qquad (\text{Taking out } (\alpha+\beta+\gamma) \text{ common from } C_1)$$

$$= (\alpha+\beta+\gamma)\begin{vmatrix} \alpha & \alpha^2 & 1 \\ \beta-\alpha & \beta^2-\alpha^2 & 0 \\ \gamma-\alpha & \gamma^2-\alpha^2 & 0 \end{vmatrix} \qquad (\text{Using } R_2 \to R_2 - R_1 \text{ and } R_3 \to R_3 - R_1)$$

Expanding along C_3, we get

$= (\alpha + \beta + \gamma)\ [(\beta - \alpha)(\gamma^2 - \alpha^2) - (\gamma - \alpha)(\beta^2 - \alpha^2)]$

$= (\alpha + \beta + \gamma)\ [(\beta - \alpha)(\gamma - \alpha)(\gamma + \alpha) - (\gamma - \alpha)(\beta - \alpha)(\beta + \alpha)]$

$= (\alpha + \beta + \gamma)\ (\beta - \alpha)(\gamma - \alpha)[\gamma + \alpha - \beta - \alpha]$

$= (\alpha + \beta + \gamma)\ (\beta - \alpha)(\gamma - \alpha)(\gamma - \beta)$

$= (\alpha + \beta + \gamma)\ (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$ = RHS

34. Using properties of determinants, prove that $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

Ans:

$$LHS = \begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} \quad \text{(using } C_1 \to C_1 + C_2 + C_3\text{)}$$

$$= (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix} \quad \text{(Taking out } (a+b+c) \text{ common from } C_1\text{)}$$

Now applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get

$$= (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix}$$

Expanding along C_1, we get

$= (a + b + c)[(2b + a)(2c + a) - (a - b)(a - c)]$

$= (a + b + c)[4bc + 2ab + 2ac + a - a + ac + ba - bc]$ 2 2

$= (a + b + c)(3ab + 3bc + 3ac) = 3(a + b + c)(ab + bc + ca) = \text{RHS}$

35. Solve the system of equations:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

Ans:

Let $\frac{1}{x} = p,\ \frac{1}{y} = q\ \ and\ \ \frac{1}{z} = r$, then the given equations become

$2p + 3q + 10r = 4$, $4p - 6q + 5r = 1$, $6p + 9q - 20r = 2$

This system can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} p \\ q \\ r \end{bmatrix}\ \ and\ \ B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix} = 2(120-45) - 3(-80-30) + 10(36+36)$$

$= 150 + 330 + 720 = 1200 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Therefore, the above system is consistent and has a unique solution given by $X = A^{-1}B$

Cofactors of A are

$A_{11} = 120 - 45 = 75$,

$A_{12} = -(-80 - 30) = 110,$
$A_{13} = (36 + 36) = 72,$
$A_{21} = -(-60 - 90) = 150,$
$A_{22} = (-40 - 60) = -100,$
$A_{23} = -(18 - 18) = 0,$
$A_{31} = 15 + 60 = 75,$
$A_{32} = -(10 - 40) = 30,$
$A_{33} = -12 - 12 = -24$

$$adj(A) = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\Rightarrow p = \frac{1}{2}, q = \frac{1}{3}, r = \frac{1}{5}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}, \frac{1}{z} = \frac{1}{5}$$

$\Rightarrow x = 2$, $y = 3$ and $z = 5$.

36. If a, b, c, are in A.P, then find the determinant of $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$

Ans:

$$Let\ \ A = \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} x+2 & x+3 & x+2a \\ 0 & 0 & 2(2b-a-c) \\ x+4 & x+5 & x+2c \end{vmatrix}$$ (using $R_2 \rightarrow 2R_2 - R_1 - R_3$)

But a,b, c are in AP. Using $2b = a + c$, we get

$$A = \frac{1}{2}\begin{vmatrix} x+2 & x+3 & x+2a \\ 0 & 0 & 0 \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$$ [Since, all elements of R_2 are zero]

37. Show that the matrix $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$, where I is 2 × 2 identity matrix and O is 2 × 2 zero matrix. Using this equation, find A^{-1}.

