CHAPTER – 5: CONTINUITY AND DIFFERENTIABILITY

MARKS WEIGHTAGE – 10 marks

Previous Years Board Exam (Important Questions)

1. For what value of *k* is the following function continuous at *x* = 2?

$$f(x)=\begin{cases}2x+1 \;;\; x<2\\ k \;;\; x=2\\ 3x-1 \;;\; x>2\end{cases}$$

2. Discuss the continuity of the following function at *x* = 0 : $f(x)=\begin{cases}\dfrac{x^4+2x^3+x^2}{\tan^{-1}x}, & x\neq 0\\ 0, & x=0\end{cases}$

3. If the function *f* (*x*) given by $f(x)=\begin{cases}3ax+b \;;\; x>1\\ 11 \;;\; x=1\\ 5ax-2b \;;\; x<1\end{cases}$ is continuous at *x* =1, find the values of *a* and *b*.

4. Find the relationship between '*a*' and '*b*' so that the function '*f*' defined by:

$f(x)=\begin{cases}ax+1, & x\le 3\\ bx+3, & x>3\end{cases}$ is continuous at *x* = 3 .

5. Show that the function *f*(*x*) = |*x* – 3| , $x\in R$, is continuous but not differentiable at *x* = 3.

6. Find the value of *k*, for which $f(x)=\begin{cases}\dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & -1\le x<0\\ \dfrac{2x+1}{x-1}, & 0\le x<1\end{cases}$ is continuous at *x* = 0.

7. Find the value of *k* so that the function *f*, defined by $f(x)=\begin{cases}kx+1, & x\le\pi\\ \cos x, & x>\pi\end{cases}$ is continuous at *x* = π.

8. Find the value of '*a*' for which the function *f* defined as

$f(x)=\begin{cases}a\sin\dfrac{\pi}{2}(x+1), & x\le 0\\ \dfrac{\tan x-\sin x}{x^3}, & x>0\end{cases}$ is continuous at *x* = 0.

9. Find all points of discontinuity of *f*, where *f* is defined as follows :

$$f(x)=\begin{cases}|x|+3 \;;\; x\le -3\\ -2x \;;\; -3<x<3\\ 6x+2 \;;\; x\ge 3\end{cases}$$

10. Show that the function *f* defined as follows, is continuous at *x* = 2, but not differentiable:

$$f(x)=\begin{cases}3x-2 \;;\; 0<x\le 1\\ 2x^2-x;\; 1<x\le 2\\ 5x-4 \;;\; x>2\end{cases}$$

11. Verify Lagrange's mean value theorem for the following function: $f(x) = x^2 + 2x + 3$, for [4, 6].

12. If $f(x)=\sqrt{\dfrac{\sec x-1}{\sec x+1}}$, then f'(x). Also find $f'\left(\dfrac{\pi}{2}\right)$.

13. Find $\dfrac{dy}{dx}$,if $(x^2+y^2)^2=xy$.

14. If sin y = x sin(a + y), prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

15. If (cos x)^y = (sin y)^x, find $\frac{dy}{dx}$.

16. If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, then prove that $(1-x^2)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=0$.

17. If $y = e^x(\sin x + \cos x)$, then show that $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$.

18. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\frac{d^2y}{dx^2}$.

19. If $\log(x^2+y^2)=2\tan^{-1}\left(\frac{y}{x}\right)$, then show that $\frac{dy}{dx}=\frac{x+y}{x-y}$

20. If $y = a \cos(\log x) + b \sin(\log x)$, then show that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$

21. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then show that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

22. Find $\frac{dy}{dx}$,if $y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]$

23. Find $\frac{dy}{dx}$,if $y=(\cos x)^x+(\sin x)^{1/x}$

24. Differentiate the following with respect to x : $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

25. If $y=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$, find $\frac{dy}{dx}$

26. Differentiate the following function w.r.t. x : $(x)^{\cos x} + (\sin x)^{\tan x}$

27. If $y=e^{a\sin^{-1}x}, -1\le x\le 1$, then show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$

28. If $y=\cos^{-1}\left(\frac{3x+4\sqrt{1-x^2}}{5}\right)$, find $\frac{dy}{dx}$

29. If $y = \text{cosec}^{-1} x$, $x > 1$. then show that $x(x^2-1)\frac{d^2y}{dx^2}+(2x^2-1)\frac{dy}{dx}=0$.

30. If $y=\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$ then show that $\frac{dy}{dx}=\sec x$. Also find $\frac{d^2y}{dx^2}$ at $x=\frac{\pi}{4}$.

31. Differentiate the following function with respect to x: $f(x)=\tan^{-1}\left(\frac{1-x}{1+x}\right)-\tan^{-1}\left(\frac{x+2}{1-2x}\right)$

32. If $x=a\left(\cos t+\log\tan\frac{t}{2}\right), y=a(1+\sin t)$, then find $\frac{d^2y}{dx^2}$.

33. If $x=a(\theta-\sin\theta), y=a(1+\cos\theta)$, then find $\frac{d^2y}{dx^2}$.

34. Differentiate $x^{\cos x}+\frac{x^2+1}{x^2-1}$ w.r.t.x

35. If $x^y=e^{x-y}$, then show that $\frac{dy}{dx}=\frac{\log x}{[\log(xe)]^2}$

36. If $x = \tan\left(\frac{1}{a}\log y\right)$, then show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$.

37. Prove that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right] = \sqrt{a^2-x^2}$

38. If $y = \log\left[x+\sqrt{x^2+1}\right]$ then show that $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

39. If $y = \log\left[x+\sqrt{x^2+a^2}\right]$ then show that $(x^2+a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

40. If $y = \sin^{-1} x$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

41. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}-1}{x}\right)$ w.r.t. x.

42. If $x = a$ (cos t + t sin t) and $y = a$ (sin t – t cos t), $0 < t < \frac{\pi}{2}$, find $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2y}{dx^2}$.

43. If $x^m y^m = (x+y)^{m+n}$, then show that $\frac{dy}{dx} = \frac{y}{x}$.

44. If $x^{16}y^9 = (x^2+y)^{17}$, then show that $\frac{dy}{dx} = \frac{2y}{x}$.

45. If $x = a\sin t, y = a\left(\cos t + \log\tan\frac{t}{2}\right)$, then find $\frac{d^2y}{dx^2}$.

46. If $y^x = e^{y-x}$, then show that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$.

47. If $x^y = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

48. Differentiate the following with respect to x : $\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$

49. If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

50. If $x \sin(a+y) + \sin a \cos(a+y) = 0$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

51. If $y = \sin(\log x)$, then prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

52. Show that the function $f(x) = 2x - |x|$ is continuous but not differentiable at $x = 0$.

53. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$.

54. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

55. If $y = x^x$, then show that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$