CHAPTER – 5: CONTINUITY AND DIFFERENTIABILITY

MARKS WEIGHTAGE – 10 marks

NCERT Important Questions & Answers

1. Find all points of discontinuity of *f*, where *f* is defined by $f(x)=\begin{cases}2x+3, & if \quad x\le 2\\ 2x-3, & if \quad x>2\end{cases}$.

Ans.

Here, $f(x)=\begin{cases}2x+3, & if \quad x\le 2\\ 2x-3, & if \quad x>2\end{cases}$.

At x =2, $LHL=\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-}(2x+3)$

Putting $x = 2 - h$ has $x \to 2^-$ when $h \to 0$

$\therefore \lim_{x\to 2^-} f(x)=\lim_{h\to 0}(2(2-h)+3)=\lim_{h\to 0}(4-2h+3)=\lim_{h\to 0}(7-2h)=7$

At x =2, $RHL=\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+}(2x-3)$

Putting $x = 2 + h$ as $x \to 2^+$ when $h \to 0$

$\therefore \lim_{x\to 2^+} f(x)=\lim_{h\to 0}(2(2+h)-3)=\lim_{h\to 0}(4+2h-3)=\lim_{h\to 0}(1+2h)=1$

$\therefore$ LHL $\neq$ RHL. Thus, $f(x)$ is discontinuous at $x = 2$.

2. Find all points of discontinuity of *f*, where *f* is defined by $f(x)=\begin{cases}\frac{|x|}{x}, & if \quad x\neq 0\\ 0, & if \quad x=0\end{cases}$

Ans. Here, $f(x)=\begin{cases}\frac{|x|}{x}, & if \quad x\neq 0\\ 0, & if \quad x=0\end{cases}$

$LHL=\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\frac{|x|}{x}$

Putting $x = 0 - h$ as $x \to 0^-$ when $h \to 0$

$\therefore \lim_{x\to 0^-} f(x)=\lim_{h\to 0}\frac{|0-h|}{0-h}=\lim_{h\to 0}\frac{h}{-h}=-1$

$RHL=\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{|x|}{x}$

Putting $x = 0 + h$ as $x \to 0^+$; $h \to 0$

$\therefore \lim_{x\to 0^+} f(x)=\lim_{h\to 0}\frac{|0+h|}{0+h}=\lim_{h\to 0}\frac{h}{h}=1$

$\therefore$ LHL $\neq$ RHL. Thus, $f(x)$ is discontinuous at $x = 0$.

3. Find all points of discontinuity of *f*, where *f* is defined by $f(x)=\begin{cases}x^3-3, & if \quad x\le 2\\ x^2+1, & if \quad x>2\end{cases}$

Ans.

For $x < 2$, $f(x) = x^3 - 3$ and for $x > 2$, $f(x) = x^2 + 1$ is a polynomial function, so f is continuous in the above interval. Therefore, we have to check the continuity at $x = 2$.

$$LHL = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x^3 - 3)$$

Putting $x = 2 - h$ has $x \to 2^-$ when $h \to 0$

$$\therefore \lim_{x \to 2^-} f(x) = \lim_{h \to 0} ((2-h)^3 - 3) = \lim_{h \to 0} (8 - 12h + 6h^2 - h^3 - 3)$$

$$= \lim_{h \to 0} (5 - 12h + 6h^2 - h^3) = 5$$

$$RHL = \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2 + 1)$$

Putting $x = 2 + h$ as $x \to 2^+$ when $h \to 0$

$$\therefore \lim_{x \to 2^+} f(x) = \lim_{h \to 0} ((2+h)^2 + 1) = \lim_{h \to 0} (4 + 4h + h^2 + 1)$$

$$= \lim_{h \to 0} (5 + 4h + h^2) = 5$$

Also, $f(2) = (2)^3 - 3 = 8 - 3 = 5$ [since $f(x) = x^3 - 3$]

$\therefore$ LHL = RHL =f(2) . Thus, $f(x)$ is continuous at $x = 2$.

Hence, there is no point of discontinuity for this function $f(x)$.

4. Find the relationship between *a* and *b* so that the function *f* defined by $f(x) = \begin{cases} ax + 1, & if \quad x \le 3 \\ bx + 3, & if \quad x > 3 \end{cases}$ **is continuous at *x* = 3.**

Ans.

Here, $f(x) = \begin{cases} ax + 1, & if \quad x \le 3 \\ bx + 3, & if \quad x > 3 \end{cases}$

$$LHL = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (ax + 1)$$

Putting $x = 3 - h$ has $x \to 3^-$ when $h \to 0$

$$\therefore \lim_{x \to 3^-} f(x) = \lim_{h \to 0} (a(3-h) + 1) = \lim_{h \to 0} (3a - ah + 1) = 3a + 1$$

$$RHL = \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (bx + 3)$$

Putting $x = 3 + h$ as $x \to 3^+$ when $h \to 0$

$$\therefore \lim_{x \to 3^+} f(x) = \lim_{h \to 0} (b(3+h) + 3) = \lim_{h \to 0} (3b + bh + 3) = 3b + 3$$

Also, f(3) =3 a +1 [since $f(x) = ax + 1$]

Since, $f(x)$ is continuous at $x = 3$.