Ans:

Given that $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$

$$A^2 = AA = \begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix} = \begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix}$$

$$Hence,\ A^2 - 4A + I = \begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix} - 4\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix} - \begin{bmatrix}8 & 12\\ 4 & 8\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}7-8+1 & 12-12+0\\ 4-4+0 & 7-8+1\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} = O$$

$Now,\ A^2 - 4A + I = O$

$\Rightarrow AA - 4A = -I$

$\Rightarrow AA(A^{-1}) - 4AA^{-1} = -IA^{-1}$ (Post multiplying by A^{-1} because $|A| \neq 0$)

$\Rightarrow A(AA^{-1}) - 4I = -A^{-1}$

$\Rightarrow AI - 4I = -A^{-1}$

$$\Rightarrow A^{-1} = 4I - A = 4\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix} = \begin{bmatrix}4-2 & 0-3\\ 0-1 & 4-2\end{bmatrix} = \begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix}$$

38. Solve the following system of equations by matrix method.

$3x - 2y + 3z = 8$
$2x + y - z = 1$
$4x - 3y + 2z = 4$

Ans:

The system of equation can be written as $AX = B$, where

$$A = \begin{bmatrix}3 & -2 & 3\\ 2 & 1 & -1\\ 4 & -3 & 2\end{bmatrix}, X = \begin{bmatrix}x\\ y\\ z\end{bmatrix}\ and\ B = \begin{bmatrix}8\\ 1\\ 4\end{bmatrix}$$

$$Here, |A| = \begin{vmatrix}3 & -2 & 3\\ 2 & 1 & -1\\ 4 & -3 & 2\end{vmatrix}$$

= 3 (2 – 3) + 2(4 + 4) + 3 (– 6 – 4) = – 17 ≠ 0

Hence, A is nonsingular and so its inverse exists. Now

$A_{11} = -1$, $A_{12} = -8$, $A_{13} = -10$
$A_{21} = -5$, $A_{22} = -6$, $A_{23} = 1$
$A_{31} = -1$, $A_{32} = 9$, $A_{33} = 7$

$$adj(A) = \begin{bmatrix}-1 & -8 & 10\\ -5 & -6 & 1\\ -1 & 9 & 7\end{bmatrix}^T = \begin{bmatrix}-1 & -5 & -1\\ -8 & -6 & 9\\ -10 & 1 & 7\end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = -\frac{1}{17}\begin{bmatrix}-1 & -5 & -1\\ -8 & -6 & 9\\ -10 & 1 & 7\end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{17}\begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence $x = 1$, $y = 2$ and $z = 3$.

39. Show that $\Delta = \begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$

Ans:

Given that $\Delta = \begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix}$

Applying $R_1 \to xR_1$, $R_2 \to yR_2$, $R_3 \to zR_3$ to Δ and dividing by xyz, we get

$$\Delta = \frac{1}{xyz}\begin{vmatrix} x(y+z)^2 & x^2y & x^2z \\ xy^2 & y(x+z)^2 & y^2z \\ xz^2 & yz^2 & z(x+y)^2 \end{vmatrix}$$

Taking common factors x, y, z from C_1, C_2 and C_3 respectively, we get

$$\Delta = \frac{xyz}{xyz}\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (x+z)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, we have

$$\Delta = \begin{vmatrix} (y+z)^2 & x^2-(y+z)^2 & x^2-(y+z)^2 \\ y^2 & (x+z)^2-y^2 & 0 \\ z^2 & 0 & (x+y)^2-z^2 \end{vmatrix}$$

Taking common factor $(x + y + z)$ from C_2 and C_3, we have

$$\Delta = (x+y+z)^2\begin{vmatrix} (y+z)^2 & x-(y+z) & x-(y+z) \\ y^2 & (x+z)-y & 0 \\ z^2 & 0 & (x+y)-z \end{vmatrix}$$

Applying $R_1 \to R_1 - (R_2 + R_3)$, we have

$$\Delta = (x+y+z)^2\begin{vmatrix} 2yz & -2z & -2y \\ y^2 & x-y+z & 0 \\ z^2 & 0 & x+y-z \end{vmatrix}$$

Applying $C_2 \to (C_2 + \frac{1}{y}C_1)$ and $C_3 \to C_3 + \frac{1}{z}C_1$, we get

$$\Delta = (x+y+z)^2 \begin{vmatrix} 2yz & 0 & 0 \\ y^2 & x+z & \frac{y^2}{z} \\ z^2 & \frac{z^2}{y} & x+y \end{vmatrix}$$

