KVS Junior Mathematics Olympiad (JMO)
SAMPLE PAPER – 3

M.M. 100 Time : 3 hours

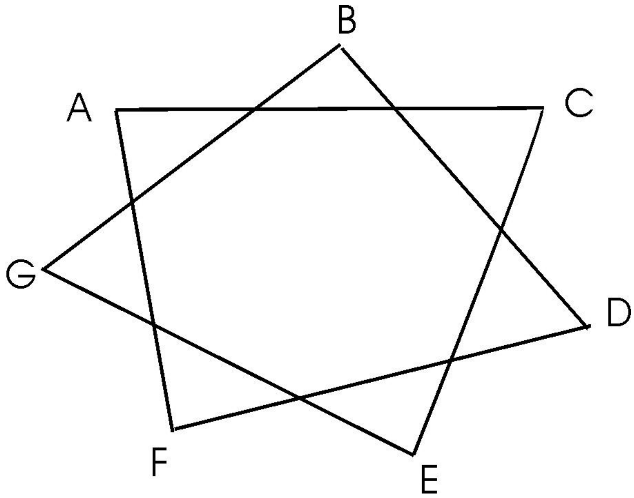
Note : Attempt all questions.

All questions carry equal marks

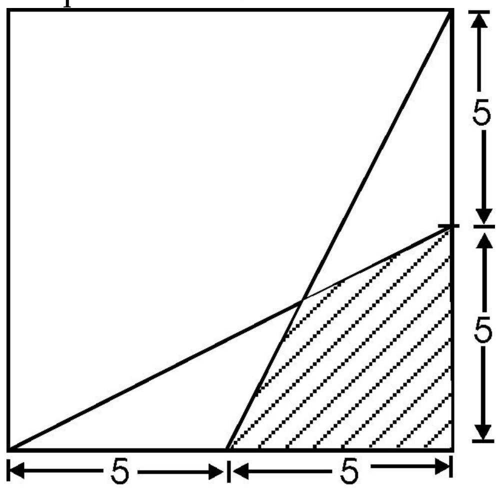
Q1. Solve into factors

$a^3 + b^3 + c^3 - 3abc$

Q2. In the star $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G = ?$



Q3. Find the area of shaded quadrilateral.



Q4. If a polynomial is divided by x-2 and x-2, we obtain remainder 2 and 1 respectively. Find the remainder if it is divided by (x-1) (x-2).

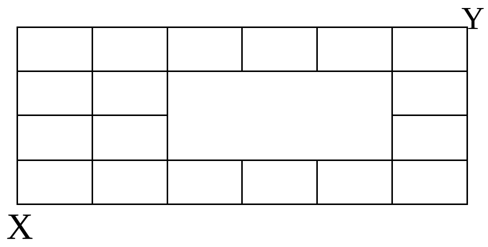
Q5. Find the respective times between 7 and 8 'o' clock, when the hour and minute hands of a watch are

(a) exactly opposite to each other.

(b) at right angles to each other.

Q6. A ladder 25m long is placed so as to reach a road side window 24m high and returning the ladder over to the other side of the road, it reaches a point 7m high. Find the breadth of the road.

Q7. In the street map shown in the diagram, each line segment represents a street which can only a travelled along in either the rightwards or upwards direction. How many paths are there from point X to Y.



Q8. Find the number of zeros that appear at the end in the representation of 158 ! in base of 10.

Q9. The sum of letters in the word MATHS = 2002
Find the base (less than 10) and the number for each letter. Assuming a set of twenty six consecutive non-negative integers represent letter of alphabet.

Q10. If p<q, find p,q. Such that the number 2p6287q is divisible by 11.