$\therefore$ LHL = RHL = f(3)

$$\Rightarrow 3a + 1 = 3b + 3 \Rightarrow 3a = 3b + 2 \Rightarrow a = b + \frac{2}{3}$$

5. For what value of λ is the function defined by $f(x) = \begin{cases} \lambda(x^2 - 2x), & if \quad x \le 0 \\ 4x + 1, & if \quad x > 0 \end{cases}$ **continuous at *x* = 0? What about continuity at *x* = 1?**

Ans.

Here, $f(x) = \begin{cases} \lambda(x^2 - 2x), & if \quad x \le 0 \\ 4x + 1, & if \quad x > 0 \end{cases}$

At x = 0, $LHL = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \lambda(x^2 - 2x)$

Putting $x = 0 - h$ as $x \to 0^-$ when $h \to 0$

$\therefore \lim_{x \to 0^-} f(x) = \lim_{h \to 0} \lambda[(0-h)^2 - 2(0-h)] = \lim_{h \to 0} \lambda(h^2 + 2h) = 0$

At x = 0, $RHL = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (4x+1)$

Putting $x = 0 + h$ as $x \to 0^+$; $h \to 0$

$\therefore \lim_{x \to 0^+} f(x) = \lim_{h \to 0} [4(0+h)+1] = \lim_{h \to 0} (4h+1) = 1$

$\therefore$ LHL ≠ RHL. Thus, $f(x)$ is discontinuous at $x = 0$ for any value of λ.

At x = 1, $LHL = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (4x+1)$

Putting $x = 1 - h$ as $x \to 1^-$ when $h \to 0$

$\therefore \lim_{x \to 1^-} f(x) = \lim_{h \to 0} (4(1+h)+1) = \lim_{h \to 0} (4+4h+1) = 5$

At x = 1, $RHL = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (4x+1)$

Putting $x = 1 + h$ as $x \to 1^+$; $h \to 0$

$\therefore \lim_{x \to 1^+} f(x) = \lim_{h \to 0} [4(1+h)+1] = \lim_{h \to 0} (4+4h+1) = 5$

$\therefore$ LHL = RHL. Thus, $f(x)$ is continuous at $x = 1$ for any value of λ.

6. Find all points of discontinuity of *f*, where $f(x) = \begin{cases} \frac{\sin x}{x}, & if \quad x < 0 \\ x+1, & if \quad x \geq 0 \end{cases}$

Ans. Here, $f(x) = \begin{cases} \frac{\sin x}{x}, & if \quad x < 0 \\ x+1, & if \quad x \geq 0 \end{cases}$

At x = 0, $LHL = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin x}{x}$

Putting $x = 0 - h$ as $x \to 0^-$ when $h \to 0$

$\therefore \lim_{x \to 0^-} f(x) = \lim_{h \to 0} \frac{\sin(0-h)}{0-h} = \lim_{h \to 0} \frac{\sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin h}{-h} = \lim_{h \to 0} \frac{\sin h}{h} = 1$

At x = 0, $RHL = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\sin x}{x}$

Putting $x = 0 + h$ as $x \to 0^+$; $h \to 0$

$\therefore \lim_{x \to 0^+} f(x) = \lim_{h \to 0} \frac{\sin(0+h)}{0+h} = \lim_{h \to 0} \frac{\sin h}{h} = 1$

Also, f(0) = 0 + 1 = 1

$\therefore$ LHL = RHL = f(0). Thus, $f(x)$ is continuous at $x = 0$.

When $x < 0$, sin x and x both are continuous. Therefore, $\frac{\sin x}{x}$ is also continuous.

When $x > 0$, $f(x) = x + 1$ is a polynomial. Therefore f is continuous.

Hence, there is no point of discontinuity for this function $f(x)$.

7. Determine if *f* defined by $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & if \quad x\neq 0 \\ 0, & if \quad x=0 \end{cases}$ **is a continuous function?**

Ans.

Here, $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & if \quad x\neq 0 \\ 0, & if \quad x=0 \end{cases}$

At x = 0, $LHL=\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-} x^2\sin\frac{1}{x}$

Putting $x = 0 - h$ as $x \to 0^-$ when $h \to 0$

$\therefore \lim_{x\to 0^-} f(x)=\lim_{h\to 0}(0-h)^2\sin\frac{1}{0-h}=\lim_{h\to 0}\left(-h^2\sin\frac{1}{h}\right)=-0\times\sin\infty$

= 0 x value between –1 and 1 (since $-1\le\sin x\le 1$, for all values of $x\in R$)

At x = 0, $RHL=\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+} x^2\sin\frac{1}{x}$

Putting $x = 0 + h$ as $x\to 0^+$; $h\to 0$

$\therefore \lim_{x\to 0^+} f(x)=\lim_{h\to 0}(0+h)^2\sin\frac{1}{0+h}=\lim_{h\to 0}\left(h^2\sin\frac{1}{h}\right)=0\times\sin\infty$

= 0 x value between –1 and 1 (since $-1\le\sin x\le 1$, for all values of $x\in R$)

$\therefore$ LHL = RHL = f(0). Thus, *f*(*x*) is continuous at $x = 0$.

