KVS Junior Mathematics Olympiad (JMO)

SAMPLE PAPER – 6

M.M. 100 Time : 3 hours

Note : Attempt all questions.
All questions carry equal marks

Q.1. Factorise :

$$(x-a)^2\ (b-c) + (x-b)^2\ (c-a) + (x-c)^2\ (a-b)$$

Q.2. The sides of a triangle are equal to a,b, and c. Compute the median m_c drawn to the side c.

Q.3. Let a+b+c =1 and $ab + bc + ca = \frac{1}{3}$ where a, b, c are real numbers.

Find the value of

(i) $\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$ (ii) $\frac{a}{b+a} + \frac{b}{c+a} + \frac{c}{a+1}$

Q.4. Reconstruct the following exact long division problem in which the digits indiscriminately replaced by x.

X is any digit

0 to 9 having same or different values.

```
        xx x8 xx
xxx ) xx xx xx xx
      x xx
    ------------
       xxxx
        xxx
    ------------
        xx xx
        xx xx
    ------------
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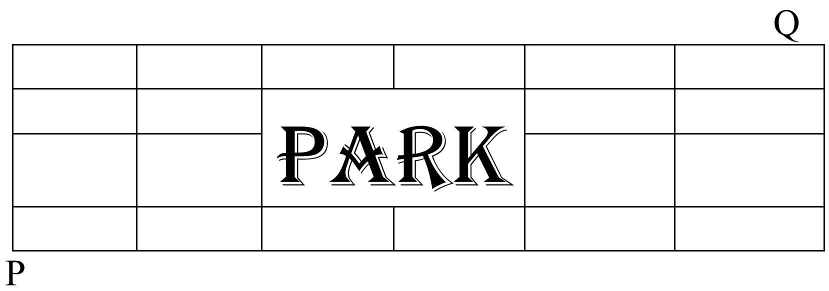
Q. 5. Find the number of digits in 2^{1000}.

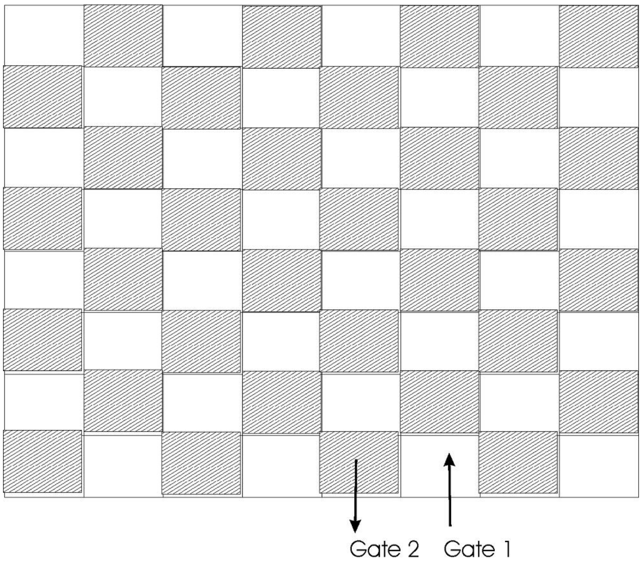
Q. 6. A road map of city is shown in the diagram. The perimeter of the park is a road but there is no road through the park. How many different shortest road routes are there from point P to Q.



Q.7. Evaluate : 1 + (1+2) + (1+2+3) + (1 +2 + 3+ 4) + … upto n terms.

Q.8. You need to find the age of a women. Take double the age and add 4 more than the square root of twice the age. This sum added to its square will be equal to 2162.

Q.9. Below is the figure of a standard chess board. The military enters gate no.1 Marches through all 64 squares. Leave by gate no. 2 making fewest possible number of move. All movers must be like a chess rook's and no square can be visited move than once.



