KVS Junior Mathematics Olympiad (JMO)
SAMPLE PAPER – 1

M.M. 100 Time : 3 hours

Note : Attempt all questions.
All questions carry equal marks

Q1. If $\sqrt{a-x}+\sqrt{b-x}+\sqrt{c-x}=0$
Prove that (a + b + c + 3x) (a + b + c - x) = 4(bc + ca + ab)

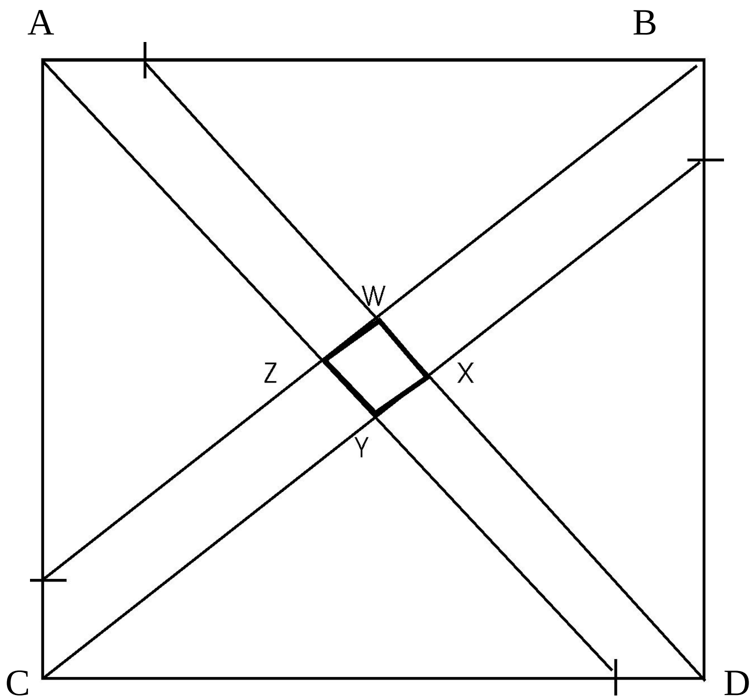
Q2. If $a=\dfrac{xy}{x+y}$, $b=\dfrac{xz}{x+z}$, and $c=\dfrac{yz}{y+z}$
a, b and c are other than zero.
Find the value of x, y, z in terms of a,b and c.

Q3. Two clocks showed correct time at 12 noon. After that one started gaining 40 seconds and other started loosing 50 seconds in every 24 hours. After what interval the difference of time shown by the two clock was 16 minutes ?
What was then the correct time ?

Q4. A triangle has sides of lengths 6, 8 and 10. Find the distance between the center of its inscribed circle and the center of the circumscribed circle.

Q5. A pair of poles are s meters apart and is supported by two cables which run from top of each pole to the bottom of other. The poles 4m and 6m tall. Determine the height of the point T.
What happens to this height if s increases.

Q6. In the figure square ABCD is having unit area. Find the value of ‘a’ such that area of wxyz – 1/2001



Q7. Solve for n :
$100^{1/n} \times 100^{2/n} \times 100^{3/n} \times \ldots\ldots\ldots \times 100^{2003/n} = 1000$

Q8. A right triangle has base and altitude of b and a. A circle of radius r touches the two sides and has its center on the hypotenuse. Show that

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{r}$$

Q9. One goes on spiraling walk on cartesian plane. Starting at (0,0). The first five steps are (1,0) (1,1) (0,1), (-1, 1) and (-1,0). Find the point on 2002nd step.

Q10. Find the ratio of sum of squares of the medians of a triangles to sum of the squares of its sides.

SOLUTION AND HINTS (SAMPLE PAPER 1)

Q1. $\sqrt{a-x}+\sqrt{b-x}+\sqrt{c-x}=0$

$\Rightarrow$ $\sqrt{a-x}+\sqrt{b-x}=-\sqrt{c-x}$

$\Rightarrow$ $(a+b)+(b-x)+2\sqrt{a-x}-\sqrt{b-x}=c-x$

$\Rightarrow$ $a + b - c - x = - 2\ \sqrt{a-x}\sqrt{b-x}$

$\Rightarrow$ $a^2 + b^2 + c^2 + 2ab - 2bc - 2ca - 2x\ (a+b-c) + x^2 = 4\ \{ab - x(a+b) + x^2\}$

$\Rightarrow$ $(a + b + c)^2 + 2x\ (a + b + c\ ) - 3x^2 = 4\ (ab + bc + ca)$

$\Rightarrow$ $(a + b + c + 3x)\ (a + b + c - x) = 4\ (bc + ca + ab)$

Q2. $\frac{xy}{x+y}=a \quad \Rightarrow \frac{1}{a}=\frac{1}{x}+\frac{1}{y}$ (i)

$\frac{xz}{x+z}=b \quad \Rightarrow \frac{1}{b}=\frac{1}{x}+\frac{1}{z}$ (ii)

$\frac{yz}{y+z}=c \quad \Rightarrow \frac{1}{c}=\frac{1}{y}+\frac{1}{z}$ (iii)

Now (i) – (iii)

$$\frac{1}{x}-\frac{1}{z}=\frac{1}{a}-\frac{1}{c}$$

and (ii) + (iv) gives

$$\frac{2}{x}=\frac{1}{a}+\frac{1}{b}-\frac{1}{c}$$

$$\frac{2}{x}=\frac{bc+ca-ab}{abc}$$

$$x=\frac{2abc}{ac+bc-ab}$$

Q3. Two clocks show a difference of 90 secs in 24 hrs.

