CHAPTER – 3: DETERMINANTS

MARKS WEIGHTAGE – 10 marks

Previous Years Board Exam (Important Questions & Answers)

1. Let A be a square matrix of order 3 × 3. Write the value of $|2A|$, where $|A| = 4$.

Ans:

Since $|2A| = 2^n|A|$ where n is order of matrix A.

Here $|A| = 4$ and $n = 3$

$\therefore |2A| = 2^3 \times 4 = 32$

2. Write the value of the following determinant: $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

Ans:

Given that $\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

Applying $R_1 \to R_1 - 6R_3$

$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 0$ (Since R_1 is zero)

3. If A is a square matrix and $|A| = 2$, then write the value of $|AA'|$, where A' is the transpose of matrix A.

Ans:

$|AA|' = |A|.\ |A'| = |A|.\ |A| = |A|^2 = 2 \times 2 = 4$.

[since, $|AB| = |A|.|B|$ and $|A| = |A'|$, where A and B are square matrices.]

4. If A is a 3 × 3 matrix, $|A| \neq 0$ and $|3A| = k\,|A|$, then write the value of k.

Ans:

Here, $|3A| = k|A|$

$\Rightarrow 3^3|A| = k|A|$ [$\because$ $|kA| = kn|A|$ where n is order of A]

$\Rightarrow 27\,|A| = k|A|$

$\Rightarrow k = 27$

5. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

Ans:

$\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}\pi = (a+ib)(a-ib)-(c+id)(-c+id)$

$= (a+ib)(a-ib)+(c+id)(c-id)$

$= a^2 - i^2b^2 + c^2 - i^2d^2$

$= a^2 + b^2 + c^2 + d^2$

6. If $\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix} = 3$, find the value of x.

Ans:

Given that $\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix} = 3$

$\Rightarrow$ $4x + 8 - 3x - 15 = 3$

$\Rightarrow x - 7 = 3$

$\Rightarrow x = 10$

7. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{23}.

Ans:

Minor of $a_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$.

8. Evaluate: $\begin{vmatrix} \cos 15^0 & \sin 15^0 \\ \sin 75^0 & \cos 75^0 \end{vmatrix}$

Ans:

Expanding the determinant, we get

$\cos 15° . \cos 75° - \sin 15° . \sin 75°$

$= \cos(15° + 75°) = \cos 90° = 0$

[since $\cos(A + B) = \cos A . \cos B - \sin A . \sin B$]

9. Using properties of determinants, prove the following: $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b)$

Ans:

$$Let \ \Delta = \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we have

$$\Delta = \begin{vmatrix} 3(a+b) & 3(a+b) & 3(a+b) \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

Taking out $3(a + b)$ from 1st row, we have

$$\Delta = 3(a+b)\begin{vmatrix} 1 & 1 & 1 \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$ and $C2 \to C_2 - C_3$

$$\Delta = 3(a+b)\begin{vmatrix} 0 & 0 & 1 \\ -2b & -b & a+b \\ -b & 2b & a \end{vmatrix}$$

Expanding along first row, we have

$D = 3(a + b) [1. (4b^2 - b^2)]$

$= 3(a + b) \times 3b^2 = 9b^2(a + b)$

10. Write the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

Ans:

Given determinant |A|= $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

$= 3x\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix} = 0(\because\ R_1 = R_3)$

11. Two schools P and Q want to award their selected students on the values of Tolerance, Kindness and Leadership. The school P wants to award Rs. x each, Rs. y each and Rs. z each for the three respective values to 3, 2 and 1 students respectively with a total award money of Rs. 2,200. School Q wants to spend Rs. 3,100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is Rs. 1,200, using matrices, find the award money for each value. Apart from these three values, suggest one more value that should be considered for award.

Ans:

According to question,

$3x + 2y + z = 2200$

$4x + y + 3z = 3100$

$x + y + z = 1200$

The above system of equation may be written in matrix form as $AX = B$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 3(1-3) - 2(4-3) + 1(4-1) = -6 - 2 + 3 = -5 \neq 0$$

$\therefore$ A^{-1} exists.

Now, $A_{11} = (1 - 3) = -2$,

$A_{12} = -(4 - 3) = -1$,

$A_{13} = (4 - 1) = 3$,

$A_{21} = -(2 - 1) = -1$,

$A_{22} = (3 - 1) = 2$,

$A_{23} = -(3 - 2) = -1$

$A_{31} = (6 - 1) = 5$,

$A_{32} = -(9 - 4) = -5$,

$A_{33} = (3 - 8) = -5$

$$adj(A) = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{-5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 4400+3100-6000 \\ 2200-6200+6000 \\ -6600+3100+6000 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1500 \\ 2000 \\ 2500 \end{bmatrix} = \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$$

$\Rightarrow$ $x = 300$, $y = 400$, $z = 500$

i.e., Rs. 300 for tolerance, Rs. 400 for kindness and Rs. 500 for leadership are awarded.
One more value like punctuality, honesty etc may be awarded.