Q.10. The centers of flour circles are situated at the vertices of a square with side a, each radius being equal to a compute the area of the intersection of the circle.

SOLUTION AND HINTS (SAMPLE PAPER 6)

Q.1 $(x-a)^2\ (b-c) + (x-b)^2\ (c-a) + (x-c)^2\ (a-b)$

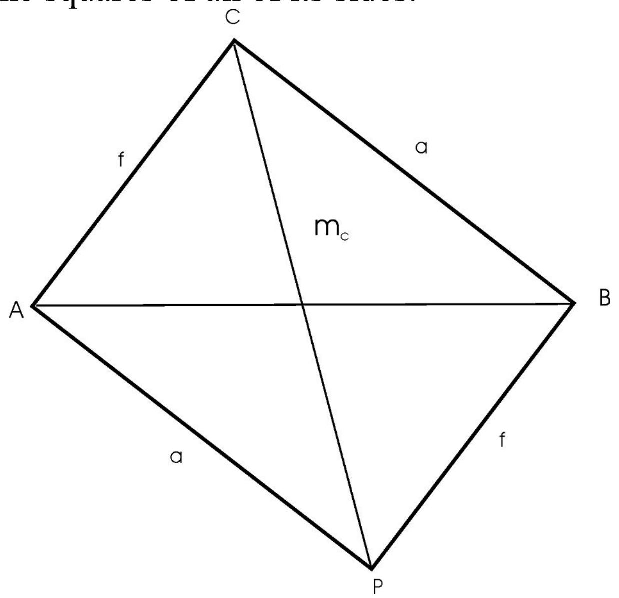
$= \left(x^2-2ax+a^2\right)(b-c) + \left(x^2-2bx+b^2\right)(c-a) + \left(x^2-2cx+c^2\right)(a-b)$

$= x^2\ \{(b-c) + (c-a) + (a-b)\}\ -2x\ \{a(b-c) + b(c-a) + c(a-b)\}$
$\quad + \left\{a^2(b-c) + b^2(c-a) + c^2(a-b)\right\}$

$= x^2 \cdot 0 - 2x \cdot 0 - (b-c)\ (c-a)\ (a-b)$

$= -(b-c)\ (c-a)\ (a-b)$

Q. 2 Let the median mc be double and construct the parallelogram ACBP. For a paralleglogram, we know : The sum of the squares of the diagonals of a parallelogram in equal to the sum of the squares of all of its sides.



We get : $CP^2 + AB^2 = 2AC^2 + 2BC^2$

$\Rightarrow\quad (2m_c)^2 + c^2 = 2b^2 + 2a^2$

$\Rightarrow\quad 4m_c^2 = 2b^2 + 2a^2 - c^2$

$\Rightarrow\quad m_c = \dfrac{\sqrt{2a^2+2b^2-C^2}}{2}$

Q.3 $(a-b)^2 + (b-c)^2 + (c-a)^2$

$= 2\left(a^2 + b^2 + c^2 - ab - bc - ca\right)$

$= 2\left[(\mathrm{a} + \mathrm{b} + \mathrm{c})^2 - 3\ (ab + bc + ca)\right]$

$= 2\left[1 - 3 \cdot \dfrac{1}{3}\right]$

$= 0$

So, a = b, b = c, and c = a

So, a+b+c=1, gives $a = b = c = \frac{1}{3}$

So, $\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$

= 1+1+1

= 3

and $\frac{a}{b+1} + \frac{b}{c+a} + \frac{c}{a+1}$

= $\frac{1/3}{4/3} + \frac{1/3}{4/3} + \frac{1/3}{4/3}$

= $\frac{3}{4}$

Q. 4 The solution is $\sqrt[124]{10020316}$

Q.5. Lets know the power and number of digits.

	No of digit