8. Find the values of *k* so that the function *f* $f(x)=\begin{cases} \frac{k\cos x}{\pi-2x}, & if \quad x\neq\frac{\pi}{2} \\ 3, & if \quad x=\frac{\pi}{2} \end{cases}$ **is continuous at point** $x=\frac{\pi}{2}$

Ans.

Here, $f(x)=\begin{cases} \frac{k\cos x}{\pi-2x}, & if \quad x\neq\frac{\pi}{2} \\ 3, & if \quad x=\frac{\pi}{2} \end{cases}$

$LHL=\lim_{x\to\frac{\pi}{2}^-} f(x)=\lim_{x\to\frac{\pi}{2}^-}\frac{k\cos x}{\pi-2x}$

Putting $x=\frac{\pi}{2}-h$ as $x\to\frac{\pi}{2}^-$ when $h\to 0$

$\therefore \lim_{x\to\frac{\pi}{2}^-} f(x)=\lim_{h\to 0}\frac{k\cos\left(\frac{\pi}{2}-h\right)}{\pi-2\left(\frac{\pi}{2}-h\right)}=\lim_{h\to 0}\frac{ksinh}{2h}=\frac{k}{2}\times\lim_{h\to 0}\frac{sinh}{h}=\frac{k}{2}\times 1=\frac{k}{2}$

Since f(x) is continuous at $x=\frac{\pi}{2}$, therefore LHL = $f\left(\frac{\pi}{2}\right)$

Also, $f\left(\frac{\pi}{2}\right)=3\Rightarrow\frac{k}{2}=3\Rightarrow k=6$

9. Find the values of *k* so that the function *f* $f(x)=\begin{cases} kx+1, & if \quad x\le 5 \\ 3x-5, & if \quad x>5 \end{cases}$ is continuous at point x = 5.

Ans.

Here, $f(x)=\begin{cases} kx+1, & if \quad x\le 5 \\ 3x-5, & if \quad x>5 \end{cases}$

At x = 5, $LHL=\lim_{x\to 5^-} f(x)=\lim_{x\to 5^-}(kx+1)$

Putting $x = 5 - h$ has $x \to 5^-$ when $h \to 0$

$\therefore \lim_{x\to 5^-} f(x)=\lim_{h\to 0}(k(5-h)+1)=\lim_{h\to 0}(5k-kh+1)=5k+1$

At x = 5, $RHL=\lim_{x\to 5^+} f(x)=\lim_{x\to 5^+}(3x-5)$

Putting $x = 5 + h$ as $x \to 5^+$; $h \to 0$

$\therefore \lim_{x\to 5^+} f(x)=\lim_{h\to 0}(3(5+h)-5)=\lim_{h\to 0}(10+3h)=10$

Also, f(5) = 5k + 1

Since f(x) is continuous at x = 5, therefore LHL = RHL = f(5)

$\Rightarrow 5k+1=10 \Rightarrow 5k=9 \Rightarrow k=\frac{9}{5}$

10. Find the values of *a* and *b* such that the function defined by $f(x)=\begin{cases} 5, & if \quad x\le 2 \\ ax+b, & if \quad 2<x<10 \\ 21, & if \quad x\ge 10 \end{cases}$ is a continuous function.

Ans.

Here, $f(x)=\begin{cases} 5, & if \quad x\le 2 \\ ax+b, & if \quad 2<x<10 \\ 21, & if \quad x\ge 10 \end{cases}$

At x =2, $LHL=\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-}(5)=5$

At x =2, $RHL=\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+}(ax+b)$

Putting $x = 2 + h$ as $x \to 2^+$ when $h \to 0$

$\therefore \lim_{x\to 2^+} f(x)=\lim_{h\to 0}(a(2+h)+b)=\lim_{h\to 0}(2a+ah+b)=2a+b$

Also, f(2) = 5

Since $f(x)$ is continuous at x = 2, therefore LHL = RHL = f(2)

$\Rightarrow 2a+b=5$ ----------------- (1)

At x =10, $LHL=\lim_{x\to 10^-} f(x)=\lim_{x\to 10^-}(ax+b)$

Putting $x = 10 - h$ has $x \to 10^-$ when $h \to 0$

$\therefore \lim_{x\to 10^-} f(x)=\lim_{h\to 0}(a(10-h)+b)=\lim_{h\to 0}(10a-ah+b)=10a+b$

At x =2, $RHL = \lim_{x \to 10^+} f(x) = \lim_{x \to 10^+} (21) = 21$

Also, f(10) = 21

Since $f(x)$ is continuous at $x = 10$, therefore LHL = RHL = f(10)

Since, $f(x)$ is continuous at $x = 10$.

LHL = RHL = f(10)

$\Rightarrow$ 10a + b = 21 ………………(2)

Subtracting Eq. (1) from Eq. (2), we get $8a = 16 \Rightarrow a = 2$

Put $a = 2$ in Eq. (1), we get $2 \times 2 + b = 5 \Rightarrow b = 1$

11. Prove that the function *f* given by *f* (*x*) = | *x* – 1 |, *x* ∈ R is not differentiable at *x* = 1.

Ans.