SOLUTIONS AND HINT (SAMPLE PAPER 3)

Q1. $a^3 + b^3 + c^3 - 3abc$

Since $b^3 + c^3 = (b + c)^3 – 3bc (b + c)$

So $a^3 + b^3 + c^3 - 3abc$

$= a^3 + \{(b+c)^3 – 3bc (b+c)\} – 3abc$

$= \{a^3 + (b+c)^3 \} – 3bc \{(b+c) + a\}$

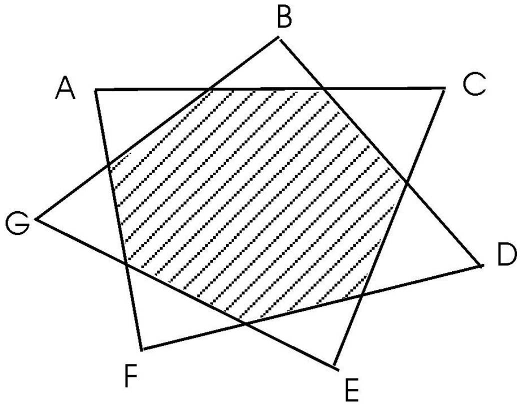
$= \{(a + (b+c)) \} \{a^2 – a(b+c) + (b+c)^2\} –3bc (a+b+c)$

$= \{(a + b + c)\} \{a^2 – ab – ac + b^2 + 2bc + c^2 – 3bc\}$

$= (a + b + c) (a^2 + b^2 + c^2 – bc – ca - ab)$

$= ½ (a + b + c) \{(b-c)^2 + (c-a)^2 + (a-b)^2\}$

Q2. In the figure there are 7 triangles and one heptagon (7-gon). Total of all the angles in triangles = 7 x 180.



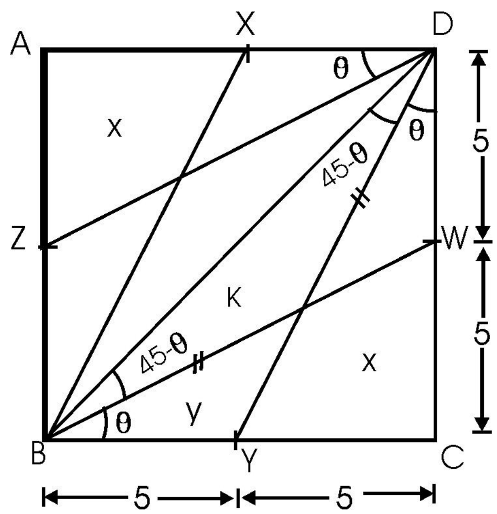
The angles of triangles besides angles

$\angle A, \angle B, \angle C, \angle D, \angle E, \angle F$ and $\angle G$ are the exterior angles of the heptagon. The total value = 2 x 360° (or 4x180°). So, the remaining angles,

$\angle A, \angle B, \angle C, \angle D, \angle E, \angle F$ and $\angle G$ have a sum of 7 x 180° – 4 x 180°

= 540°.

Q3.

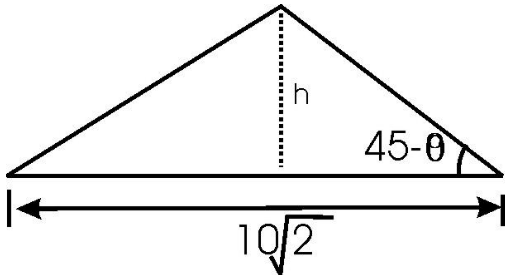


from the figure

area (ΔYDC) = ½ x 5 x 10

area (ΔBWC) = ½ x 5 x 10

let, area (ΔYDC - ΔBKY)
= area x
$\Rightarrow$ 25 – y = x
Further, area (ΔBDY) = ½ x 5 x 10
So area (ΔBDK) = x
So, x = ½ x $10\sqrt{2}$ x height
The height of triangle BDK needs to be calculated



Here tan θ = ½

Tan (45 - θ) = $\frac{h}{5\sqrt{2}}$

$\Rightarrow 5\sqrt{2}$ x tan (45 - θ)