$2^1 = 2$	1
$2^2 = 4$	1
$2^3 = 8$	1
$2^4 = 16$	2
$2^5 = 32$	2
$2^6 = 64$	2
$2^7 = 128$	2
$2^8 = 256$	3
$2^9 = 512$	3
$2^{10} = 1024$	4
$2^{11} = 2048$	4
$2^{12} = 4096$	4
$2^{13} = 8192$	4
$2^{14} = 16384$	5
$2^{15} = 32768$	5
$2^{16} = 65536$	5
$2^{17} = 131072$	6
$2^{18} = 262144$	6
$2^{19} = 524288$	6
$2^{20} = 1048576$	7
$2^{21} = 2097152$	7
$2^{22} = 4194304$	7
$2^{23} = 838868$	7
$2^{24} = 16777216$	8
$2^{25} = 33554432$	8
$2^{26} = 67108864$	8
2^{27} = 134xxxxxx	9
2^{28} = 268 xx xx xx	9

2^{29}	=	536 xx xx xx	9
2^{30}	=	1073 xx xx xx	10
2^{31}	=	2147 xx xx xx	10
2^{32}	=	4294 xx xx xx	10
2^{33}	=	8589 xx xx xx	10
2^{34}	=	17179 xx xx xx	11
2^{35}	=	34315 xx xx xx	11
2^{36}	=	68631 xx xx xx	11
2^{37}	=	137263 xx xx xx	12
2^{38}	=	27 xx xx xx xx xx	12
2^{39}	=	54 xx xx xx xx xx	12
2^{40}	=	108 xx xx xx xx xx	13
2^{41}	=	216 xx xx xx xx xx	13
2^{42}	=	432 xx xx xx xx xx	13
2^{43}	=	864 xx xx xx xx xx	13
2^{44}	=	1728 xx xx xx xx xx	14

The pattern

Power	1	4	8	12	16
No. of digit	1	2	3	4	5

Power	20	24	28	32	36
No. of digit	7	8	9	10	11

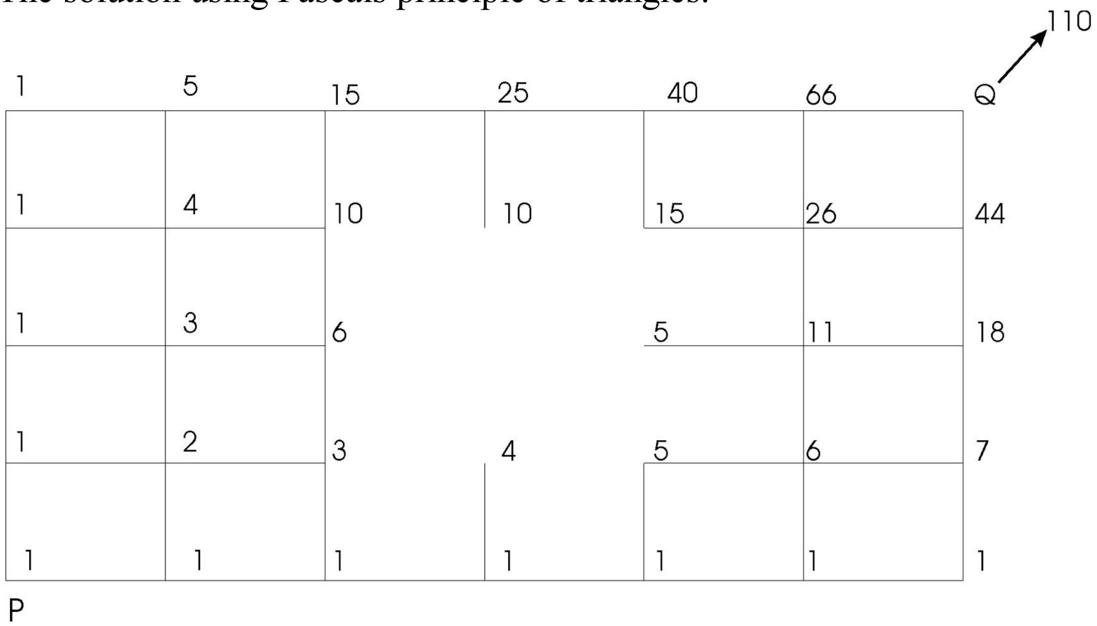
Power	40	44	48	52	56
No. of digit	13	14	15	16	17

Power	60	64	68	72	76
No. of digit	19	20	21	22	23

Power	1000	1004	1008	1012	1016
No. of digit	301	302	303	304	305

So 2^{1000} have 301 digits.

Q. 6 The solution using Pascals principle of triangles.



So numbers of routes – 110.

Q.7 We know

$$1 + 2 + 3 + 4 + ... + n = \frac{n(n+1)}{2}$$

The nth term of series

$$= \frac{n\ (n+1)}{2}$$

$$= \frac{n^2}{2} + \frac{n}{2}$$

Now $t_2 = \dfrac{n^2}{2} + \dfrac{n}{2}$

$$t_1 = \frac{1^2}{2} + \frac{1}{2}$$

$$t_2 = \frac{2^2}{2} + \frac{2}{2}$$

$$t_3 = \frac{3^2}{2} + \frac{3}{2}$$