Given, $f(x) = |x-1| = \begin{cases} x-1, & if \quad x-1 \geq 0 \\ -(x-1), & if \quad x-1 < 0 \end{cases}$

We have to check the differentiability at $x = 1$

Here, $f(1) = 1 - 1 = 0$

$$Lf'(1) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \to 0} \frac{1-(1-h)-0)}{-h}$$

$$= \lim_{h \to 0} \frac{h}{-h} = -1$$

and

$$Rf'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(1+h)-1-0}{h}$$

$$= \lim_{h \to 0} \frac{h}{h} = 1$$

$\therefore Lf'(1) \neq Rf'(1).$

Hence, f(x) is not differentiable at x = 1

12. Find $\frac{dy}{dx}$ if $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Ans.

Let x = $\tan\theta \Rightarrow \theta = \tan^{-1} x$, then we have

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \cos^{-1}\cos 2\theta = 2\theta$$

$$\Rightarrow y = 2\tan^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = 2 \times \frac{1}{1+x^2} = \frac{2}{1+x^2}$$

13. Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Ans.

Let x = $\tan\theta \Rightarrow \theta = \tan^{-1} x$, then we have

$$y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \sin^{-1}\cos 2\theta = \sin^{-1}\sin\left(\frac{\pi}{2} - 2\theta\right)$$

$$\Rightarrow y = \frac{\pi}{2} - 2\theta \Rightarrow y = \frac{\pi}{2} - 2\tan^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = 0 - 2 \times \frac{1}{1+x^2} = \frac{-2}{1+x^2}$$

14. Find $\frac{dy}{dx}$ if $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$

Ans.

Let x = $\tan\theta \Rightarrow \theta = \tan^{-1} x$, then we have

$$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \cos^{-1}\sin 2\theta = \cos^{-1}\cos\left(\frac{\pi}{2} - 2\theta\right)$$

$$\Rightarrow y = \frac{\pi}{2} - 2\theta \Rightarrow y = \frac{\pi}{2} - 2\tan^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = 0 - 2 \times \frac{1}{1+x^2} = \frac{-2}{1+x^2}$$

15. Find $\frac{dy}{dx}$ if $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$

Ans.

Let $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$, then we have

$$y = \sec^{-1}\left(\frac{1}{2x^2-1}\right) = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = \sec^{-1}\sec 2\theta = 2\theta$$

$$\Rightarrow y = 2\cos^{-1} x = 2 \times \frac{-1}{\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}}$$

16. Differentiate sin ($\tan^{-1} e^{-x}$) with respect to x.

Ans.

Let y = sin ($\tan^{-1} e^{-x}$)

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin(\tan^{-1} e^{-x})\right] = \cos(\tan^{-1} e^{-x})\frac{d}{dx}(\tan^{-1} e^{-x})$$

$$= \cos(\tan^{-1} e^{-x})\frac{1}{1+(e^{-x})^2}\frac{d}{dx}(e^{-x})$$

$$= \cos(\tan^{-1} e^{-x})\frac{1}{1+e^{-2x}}(-e^{-x}) = -\frac{e^{-x}\cos(\tan^{-1} e^{-x})}{1+e^{-2x}}$$

17. Differentiate log (cos e^x) with respect to x.

Ans.

Let y = log (cos e^x)

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log(\cos e^x)\right] = \frac{1}{\cos e^x}\frac{d}{dx}(\cos e^x)$$

$$= \frac{1}{\cos e^x}(-\sin e^x)\frac{d}{dx}(e^x) = (-\tan e^x).e^x = -e^x \tan e^x$$

18. Differentiate cos (log x + e^x), x > 0 with respect to x.

Ans.

Let y = cos (log x + e^x)

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\cos(\log x + e^x)\right] = -\sin(\log x + e^x)\frac{d}{dx}(\log x + e^x)$$

$$= -\sin(\log x + e^x)\left(\frac{1}{x} + e^x\right) = -\sin(\log x + e^x)\left(\frac{1 + xe^x}{x}\right)$$

$$= \frac{-(1 + xe^x)\sin(\log x + e^x)}{x}$$

19. Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$.

Ans.

Given that $y^x + x^y + x^x = a^b$

Putting $u = y^x$, $v = x^y$ and $w = x^x$, we get $u + v + w = a^b$

Therefore, $\frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$ ------------------ (1)

Now, $u = y^x$. Taking logarithm on both sides, we have $\log u = x \log y$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x) = x\frac{1}{y}.\frac{dy}{dx} + \log y.1$$

$$\Rightarrow \frac{du}{dx} = u\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) \text{ -------------------- (2)}$$

Also $v = x^y$

Taking logarithm on both sides, we have $\log v = y \log x$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{v}\frac{dv}{dx} = y\frac{d}{dx}(\log x) + \log x\frac{dy}{dx} = y\frac{1}{x} + \log x\frac{dy}{dx}$$

$$\Rightarrow \frac{dv}{dx} = v\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) = x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) \text{ ------------------ (3)}$$

Again $w = x^x$

Taking logarithm on both sides, we have $\log w = x \log x$.