Finally expanding along R1, we have

$\Delta = (x + y + z)^2 (2yz) [(x + z) (x + y) – yz] = (x + y + z)^2 (2yz) (x^2 + xy + xz)$

$= (x + y + z)^3 (2xyz)$

40. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ **to solve the system of equations**

$x – y + 2z = 1$

$2y – 3z = 1$

$3x – 2y + 4z = 2$

Ans:

Consider the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

$$= \begin{bmatrix} -2-9+12 & 0-2+2 & 1+3-4 \\ 0+18-18 & 0+4-3 & 0-6+6 \\ -6-18+24 & 0-4+4 & 3+6-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

Now, given system of equations can be written, in matrix form, as follows

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

Hence $x = 0$, $y = 5$ and $z = 3$

CHAPTER – 3: DETERMINANTS

MARKS WEIGHTAGE – 10 marks

Previous Years Board Exam (Important Questions & Answers)

19. Let A be a square matrix of order 3 × 3. Write the value of $|2A|$, where $|A| = 4$.

Ans:

Since $|2A| = 2^n|A|$ where n is order of matrix A.

Here $|A| = 4$ and $n = 3$

$\therefore\ |2A| = 2^3 \times 4 = 32$

20. Write the value of the following determinant: $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

Ans:

Given that $\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

Applying $R_1 \to R_1 - 6R_3$

$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 0$ (Since R_1 is zero)

21. If A is a square matrix and $|A| = 2$, then write the value of $|AA'|$, where A' is the transpose of matrix A.

Ans:

$|AA|' = |A|.\ |A'| = |A|.\ |A| = |A|^2 = 2 \text{ x } 2 = 4.$

[since, $|AB| = |A|.|B|$ and $|A| = |A'|$, where A and B are square matrices.]

22. If A is a 3 × 3 matrix, |A| ≠ 0 and |3A| = k |A|, then write the value of k.

Ans:

Here, $|3A| = k|A|$

$\Rightarrow 3^3|A| = k|A|$ [$\because$ $|kA| = kn|A|$ where n is order of A]

$\Rightarrow 27\ |A| = k|A|$

$\Rightarrow k = 27$

23. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

Ans:

$\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib\pi \end{vmatrix} = (a+ib)(a-ib)-(c+id)(-c+id)$

$= (a+ib)(a-ib)+(c+id)(c-id)$

$= a^2 - i^2b^2 + c^2 - i^2d^2$

$= a^2 + b^2 + c^2 + d^2$

24. If $\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix} = 3$, find the value of x.

Ans:

Given that $\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix} = 3$

$\Rightarrow$ $4x + 8 - 3x - 15 = 3$

$\Rightarrow x - 7 = 3$

$\Rightarrow x = 10$

25. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{23}.

Ans:

Minor of $a_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$.

26. Evaluate: $\begin{vmatrix} \cos 15^0 & \sin 15^0 \\ \sin 75^0 & \cos 75^0 \end{vmatrix}$

Ans:

Expanding the determinant, we get

cos 15° . cos 75° - sin 15° . sin 75°

= cos (15° + 75°) = cos 90° = 0

[since cos (A + B) = cos A. cos B – sinA . sin B]

27. Using properties of determinants, prove the following: $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b)$

Ans:

$Let\ \ \Delta = \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have

$\Delta = \begin{vmatrix} 3(a+b) & 3(a+b) & 3(a+b) \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$

Taking out $3(a + b)$ from 1st row, we have

$\Delta = 3(a+b)\begin{vmatrix} 1 & 1 & 1 \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$

Applying $C_1 \rightarrow C_1 - C_2$ and $C2 \rightarrow C_2 - C_3$

$\Delta = 3(a+b)\begin{vmatrix} 0 & 0 & 1 \\ -2b & -b & a+b \\ -b & 2b & a \end{vmatrix}$

Expanding along first row, we have

D = $3(a + b)$ [1. $(4b^2 - b^2)$]

= $3(a + b) \times 3b^2 = 9b^2(a + b)$

28. Write the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

Ans:

Given determinant |A|= $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

$= 3x\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix} = 0 (\because R_1 = R_3)$

29. Two schools P and Q want to award their selected students on the values of Tolerance, Kindness and Leadership. The school P wants to award Rs. x each, Rs. y each and Rs. z each for the three respective values to 3, 2 and 1 students respectively with a total award money of Rs. 2,200. School Q wants to spend Rs. 3,100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is Rs. 1,200, using matrices, find the award money for each value. Apart from these three values, suggest one more value that should be considered for award.