$\Rightarrow$ $\frac{3}{2}$ minutes in 24 hrs.

$\therefore$ two clocks shows a difference of 16 minutes in 2/3 x 24 x 16 = 256 hrs.

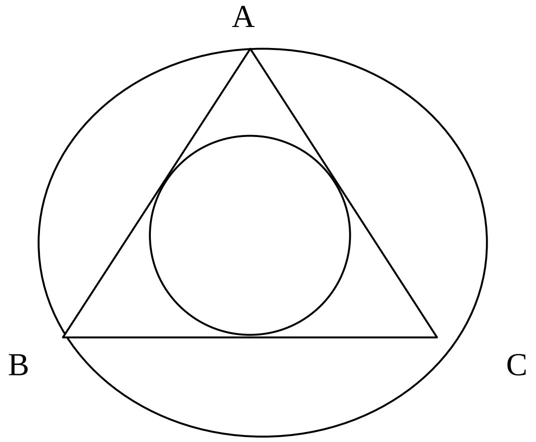
256 hrs = 10 days 16 hrs

so the actual time at the moment

= 4 o'clock in the morning

The correct time was at 12 noon.

Q4.



Let AB = 8
BC = 10
AC = 6
As $AB^2 + AC^2 = BC^2$
$\therefore \angle BAC = 90^o$

So area (ΔABC) = $\frac{1}{2}.AB.AC$

= 24

Let r and R is radii of inscribed and circumscribed circles respectively. d is distance between incentre and circumcentre.

Then, $d^2 = R^2 - 2Rr$ [Formulae]

$r = \frac{\Delta}{2}$

$= \frac{24}{24/2}$

= 2

$4R = \frac{abc}{\Delta}$

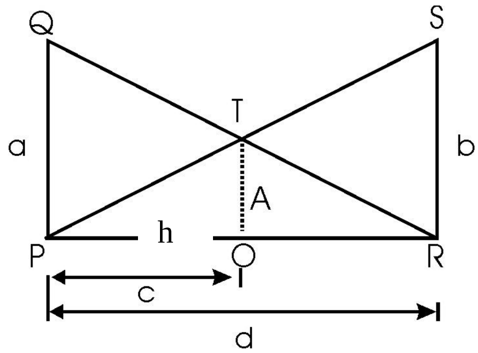
$\Rightarrow$ R = 5

So, $d^2 = 25 - 2 \times 5 \times 2$

= 5

$\Rightarrow d = \sqrt{5}$

Q5.



Since, ΔQPR and ΔTOR are similar,

$$\frac{a}{d} = \frac{h}{d-c}$$

$\Rightarrow$ d-c = $\frac{dh}{a}$ (i)

ΔSRP and ΔTOP are similar,

$$\Rightarrow \frac{b}{d} = \frac{h}{c}$$

$\Rightarrow c = \frac{dh}{b}$ (ii)

so from (i) and (ii) we get,

$$d = dh\left(\frac{1}{a} + \frac{1}{b}\right)$$

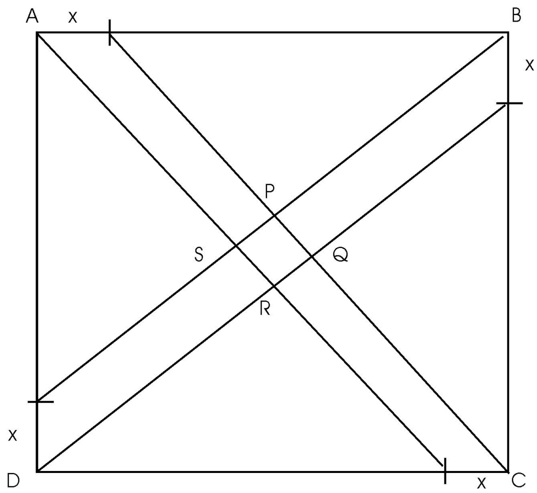
$$\Rightarrow \quad h = \frac{ab}{a+b}$$

Here a=4, b=6 $\therefore$ h = $\frac{12}{5}$

Height of T = $\frac{12}{5}$ meters.

Height h is not dependent on d.