Differentiating both sides w.r.t. x, we have

$$\frac{1}{w}\frac{dw}{dx} = x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x) = x\frac{1}{x} + \log x.1$$

$$\Rightarrow \frac{dw}{dx} = w(1 + \log x) = x^x(1 + \log x) \text{ -------------------- (4)}$$

From (1), (2), (3), (4), we have

$$y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) + x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) + x^x(1 + \log x) = 0$$

$$(x.y^{x-1} + x^y.\log x)\frac{dy}{dx} = -x^x(1 + \log x) - y.x^{y-1} - y^x \log y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\left[x^x(1 + \log x) + y.x^{y-1} + y^x \log y\right]}{x.y^{x-1} + x^y.\log x}$$

20. Differentiate $x^x - 2^{\sin x}$ with respect to x.

Ans.

Let $y = x^x - 2^{\sin x}$

Let $u = x^x$ and $v = 2^{\sin x}$ then we have $y = u - v$

Therefore, $\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$ -------------------- (1)

Now, $u = x^x$

Taking logarithm on both sides, we have $\log u = x \log x$.

Differentiating both sides w.r.t. x, we have

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x) = x\frac{1}{x} + \log x.1$$

$$\Rightarrow \frac{du}{dx} = u\left(1+\log x\right) = x^x\left(1+\log x\right) \qquad \text{--------------------} \quad (2)$$

Again $v = 2^{sinx}$

Taking logarithm on both sides, we have log v = (sin$x$) log *2*.

Differentiating both sides w.r.t. x, we have

$$\frac{1}{v}\frac{dv}{dx} = \cos x(\log 2) \Rightarrow \frac{dv}{dx} = v[\cos x(\log 2)] = 2^{\sin x}[\cos x(\log 2)]$$

From (1), (2) and (3), we get

$$\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx} = x^x\left(1+\log x\right) - 2^{\sin x}[\cos x(\log 2)]$$

21. Differentiate (log x)$^{\mathbf{x}}$ + x$^{\mathbf{\log x}}$ with respect to x.

Ans.

Let y = $(\log x)^x + x^{\log x}$

Let u = $(\log x)^x$ and v = $x^{\log x}$ then we have y = u + v

Therefore, $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ -------------------- (1)

Now, $u = (\log x)^x$

Taking logarithm on both sides, we have log u = xlog(logx).

Differentiating both sides w.r.t. x, we have

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log(\log x) + \log(\log x)\frac{d}{dx}(x) = \frac{x}{\log x} \times \frac{1}{x} + \log(\log x)$$

$$\Rightarrow \frac{du}{dx} = u\left[\frac{1}{\log x} + \log(\log x)\right] = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] \qquad \text{----------------------} \ (2)$$

Again $v = x^{logx}$

Taking logarithm on both sides, we have log v = (log$x$) logx = (logx)^2

Differentiating both sides w.r.t. x, we have

$$\frac{1}{v}\frac{dv}{dx} = 2\log x\frac{d}{dx}(\log x) = 2\log x \times \frac{1}{x}$$

$$\Rightarrow \frac{dv}{dx} = v\left[\frac{2\log x}{x}\right] = x^{\log x}\left[\frac{2\log x}{x}\right] \qquad \text{----------------------} \ (3)$$

From (1), (2) and (3)

$$\frac{dy}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] + x^{\log x}\left[\frac{2\log x}{x}\right]$$

$$\Rightarrow \frac{dy}{dx} = (\log x)^{x-1}\left[1 + \log x\log(\log x)\right] + 2x^{\log x - 1}.\log x$$

22. Differentiate $(\sin x)^x + \sin^{-1}\sqrt{x}$ **with respect to x.**

Ans.

Let y = $(\sin x)^x + \sin^{-1}\sqrt{x}$

Let u = $(\sin x)^x$, v = $\sin^{-1}\sqrt{x}$ then we have y = u + v

Therefore, $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ -------------------- (1)

Now, $u = (\sin x)^x$

Taking logarithm on both sides, we have log u = xlog(sin x).

Differentiating both sides w.r.t. x, we have

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log(\sin x) + \log(\sin x)\frac{d}{dx}(x) = \frac{x}{\sin x}\times\cos x + \log(\sin x)$$

$$\Rightarrow \frac{du}{dx} = u\left[x\cot x + \log(\sin x)\right] = (\sin x)^x\left[x\cot x + \log(\sin x)\right] \quad \text{------------- (2)}$$

Again $v = \sin^{-1}\sqrt{x}$

Differentiating both sides w.r.t. *x*, we have

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}\frac{d}{dx}(\sqrt{x}) = \frac{1}{\sqrt{1-x}}\times\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x-x^2}} \quad \text{---------------- (3)}$$

From (1), (2) and (3)

$$\frac{dy}{dx} = (\sin x)^x\left[x\cot x + \log(\sin x)\right] + \frac{1}{2\sqrt{x-x^2}}$$

23. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

Ans.