Ans:

According to question,

$3x + 2y + z = 2200$

$4x + y + 3z = 3100$

$x + y + z = 1200$

The above system of equation may be written in matrix form as $AX = B$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \; and \; B = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 3(1-3) - 2(4-3) + 1(4-1) = -6 - 2 + 3 = -5 \neq 0$$

$\therefore$ A^{-1} exists.

Now, $A_{11} = (1 - 3) = -2$,

$A_{12} = -(4 - 3) = -1$,

$A_{13} = (4 - 1) = 3$,

$A_{21} = -(2 - 1) = -1$,

$A_{22} = (3 - 1) = 2$,

$A_{23} = -(3 - 2) = -1$

$A_{31} = (6 - 1) = 5$,

$A_{32} = -(9 - 4) = -5$,

$A_{33} = (3 - 8) = -5$

$$adj(A) = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{-5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 4400+3100-6000 \\ 2200-6200+6000 \\ -6600+3100+6000 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1500 \\ 2000 \\ 2500 \end{bmatrix} = \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$$

$\Rightarrow$ $x = 300$, $y = 400$, $z = 500$

i.e., Rs. 300 for tolerance, Rs. 400 for kindness and Rs. 500 for leadership are awarded.

One more value like punctuality, honesty etc may be awarded.

30. Using properties of determinants, prove that

Ans:

$$LHS = \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$= \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & y & a+z \end{vmatrix}$$

Apply $R_1 \to R_1 - R_2$, we get

$$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & y & a+z \end{vmatrix}$$

$$= (a+x+y+z)\begin{vmatrix} 0 & -a & 0 \\ 1 & a+y & z \\ 1 & y & a+z \end{vmatrix}$$

Expanding along R_1, we get

$= (a + x + y + z) \{0 + a (a + z - z)\} = a^2(a + x + y + z) =$ RHS

31. 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students and the third group contains vigilant and obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of first and second group is four times that of the third group. Using matrix method, find the number of students in each group. Apart from the values, hard work, honesty and respect for law, vigilance and obedience, suggest one more value, which in your opinion, the school should consider for awards.

Ans:

Let no. of students in Ist, 2nd and 3rd group to x, y, z respectively.

From the statement we have

$$x + y + z = 10$$
$$2x + y = 13$$
$$x + y - 4z = 0$$

The above system of linear equations may be written in matrix form as $AX = B$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 10 \\ 13 \\ 0 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & -4 \end{vmatrix} = 1(-4-0) - 1(-8-0) + 1(2-1) = -4 + 8 + 1 = 5 \neq 0$$

$\therefore$ A^{-1} exists.

Now, $A_{11} = -4 - 0 = -4$

$A_{12} = -(-8 - 0) = 8$

$A_{13} = 2 - 1 = 1$

$A_{21} = -(-4 - 1) = 5$

$A_{22} = -4 - 1 = -5$

$A_{23} = -(1 - 1) = 0$

$A_{31} = 0 - 1 = -1$

$A_{32} = -(0 - 2) = 2$

$A_{33} = 1 - 2 = -1$

$$adj(A) = \begin{bmatrix} -4 & 8 & 1 \\ 5 & -5 & 0 \\ -1 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{5}\begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 10 \\ 13 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -40+65 \\ 80-65 \\ 10 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 25 \\ 15 \\ 10 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$$

$\Rightarrow x = 5, y = 3, z = 2$

32. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.

Ans:

According to question

$x + y + z = 12$

$2x + 3y + 3z = 33$

$x - 2y + z = 0$

The above system of linear equation can be written in matrix form as $AX = B$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{vmatrix}$$

= 1 (3 + 6) –1 (2–3) + 1 (– 4 –3) = 9 + 1 – 7 = 3

$\therefore$ A^{-1} exists.

