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<th>Short answer (2 marks)</th>
<th>Long answer - I (4 marks)</th>
<th>Long answer - II (6 marks)</th>
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KENDRIYA VIDYALAYA SANGATHAN, HYDERABAD REGION
SAMPLE PAPER – 01 (2017-18)

SUBJECT: MATHEMATICS
MAX. MARKS : 100
CLASS : XII
DURATION : 3 HRS

General Instruction:
(i) All questions are compulsory.
(ii) This question paper contains 29 questions.
(iii) Question 1-4 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
(v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
(vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION – A
Questions 1 to 4 carry 1 mark each.

1. If the binary operation * on the set of integers Z, is defined by \( a * b = a + 3b^2 \), then find the value of \( 2 * 4 \).
2. If \( f : R \rightarrow R \) be defined by \( f(x) = (3 - x^3)^{1/3} \), then find \( fof(x) \).
3. If A is a 3 \times 3 matrix, \( |A| \neq 0 \) and \( |3A| = k |A| \), then write the value of \( k \).
4. Find a vector in the direction of vector \( \vec{2i} - 3\vec{j} + 6\vec{k} \) which has magnitude 21 units.

SECTION – B
Questions 5 to 12 carry 2 marks each.

5. Prove that \( \tan^{-1}(\sqrt[3]{\frac{1}{x}}) = \frac{1}{2} \cos^{-1}\left(\frac{1-x}{1+x}\right), x \in (0,1) \)
6. If \( \begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix} \), find the value of \( x + y \).
7. If \( y = \cot^{-1}\left(\frac{1-\sin x}{1+\sin x}\right) \), find \( \frac{dy}{dx} \).
8. Evaluate: \( \int xe^x \frac{1}{(1+x)^2} \, dx \)
9. Find the approximate change in the volume \( V \) of a cube of side \( x \) meters caused by increasing the side by 2%.
10. If \( \vec{a} \) and \( \vec{b} \) are perpendicular vectors, \( |\vec{a} + \vec{b}| = 13 \) and \( |\vec{a}| = 5 \) find the value of \( |\vec{b}| \).
11. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that the youngest is a girl?
12. Form the differential equation representing the family of curves \( y = a \sin(x + b) \), where \( a, b \) are arbitrary constants.

SECTION – C
Questions 13 to 23 carry 4 marks each.

13. Evaluate: \( \int \frac{\cos x}{(1 - \sin x)(2 - \sin x)} \, dx \)
14. Determine the value of k so that the function
\[ f(x) = \begin{cases} 
   kx^2, & \text{if } x \leq 2; \\
   3, & \text{if } x > 2
\end{cases} \]
is continuous.

**OR**
Discuss the differentiability of the function \[ f(x) = \begin{cases} 
   1 + x, & \text{if } x \leq 2; \\
   5 - x, & \text{if } x > 2
\end{cases} \] at \( x = 2 \).

15. Show that the matrix \( A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \) satisfies the equation \( A^2 - 4A + I = O \), where \( I \) is a 2 × 2 identity matrix and \( O \) is a 2 × 2 zero matrix. Using this equation, find \( A^{-1} \).

16. An experiment succeeds thrice as often as it fails. Find the probability that in the next five trials, there will be at least 3 successes.

17. Solve the differential equation \( (\tan^{-1} y - x) \ dy = (1 + y^2) \ dx \).

**OR**
Find the particular solution of the differential equation \( \frac{dx}{dy} + y \cot x = 2x + x^2 \cot x, x \neq 0 \) given that \( y = 0 \) when \( x = \frac{\pi}{2} \).

18. The probabilities of two students A and B coming to the school in time are \( \frac{3}{7} \) and \( \frac{5}{7} \) respectively. Assuming that the events, ‘A coming in time’ and ‘B coming in time’ are independent, find the probability of only one of them coming to the school in time. Write at least one advantage of coming to school in time.

19. If \( x = 3 \sin t - \sin 3t, y = 3 \cos t - \cos 3t \), find \( \frac{d^2y}{dx^2} \) at \( t = \frac{\pi}{3} \).

20. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

21. Find the angle between the line \( \frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6} \) and the plane \( 10x + 2y - 11z = 3 \).

22. Prove that, \( [\vec{a} + \vec{b} + \vec{c} + \vec{d}] = 2[\vec{a} \vec{b} \vec{c}] \) for any three vectors \( \vec{a}, \vec{b}, \vec{c} \).

23. Find the equation of the normals to the curve \( y = x^3 + 2x + 6 \) which are parallel to the line \( x + 14y + 4 = 0 \).

**OR**
Find the intervals in which the function \( f \) given by \( f(x) = \sin x + \cos x, \ 0 \leq x \leq 2\pi \) is strictly increasing or strictly decreasing.

**SECTION – D**
Questions 24 to 29 carry 6 marks each.

24. Let \( A = \{1, 2, 3, \ldots, 9\} \) and \( R \) be the relation in \( A \times A \) defined by \( (a, b) R (c, d) \) if \( a + d = b + c \) for \( (a, b), (c, d) \) in \( A \times A \). Prove that \( R \) is an equivalence relation. Also obtain the equivalence class \([2, 5] \).
OR

Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \frac{x-2}{x-3}$.

Show that $f$ is one-one and onto and hence find $f^{-1}$.

25. Using properties of determinants, prove that

$$
\begin{vmatrix}
a & a+b & a+b+c \\
2a & 3a+2b & 4a+3b+2c \\
3a & 6a+3b & 10a+6b+3c \\
\end{vmatrix} = a^3
$$

OR

Show that

$$
\Delta = \begin{vmatrix}
(y+z)^2 & xy & zx \\
xy & (x+z)^2 & yz \\
xz & yz & (x+y)^2 \\
\end{vmatrix} = 2xyz(x+y+z)^3
$$

26. Evaluate:

$$
\int_{0}^{\pi/2} \frac{x \sin x}{1 + \cos^2 x} \, dx
$$

OR

Find $\int_{0}^{2} (x^2 + 1) \, dx$ as the limit of a sum.

27. Find the area of the region enclosed between the two circles: $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

28. Find the equation of the plane which contains the line of intersection of the planes

$\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$, $\vec{r}.(2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.(5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

29. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs. 300 and that on a chain is Rs 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an L.P.P. and solve it graphically.