Let $y = x^{\sin x} + (\sin x)^{\cos x}$

Let $u = x^{\sin x}$, $v = (\sin x)^{\cos x}$ then we have y = u + v

Therefore, $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ -------------------- (1)

Now, $u = x^{sinx}$

Taking logarithm on both sides, we have log u = (sinx) logx.

Differentiating both sides w.r.t. *x*, we have

$$\frac{1}{u}\frac{du}{dx} = \sin x\frac{d}{dx}\log x + \log x\frac{d}{dx}(\sin x) = \sin x\frac{1}{x} + \log x\cos x$$

$$\Rightarrow \frac{du}{dx} = u\left[\frac{\sin x}{x} + \log x\cos x\right] = x^{\sin x}\left[\frac{\sin x}{x} + \log x\cos x\right] \quad \text{---------------------- (2)}$$

Again $v = \sin x^{\cos x}$

Taking logarithm on both sides, we have log v = (cos*x*) log(sinx)

Differentiating both sides w.r.t. *x*, we have

$$\frac{1}{v}\frac{dv}{dx} = cosx\frac{d}{dx}\log(\sin x) + \log(\sin x)\frac{d}{dx}(\cos x) = \cos x\frac{1}{\sin x}\times\cos x + \log(\sin x)(-\sin x)$$

$$\Rightarrow \frac{dv}{dx} = v\left[\cot x\cos x - \sin x\log(\sin x)\right] = \sin x^{\cos x}\left[\cot x\cos x - \sin x\log(\sin x)\right] \quad \text{----------- (3)}$$

From (1), (2) and (3)

$$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log x\cos x\right] + \sin x^{\cos x}\left[\cot x\cos x - \sin x\log(\sin x)\right]$$

24. Find $\frac{dy}{dx}$ if $x = a\,(\theta + \sin\theta)$, $y = a\,(1 - \cos\theta)$.

Ans.

Given that $x = a\,(\theta + \sin\theta)$, $y = a\,(1 - \cos\theta)$

Differentiating w.r.t. θ, we get

$$\frac{dx}{d\theta} = a(1+\cos\theta), \frac{dy}{d\theta} = a(\sin\theta)$$

Therefore, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a(\sin\theta)}{a(1+\cos\theta)} = \frac{\sin\theta}{1+\cos\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}$

25. Find $\frac{dy}{dx}$ if $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$

Ans.

Given that $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$
Differentiating w.r.t. θ, we get

$$\frac{dx}{d\theta} = -\sin\theta - (-\sin 2\theta)\times 2 = -\sin\theta + 2\sin 2\theta$$

$$\frac{dy}{d\theta} = \cos\theta - (\cos 2\theta)\times 2 = \cos\theta - 2\cos 2\theta$$

Therefore, $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{-\sin\theta + 2\sin 2\theta}{\cos\theta - 2\cos 2\theta}$

26. Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$

Ans.
Given that $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$
Differentiating w.r.t. θ, we get

$$\frac{dx}{d\theta} = a(1-\cos\theta), \frac{dy}{d\theta} = a(0-\sin\theta) = -a\sin\theta$$

Therefore, $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{-a\sin\theta}{a(1-\cos\theta)} = \dfrac{-\sin\theta}{1-\cos\theta} = \dfrac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \dfrac{-\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = -\cot\dfrac{\theta}{2}$

27. If $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$

Ans.
Given that $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$
Multiplying both we get,

$$xy = \sqrt{a^{\sin^{-1}t}}\sqrt{a^{\cos^{-1}t}} = \sqrt{a^{\sin^{-1}t}.a^{\cos^{-1}t}} = \sqrt{a^{\sin^{-1}t+\cos^{-1}t}} = \sqrt{a^{\pi/2}}$$

Differentiating both sides w.r.t. x, we get

$$x\frac{dy}{dx} + y = 0 \Rightarrow x\frac{dy}{dx} = -y \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

28. If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

Ans.
Given that $y = 3e^{2x} + 2e^{3x}$
Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = 6e^{2x} + 6^{3x} = 6(e^{2x} + e^{3x})$$

Again, Differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = 6(2e^{2x} + 3e^{3x})$$

Now, $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 6(2e^{2x} + 3e^{3x}) - 5(6(e^{2x} + e^{3x})) + 6(3e^{2x} + 2e^{3x})$

$$= 12e^{2x} + 18e^{3x} - 30e^{2x} - 30e^{3x} + 18e^{2x} + 12e^{3x} = 0$$

29. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2 y_2 + xy_1 + y = 0$

Ans.
Given that $y = 3\cos(\log x) + 4\sin(\log x)$
Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = y_1 = -3\sin(\log x)\frac{d}{dx}(\log x) + 4\cos(\log x)\frac{d}{dx}(\log x)$$

$$= -3\sin(\log x)\frac{1}{x} + 4\cos(\log x)\frac{1}{x} = \frac{1}{x}\left[-3\sin(\log x) + 4\cos(\log x)\right]$$

$$\Rightarrow xy_1 = -3\sin(\log x) + 4\cos(\log x)$$

Again, Differentiating both sides w.r.t. x, we get

$$xy_2 + y_1.1 = -3\cos(\log x)\frac{d}{dx}(\log x) - 4\sin(\log x)\frac{d}{dx}(\log x)$$