$A_{11} = 9$, $A_{12} = 1$, $A_{13} = -7$

$A_{21} = -3$, $A_{22} = 0$, $A_{23} = 3$

$A_{31} = 0$, $A_{32} = -1$, $A_{33} = 1$

$$adj(A) = \begin{bmatrix} 9 & 1 & -7 \\ -3 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 108-99 \\ 12+0+0 \\ -84+99 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$\Rightarrow x = 3$, $y = 4$, $z = 5$

No. of awards for honesty = 3

No. of awards for helping others = 4

No. of awards for supervising = 5.

The persons, who work in the field of health and hygiene should also be awarded.

33. Using properties of determinants, prove the following:

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx)$$

Ans:

$$LHS = \begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} x+y+z & -x+y & -x+z \\ x+y+z & 3y & z-y \\ x+y+z & y-z & 3z \end{vmatrix}$$

Taking out $(x + y + z)$ along C_1, we get

$$= (x+y+z)\begin{vmatrix} 1 & -x+y & -x+z \\ 1 & 3y & z-y \\ 1 & y-z & 3z \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$

$$= (x+y+z)\begin{vmatrix} 1 & -x+y & -x+z \\ 0 & 2y+x & x-y \\ 0 & x-z & x+2z \end{vmatrix}$$

Applying $C_2 \to C_2 - C_3$

$$= (x+y+z)\begin{vmatrix} 1 & y-z & -x+z \\ 0 & 3y & x-y \\ 0 & -3z & x+2z \end{vmatrix}$$

Expanding along I column, we get

D = $(x + y + z)[(3y (x + 2z) + 3z (x – y)]$

= $3(x + y + z)[xy + 2z + 2yz + xz – yz]$

= $3(x + y + z)(xy + yz + zx)$ = R.H.S.

34. A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of Rs. **6,000. Three times the award money for Hardwork added to that given for honesty amounts to ` 11,000. The award money given for Honesty and Hardwork together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hardwork, suggest one more value which the school must include for awards.**

Ans:

Let x, y and z be the awarded money for honesty, Regularity and hardwork.

From the statement

$x + y + z = 6000$ …(i)

$x + 3z = 11000$ …(ii)

$x + z = 2y \Rightarrow x – 2y + z = 0$ …(iii)

The above system of three equations may be written in matrix form as $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 0 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 0 & -2 & 1 \end{vmatrix} = 1(0+6) - 1(1-3) + 1(-2-0) = 6+2-2 = 6 \neq 0$$

Hence A^{-1} exist

If Aij is co-factor of aij then

$A_{11} = 0 + 6 = 6$

$A_{12} = – (1 – 3) = 2$;

$A_{13} = (–2 – 0) = –2$

$A_{21} = –(1 + 2) = –3$

$A_{22} = 0$

$A_{23} = (–2 – 1) = –3$

$A_{31} = 3 – 0 = 3$

$A_{32} = – (3 – 1) = – 2$;

$A_{33} = 0 – 1 = – 1$

$$adj(A) = \begin{bmatrix} 6 & 2 & -2 \\ -3 & 0 & 3 \\ 3 & -2 & -1 \end{bmatrix}^T = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 36000-33000+0 \\ 12000+0+0 \\ -12000+33000+0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 3000 \\ 12000 \\ 21000 \end{bmatrix} = \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$$

$\Rightarrow x = 500$, $y = 2000$, $z = 3500$

Except above three values, school must include discipline for award as discipline has great importance in student's life.

35. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$**, then write the value of** x**.**

Ans:

Given that $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

$\Rightarrow (x+1)(x+2) - (x-1)(x-3) = 12 + 1$

$\Rightarrow x^2 + 2x + x + 2 - x^2 + 3x + x - 3 = 13$

$\Rightarrow 7x - 1 = 13$

$\Rightarrow 7x = 14$

$\Rightarrow x = 2$

36. Using properties of determinants, prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

Ans:

$$LHS = \begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$$

$$= \begin{vmatrix} a & a & a+b+c \\ 2a & 3a & 4a+3b+2c \\ 3a & 6a & 10a+6b+3c \end{vmatrix} + \begin{vmatrix} a & b & a+b+c \\ 2a & 2b & 4a+3b+2c \\ 3a & 3b & 10a+6b+3c \end{vmatrix}$$

$$= a^2\begin{vmatrix} 1 & 1 & a+b+c \\ 2 & 3 & 4a+3b+2c \\ 3 & 6 & 10a+6b+3c \end{vmatrix} + ab\begin{vmatrix} 1 & 1 & a+b+c \\ 2 & 2 & 4a+3b+2c \\ 3 & 3 & 10a+6b+3c \end{vmatrix}$$