$= 5\sqrt{2}$ x $\frac{\tan 45 - \tan\theta}{1 + \tan 45 \tan\theta}$

$= 5\sqrt{2}$ x $\frac{1}{3} = \frac{5\sqrt{2}}{3}$

So x = ½ x $10\sqrt{2}$ x $\frac{5\sqrt{2}}{3}$

= $\frac{25}{3}$ sq units

Q4. As, on dividing by (x-1) the polynomial leaves a remainder of 2.
So, the polynomial must be, p(x) = k (x-1) + 2 for a real K.
$\Rightarrow$ p(x) = kx – k + 2
Now, dividing p(x) by (x-2)

$$\begin{array}{r} K \\ x-2\overline{)kx-k+2} \\ kx-2k \\ k=2 \end{array}$$

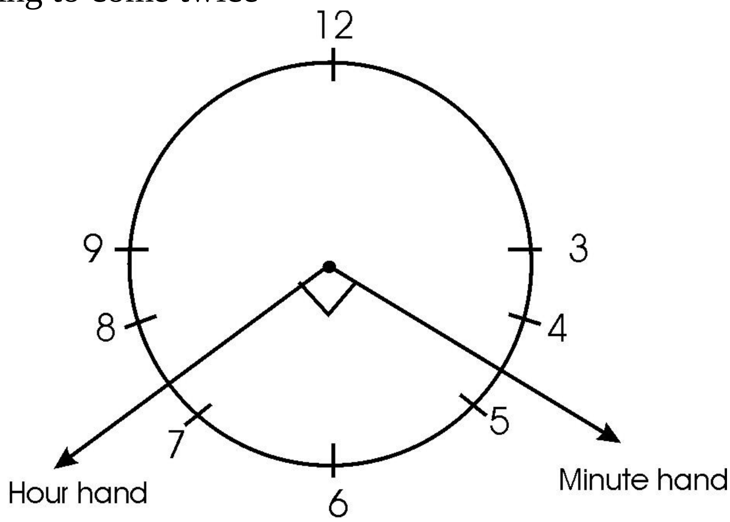
i.e. remainder = k + 2
But, remainder must be 1
$\therefore$ k+2 = 1
$\Rightarrow$ k = -1
$\therefore$ p(x) = (-1) (x-1) + 2
= - x + 3
Now (x-1) (x-2) = x^2 - 3x + 2
Dividing p(x) by x^2 – 3x + 2, we get $x^2 - 3x + 2\overline{)-x+3}$
i.e. the remainder is (-x+3) only
= -x+3

Q5. (a) Let time is x minutes past 7

So $180^o = (x+25-\frac{x}{12}) \times 6$

$\Rightarrow x = 5\frac{5}{11}$

(b) This point is going to come twice



(i) Iin this case say after x minute the situation comes.

Then, for x minutes the degrees deflection from 12 o' clock point = $6x^o$

For x minutes the degree deflection of hour hand from 7o' clock point = $\frac{x^o}{2}$

From question and figure

$6x^o + 90^o = 210^o + \frac{x}{2}$

$\Rightarrow x = 21\frac{9}{11}$ minutes

(ii) In second situation, the situation will come again, say after y minutes then,

$210 + \frac{x}{2} + 90^o = 6x$

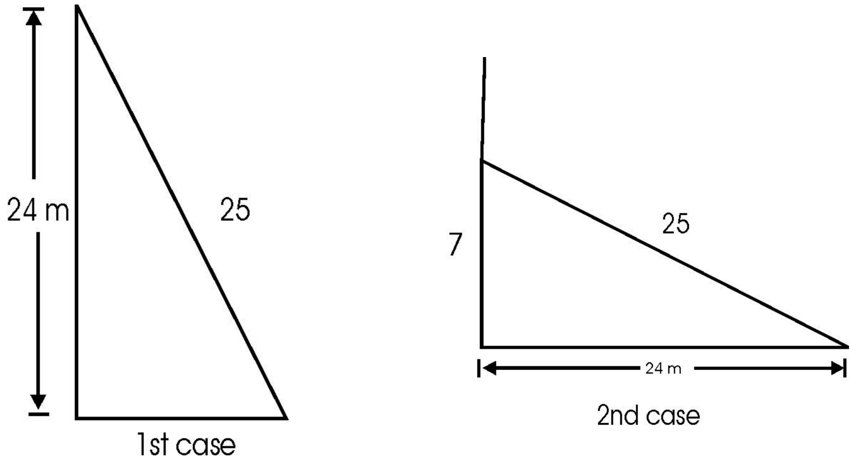
$\Rightarrow x = 54\frac{6}{11}$ minutes