$$= -3\cos(\log x)\frac{1}{x} - 4\sin(\log x)\frac{1}{x} = -\frac{1}{x}\left[3\cos(\log x) + 4\sin(\log x)\right] = -\frac{y}{x}$$

$$\Rightarrow x^2 y_2 + xy_1 = -y$$

$$\Rightarrow x^2 y_2 + xy_1 + y = 0$$

30. If $e^y (x + 1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

Ans.
Given that $e^y (x + 1) = 1$

$$\Rightarrow e^y = \frac{1}{x+1}$$

Differentiating both sides w.r.t. x, we get

$$e^y \frac{dy}{dx} = -\frac{1}{(x+1)^2} \Rightarrow \frac{1}{x+1}\frac{dy}{dx} = -\frac{1}{(x+1)^2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{x+1}$$

Again, Differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{1}{(x+1)^2} = \left(-\frac{1}{x+1}\right)^2 = \left(\frac{dy}{dx}\right)^2$$

31. If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)^2 y_2 + 2x (x^2 + 1) y_1 = 2$
Ans.
Given that $y = (\tan^{-1} x)^2$
Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = 2\tan^{-1} x \frac{d}{dx}(\tan^{-1} x) = 2\tan^{-1} x \times \frac{1}{1+x^2}$$

$$\Rightarrow y_1 = \frac{2\tan^{-1} x}{1+x^2} \Rightarrow (1+x^2)y_1 = 2\tan^{-1} x$$

Again, Differentiating both sides w.r.t. x, we get

$$(1+x^2)y_2 + 2xy_1 = 2\times\frac{1}{1+x^2} \Rightarrow (1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2$$

32. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8,\ x \in [-4, 2]$.
Ans.
Given function is $f(x) = x^2 + 2x - 8,\ x \in [-4, 2]$.
Since, a polynomial function is continuous and derivable on R, therefore
(i) $f(x)$ is continuous on $[-4,2]$.
(ii) $f(x)$ is derivable on $(-4,2)$.
Also, $f(-4) = (-4)^2 + 2(-4) - 8 = 0$ (since $f(x) = x^2 + 2x - 8$)

and $f(2) = 2^2 + 2 \text{ x } 2 - 8 = 0 \quad \Rightarrow f(-4) = f(2)$

This means that all the conditions of Rolle's theorem are satisfied by $f(x)$ in $[-4,2]$.

Therefore, it exists at least one real $c \in [-4, 2]$ such that $f'(c) = 0$.

Now, $f(x) = x^2 + 2x - 8 \Rightarrow f'(x) = \frac{d}{dx}(x^2 + 2x - 8) = 2x + 2$

Putting $f'(c) = 0 \Rightarrow 2c + 2 = 0 \Rightarrow c = -1$.

Thus, $f'(-1) = 0$ and $-1 \in (-4,2)$.

Hence, Rolle's theorem is verified with $c = -1$.

33. Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$ in the interval $[a, b]$, where $a = 1$ and $b = 4$.

Ans.

Here, $f(x) = x^2 - 4x - 3$, $x \in [1, 4]$ which is a polynomial function, so it is continuous and derivable at all $x \in R$, therefore

(i) $f(x)$ is continuous on [1, 4] (ii) $f(x)$ is derivable on (1, 4).

Therefore, Conditions of Lagrange's theorem are satisfied on [1, 4].

Hence, there is atleast one real number. $c \in (1, 4)$ such that

Now, $f'(x) = \frac{d}{dx}(x^2 - 4x - 3) = 2x - 4$

$\Rightarrow f(4) = 4^2 - 4(4) - 3 = 16 - 16 - 3 = -3$

and $f(1) = 1 - 4 - 3 = -6$

$\therefore f'(c) = \frac{f(4) - f(1)}{4 - 1} = \frac{-3 - (-6)}{3} = \frac{-3 + 6}{3} = 1$

$\Rightarrow 2c - 4 = 1 \Rightarrow 2c = 1 + 4 = 5 \Rightarrow c = \frac{5}{2} \in (1, 4)$

Hence Mean Value Theorem is verified.

34. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval $[a, b]$, where $a = 1$ and $b = 3$. Find all $c \in (1, 3)$ for which $f'(c) = 0$.

Ans.

Given, $f(x) = x^3 - 5x^2 - 3x$, $x \in (1, 3)$, which is a polynomial function. Since, a polynomial function is continuous and derivable at all
$x \in R$, therefore

(i) $f(x)$ is continuous on [1, 3]. (ii) $f(x)$ is derivable on (1, 3).

Therefore, Condition of Lagrange's MVT are satisfied on [1, 3].

Hence, there exists aleast one real $c \in (1, 3)$.