$$= a^2\begin{vmatrix} 1 & 1 & a+b+c \\ 2 & 3 & 4a+3b+2c \\ 3 & 6 & 10a+6b+3c \end{vmatrix} + ab.0$$ [since $C_1 = C_2$ in second determinant]

$$= a^2\begin{vmatrix} 1 & 1 & a+b+c \\ 2 & 3 & 4a+3b+2c \\ 3 & 6 & 10a+6b+3c \end{vmatrix}$$

$$= a^2\begin{vmatrix} 1 & 1 & a \\ 2 & 3 & 4a \\ 3 & 6 & 10a \end{vmatrix} + \begin{vmatrix} 1 & 1 & b \\ 2 & 3 & 3b \\ 3 & 6 & 6b \end{vmatrix} + \begin{vmatrix} 1 & 1 & c \\ 2 & 3 & 2c \\ 3 & 6 & 3c \end{vmatrix}$$

$$= a^2.a\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 10 \end{vmatrix} + b\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 3 & 6 & 6 \end{vmatrix} + c\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 6 & 3 \end{vmatrix}$$

$$= a^2.a\begin{vmatrix}1 & 1 & 1\\2 & 3 & 4\\3 & 6 & 10\end{vmatrix} + b.0 + c.0$$ [since $C_2 = C_3$ in second determinant and $C_1 = C_3$ in third determinant]

$$= a^3\begin{vmatrix}1 & 1 & 1\\2 & 3 & 4\\3 & 6 & 10\end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$ we get

$$= a^3\begin{vmatrix}1 & 0 & 0\\2 & 1 & 2\\3 & 3 & 7\end{vmatrix}$$

Expanding along R_1 we get

$= a^3 .1(7 - 6) - 0 + 0$

$= a^3$.

37. Using matrices, solve the following system of equations:

$x - y + z = 4; 2x + y - 3z = 0; x + y + z = 2$

Ans:

Given equations

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

We can write this system of equations as AX = B where

$$A = \begin{bmatrix}1 & -1 & 1\\2 & 1 & -3\\1 & 1 & 1\end{bmatrix}, X = \begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B = \begin{bmatrix}4\\0\\2\end{bmatrix}$$

$$Here, |A| = \begin{vmatrix}1 & -1 & 1\\2 & 1 & -3\\1 & 1 & 1\end{vmatrix}$$

= 1 (1 + 3) - (- 1) (2 + 3) + 1 (2 - 1) = 4 + 5 + 1 = 10

$\therefore$ A^{-1} exists.

$A_{11} = 4, A_{12} = -5, A_{13} = 1$

$A_{21} = 2, A_{22} = 0, A_{23} = -2$

$A_{31} = 2, A_{32} = 5, A_{33} = 3$

$$adj(A) = \begin{bmatrix}4 & -5 & 1\\2 & 0 & -2\\2 & 5 & 3\end{bmatrix}^T = \begin{bmatrix}4 & 2 & 2\\-5 & 0 & 5\\1 & -2 & 3\end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{10}\begin{bmatrix}4 & 2 & 2\\-5 & 0 & 5\\1 & -2 & 3\end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{10}\begin{bmatrix}4 & 2 & 2\\-5 & 0 & 5\\1 & -2 & 3\end{bmatrix}\begin{bmatrix}4\\0\\2\end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4-0+6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

The required solution is

$\therefore$ $x = 2$, $y = -1$, $z = 1$

38. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$**, find** $(AB)^{-1}$**.**

Ans:

For B^{-1}

$$|B| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix} = 1(3 - 0) - 2(-1 - 0) - 2(2 - 0) = 3 + 2 - 4 = 1 \neq 0$$

i.e., B is invertible matrix

$\Rightarrow B^{-1}$ exist.

$A_{11} = 3$, $A_{12} = 1$, $A_{13} = 2$

$A_{21} = 2$, $A_{22} = 1$, $A_{23} = 2$

$A_{31} = 6$, $A_{32} = 2$, $A_{33} = 5$

$$adj(B) = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}^T = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{1}{|B|}(adjb) = \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now $(AB)^{-1} = B^{-1}. A^{-1}$

$$\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$