$$t_4 = \frac{4^2}{2} + \frac{4}{2}$$

… … …

$$t_n = \frac{n^2}{2} + \frac{n}{2}$$

$$S = \frac{1}{2}\left(1^2 + 2^2 + 3^2 + 4^4 + \cdot\cdot + n^2\right) + \frac{1}{2}\left(1 + 2 + 3 + 4 + \cdots + n\right)$$

$= \frac{1}{2} \times \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \times \frac{n(n+1)}{2}$

$= \frac{n(n+1)}{2} \left[\frac{2n+1}{3} + 1\right]$

$= \frac{n(n+1)}{2} \frac{2n+4}{3}$

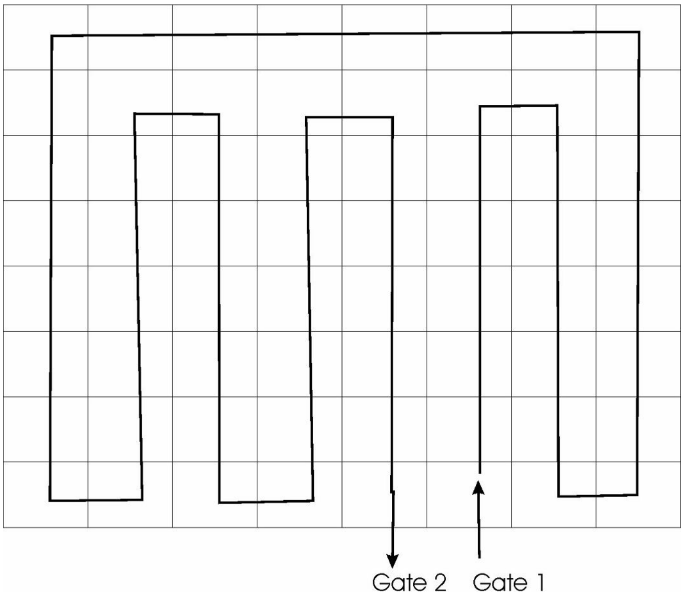
$= \frac{n\ (n+1)\ (n+2)}{6}$

Q.8 Let the age of woman = x From the question :

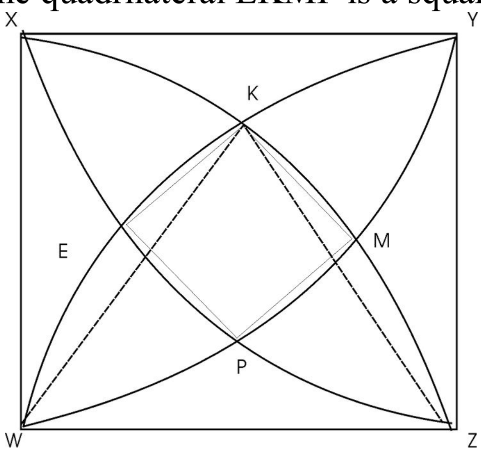
$2x + \sqrt{2x} + 4 + (2x + \sqrt{2x} + 4)^2 = 216_2$

it can be solved to get x = 18

Q.9 Solution :



Q. 10 From the symmetry the quadrilateral EKMP is a square



So, the desired are to be calculated in square EKMP plus four equal segments

ΔWKZ is equilateral,

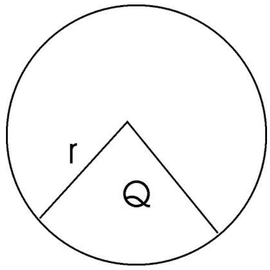
i.e. $\angle$ KWZ = 60^0

$\therefore$ $\angle$ XWK = 33^0

$\angle$MWZ = 33^0

So $\angle$KWM = 330

We have, area of the segment



$S = \frac{1}{2} r^2 \left(\theta - \sin\theta\right)$

θ is measured in radians $\left[\pi \quad readian = 180^0\right]$

So, area of the segment = $\frac{1}{2} a^2 \left(\frac{\pi}{6} - \frac{1}{2}\right)$ …(1)

For Δ WKM, by cosines law :

$KM^2 = WK^2 + WM^2 - 2WK \cdot WM \cdot Cos\, 30^0$

$\Rightarrow \quad KM^2 = a^2 + a^2 - 2a^2 \cdot \frac{\sqrt{3}}{2}$

$= a^2 \left(2 - \sqrt{3}\right)$

So, the desired area = Area of square + 4 x area of segment

$= a^2 \left(2 - \sqrt{3}\right) + 4 \ x \ \frac{1}{2} a^2 \left(\frac{\pi}{6} - \frac{1}{2}\right)$

$= a^2 \left(1 + \frac{\pi}{3} - \sqrt{3}\right)$

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