Now, $f'(x) = \frac{d}{dx}(x^3 - 5x^2 - 3x) = 3x^2 - 10x - 3$

$\Rightarrow f(3) = (3)^3 - 5(3)^2 - 3(3) = 27 - 45 - 9 = -27$

and $f(1) = 1 - 5 - 3 = -7$

$\therefore f'(c) = \frac{f(3) - f(1)}{3 - 1} = \frac{-27 - (-7)}{2} = \frac{-27 + 7}{2} = -10$

$\Rightarrow 3c^2 - 10c - 3 = -10 \Rightarrow 3c^2 - 10c - 3 + 10 = 0$

$\Rightarrow 3c^2 - 10c + 7 = 0$

$\Rightarrow c = \frac{10 \pm \sqrt{100 - 84}}{6} = \frac{10 \pm 4}{6} = 1, \frac{7}{3}$ out of which $\frac{7}{3} \in (1,3)$

Hence Mean Value Theorem is verified.

35. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for , $-1 < x < 1$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

Ans.

Given that $x\sqrt{1+y}+y\sqrt{1+x}=0 \Rightarrow x\sqrt{1+y}=-y\sqrt{1+x}$

Squaring both sides, we get

$x^2(1+y)=y^2(1+x) \qquad \Rightarrow x^2-y^2+x^2y-y^2x=0$

$\Rightarrow (x-y)(x+y)+xy(x-y)=0 \quad \Rightarrow (x-y)(x+y+xy)=0$

$\Rightarrow x-y=0 \ \ or \ \ x+y+xy=0$

$\Rightarrow y=x \ \ or \ \ y(1+x)=-x \Rightarrow y=x \ \ or \ \ y=\dfrac{-x}{1+x}$

But y =x does not satisfy the given equation

So, we consider, $y=\dfrac{-x}{1+x}$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{-x}{x+1}\right)=-\frac{(1+x)\frac{d}{dx}(x)-x\frac{d}{dx}(1+x)}{(1+x)^2}=-\frac{(1+x)-x}{(1+x)^2}=-\frac{1}{(1+x)^2}$$

36. If cos *y* = *x* cos (*a* + *y*), with cos *a* ≠ ± 1, prove that $\dfrac{dy}{dx}=\dfrac{\cos^2(a+y)}{\sin a}$

Ans.

Given that cos *y* = *x* cos (*a* + *y*)$\Rightarrow x=\dfrac{\cos y}{\cos(a+y)}$

Differentiating both sides w.r.t. y, we get

$$\frac{dx}{dy}=\frac{d}{dy}\left(\frac{\cos y}{\cos(a+y)}\right)=\frac{\cos(a+y)(-\sin y)-\cos y[-\sin(a+y)]}{\cos^2(a+y)}$$

$$=\frac{\sin(a+y)\cos y-\cos(a+y)\sin y}{\cos^2(a+y)}=\frac{\sin(a+y-y)}{\cos^2(a+y)}=\frac{\sin a}{\cos^2(a+y)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$$

37. If *x* = *a* (cos *t* + *t* sin *t*) and *y* = *a* (sin *t* – *t* cos *t*), find $\dfrac{d^2y}{dx^2}$

Ans.

Given that *x* = *a* (cos *t* + *t* sin *t*) and *y* = *a* (sin *t* – *t* cos *t*)

Differentiating both sides w.r.t. t, we get

$$\frac{dx}{dt}=a\left[\frac{d}{dt}\cos t+\left(t\frac{d}{dt}\sin t+\sin t\frac{d}{dt}(t)\right)\right]=a[-\sin t+(t\sin t+\sin t)]=at\cos t$$

and $\dfrac{dy}{dt}=a\left[\dfrac{d}{dt}\sin t+\left(t\dfrac{d}{dt}\cos t+\cos t\dfrac{d}{dt}(t)\right)\right]=a[\cos t-(-t\sin t+\cos t)]=at\sin t$

Now, $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{at\sin t}{at\cos t}=\tan t$

Again, Differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\tan t=\sec^2 t\times\frac{dt}{dx}=\frac{\sec^2 t}{at\cos t}=\frac{1}{at}\sec^3 t$$

38. Differentiate $\tan^{-1}\left(\dfrac{\sin x}{1+\cos x}\right)$ **w.r.t. x**

Ans.

Let y $= \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right) = \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$

$= \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right) = \tan^{-1}(\tan\frac{x}{2}) = \frac{x}{2}$

Differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{2}$

39. Differentiate $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ **w.r.t. x**

Ans.

Let y = $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right) = \sin^{-1}\left(\frac{2^x \times 2}{1+(2^x)^2}\right)$

Let $2^x = \tan\theta \Rightarrow \theta = \tan^{-1} 2^x$ then we have

$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1} 2^x$

Differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = 2 \times \frac{1}{1+(2^x)^2}\frac{d}{dx}(2^x) = \frac{2}{1+4^x}2^x \log 2 = \frac{2^{x+1}\log 2}{1+4^x}$

40. Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.

Ans.

Let u = $\sin^2 x$ and v = $e^{\cos x}$

Differentiating u and v w.r.t. x, we get

$\frac{du}{dx} = 2\sin x \frac{d}{dx}(\sin x) = 2\sin x\cos x$

and $\frac{dv}{dx} = e^{\cos x}\frac{d}{dx}(\cos x) = e^{\cos x}(-\sin x) = (-\sin x)e^{\cos x}$

Now, $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2\sin x\cos x}{(-\sin x)e^{\cos x}} = -\frac{2\cos x}{e^{\cos x}}$