Q6. Let's construct a coordinate plane
A and (1,0)
B (1,1)
C (0,1) and D at (0,0)



Then the slope of DR = 1 – x and slope of AS = $\frac{-1}{1-x}$

Thus DR and AS is $\perp$ and others.
Hence PQRS is a rectangle.
By symmetry PQRS is a square
PQ = x sin (<CBP) Now $\angle$CBP = 90^o - $\angle$PCB.
Let CP meet AB at T. Then 90^o - $\angle$PCB = $\angle$BTC = $\angle$CBP

$\therefore$ Sin $\angle$CBP = sin $\angle$BTC = $\dfrac{1}{\sqrt{x^2 - 2x + 2}}$

then PQ^2 = $\dfrac{x^2}{x^2 - 2x + 2} = \dfrac{1}{200}$

$\Rightarrow$ 2001 x^2 = x^2 – 2x + 2 $\Rightarrow$ x = $\dfrac{\sqrt{4001} - 1}{2000}$

Q7. Since,

$100^{1/n}$ x $100^{2/n}$ x ……..x 100 $^{2003/n}$

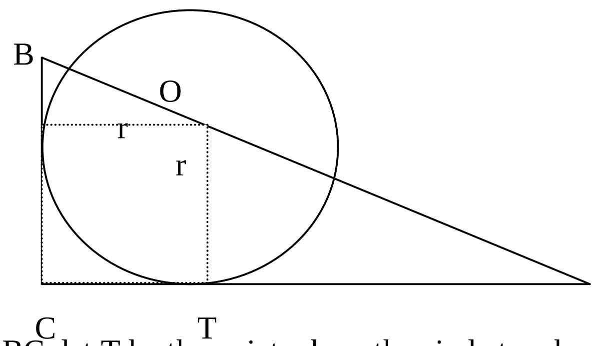
$= (10^2)\ \dfrac{1+2+3+....+2003}{n}$

$\Rightarrow$ $10\ \dfrac{2003.2004}{n}$ = 10^3

$\Rightarrow$ $n\ \dfrac{2003.2004}{3}$

$\Rightarrow$ = 1338004

Q8.

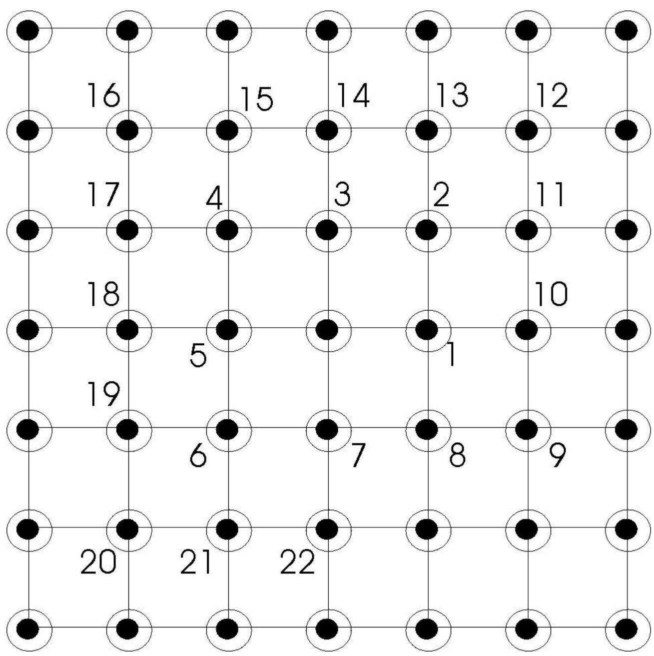


In Δ ABC, let T be the point where the circle touches the side AC.

Since Δ ABC and Δ AOT are similar,

$\Rightarrow \quad \dfrac{a}{b} = \dfrac{r}{b - r} \quad \Rightarrow \quad ab - ar = rb \quad \Rightarrow \quad ab - r = (b + a) \quad \Rightarrow \quad \dfrac{1}{r} = \dfrac{1}{a} + \dfrac{1}{b}$

Q9.



For the figure,
7^{th} step is (0,-1)
22^{th} step is (0,-2)
n^{th} point is (0, - n) at slep no. $(2n+1)^2 - (n+1) = 4n^2 + 3n$
Let us see it
$4n^2 + 3n = 2002$
$\Rightarrow (n-22)(4n+91) = 0$
so 22^{nd} point on y-axis
i.e. (0, -22) is the 2002^{nd} step.

Q10. Find the ratio of sum of squares of the medians of a triangle to sum of the squares of its sides.
We know

$$m_a = \sqrt{\frac{2b^2 + 2c^2 - a^2}{2}}$$

$$\Rightarrow ma^2 = \frac{2b^2 + 2c^2 - a^2}{4}$$

$$mb^2 = \frac{2a^2 + 2c^2 - b^2}{4}$$

$$mc^2 = \frac{2a^2 + 2b^2 - c^2}{4}$$

$$\Rightarrow ma^2 + mb^2 + mc^2 \quad \frac{2b^2 + 2c^2 - a^2 + 2a^2 + 2c^2 - b^2 + 2a^2 + 2b^2 - c^2}{4}$$

$$\Rightarrow ma^2 + mb^2 + mc^2 = \frac{3}{4}(a^2 + b^2 + c^2)$$

$$\Rightarrow \frac{m_a^2 + m_b^2 + m_c^2}{a^2 + b^2 + c^2} = \frac{3}{4}$$