So, hour and minutes hands of watch is going to be at right angle

At 7 past $21\frac{9}{11}$ minutes and

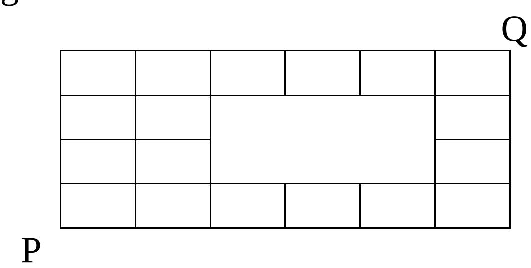
7 past $54\frac{6}{11}$ minutes

Q6. In first case the ladder is on nearer side of road.



In second case ladder is on far side of road so from Pythagoras theorem the base in 1st case = 7m, and
the base in 2nd case = 2m
so road width = 24 – 7 = 17 meters.

Q7. Using Pascal triangles method :



There are 80 paths from x to y

Q8. 158!
For one zero i.e. 10 = 2 x 5
So we need to calculate numbers of 5(s) and 2(s) in 158. Further which ever is lesser, certainly nos of 5 is lesser than 2. The highest power of 5 less than 158 is 125 = 5^2. So, the highest power of 5 that divides 158! is

$$\frac{158}{5}+\frac{158}{25}+\frac{158}{125}=31+6+1=38$$

Similarly, the highest power of 2 that divides 158 !

$$\frac{158}{2}+\frac{158}{4}+\frac{158}{8}+\frac{158}{16}+\frac{158}{32}+\frac{158}{64}+\frac{158}{128}$$

= 79 + 39 + 9 + 9 + 4 + 2 + 1 = 153
So, the highest power of 10 that divides 158 !
= minimum of (153, 38)
= 38
Hence there are 38 zeros at the end of 158 !

Q9. M + A + T + H + S = 2002
$(2002)_3 = 2 \times 2^3 + 2 = 56$
$(2002)_4 = 2 \times 4^3 + 2 = 130$
$(2002)_5 = 2 \times 5^3 + 2 = 252$
$(2002)_6 = 2 \times 6^3 + 2 = 434$

$(2002)_7 = 2 \times 7^3 + 2 = 688$
$(2002)_8 = 2 \times 8^3 + 2 = 1026$
$(2002)_9 = 2 \times 9^3 + 2 = 1460$
MATHS = (A + 12) + A + (A + 19) + (A + 7) + (A + 18) = 5A + 56
The possible form is only
56 and 1026
for integral A.
So, we can have base 3, with A = 0, Base 8 with A = 194.

Q10. Sum of digits at even places = 7 + 2 + p = 9 + p
Sum of digits at odd places = q + 16
For divisibility by 11 the difference should be 0 or divisible by 11. $p < q$
For, and
p = 1 q = 5, {q + 16 - (9 + p)} = 21 – 10 = 11 i.e. divisible by 11
p = 2 q = 6, {q + 16 - (9 + p)} = 22 – 11 = 11 i.e. divisible by 11
p = 3 q = 7, {q + 16 - (9 + p)} = 23 – 12 = 11 i.e. divisible by 11
p = 4 q = 9, {q + 16 - (9 + p)} = 24 – 13 = 11 i.e. divisible by 11
p = 5 q = 9, {q + 16 - (9 + p)} = 11 i.e. divisible by 11
so the possible values of p and q (p<q)
is p = 1, 2, 4, 5
q = 5, 6, 7, 8 and 9