12. Using properties of determinants, prove that

Ans:

$$LHS = \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$= \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & y & a+z \end{vmatrix}$$

Apply $R_1 \to R_1 - R_2$, we get

$$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & y & a+z \end{vmatrix}$$

$$= (a+x+y+z)\begin{vmatrix} 0 & -a & 0 \\ 1 & a+y & z \\ 1 & y & a+z \end{vmatrix}$$

Expanding along R_1, we get

$= (a + x + y + z) \{0 + a (a + z - z)\} = a^2(a + x + y + z) = \text{RHS}$

13. 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students and the third group contains vigilant and obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of first and second group is four times that of the third group. Using matrix method, find the number of students in each group. Apart from the values, hard work, honesty and respect for law, vigilance and obedience, suggest one more value, which in your opinion, the school should consider for awards.

Ans:

Let no. of students in Ist, 2nd and 3rd group to x, y, z respectively.

From the statement we have

$$x + y + z = 10$$
$$2x + y = 13$$
$$x + y - 4z = 0$$

The above system of linear equations may be written in matrix form as $AX = B$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 10 \\ 13 \\ 0 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & -4 \end{vmatrix} = 1(-4-0) - 1(-8-0) + 1(2-1) = -4 + 8 + 1 = 5 \neq 0$$

$\therefore$ A^{-1} exists.

Now, $A_{11} = -4 - 0 = -4$

$A_{12} = -(-8 - 0) = 8$

$A_{13} = 2 - 1 = 1$

$A_{21} = -(-4 - 1) = 5$

$A_{22} = -4 - 1 = -5$

$A_{23} = -(1 - 1) = 0$

$A_{31} = 0 - 1 = -1$

$A_{32} = -(0 - 2) = 2$

$A_{33} = 1 - 2 = -1$

$$adj(A) = \begin{bmatrix} -4 & 8 & 1 \\ 5 & -5 & 0 \\ -1 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{5}\begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 10 \\ 13 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -40+65 \\ 80-65 \\ 10 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 25 \\ 15 \\ 10 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$$

$\Rightarrow x = 5, y = 3, z = 2$

14. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.

Ans:

According to question

$x + y + z = 12$

$2x + 3y + 3z = 33$

$x - 2y + z = 0$

The above system of linear equation can be written in matrix form as $AX = B$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{vmatrix}$$

= 1 (3 + 6) –1 (2–3) + 1 (– 4 –3) = 9 + 1 – 7 = 3

$\therefore$ A^{-1} exists.

$A_{11} = 9, A_{12} = 1, A_{13} = -7$

$A_{21} = -3, A_{22} = 0, A_{23} = 3$

$A_{31} = 0, A_{32} = -1, A_{33} = 1$

$$adj(A) = \begin{bmatrix} 9 & 1 & -7 \\ -3 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

Now, $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 108-99 \\ 12+0+0 \\ -84+99 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$\Rightarrow x = 3, y = 4, z = 5$

No. of awards for honesty = 3

No. of awards for helping others = 4

No. of awards for supervising = 5.

The persons, who work in the field of health and hygiene should also be awarded.

15. Using properties of determinants, prove the following:

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx)$$

Ans:

$$LHS = \begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} x+y+z & -x+y & -x+z \\ x+y+z & 3y & z-y \\ x+y+z & y-z & 3z \end{vmatrix}$$

Taking out $(x + y + z)$ along C_1, we get

$$= (x+y+z)\begin{vmatrix} 1 & -x+y & -x+z \\ 1 & 3y & z-y \\ 1 & y-z & 3z \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$

$$= (x+y+z)\begin{vmatrix} 1 & -x+y & -x+z \\ 0 & 2y+x & x-y \\ 0 & x-z & x+2z \end{vmatrix}$$

Applying $C_2 \to C_2 - C_3$

$$= (x+y+z)\begin{vmatrix} 1 & y-z & -x+z \\ 0 & 3y & x-y \\ 0 & -3z & x+2z \end{vmatrix}$$

Expanding along I column, we get

D = $(x + y + z)[(3y\,(x + 2z) + 3z\,(x - y)]$

= $3(x + y + z)[xy + 2z + 2yz + xz - yz]$

= $3(x + y + z)(xy + yz + zx)$ = R.H.S.

16. A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of Rs. **6,000. Three times the award money for Hardwork added to that given for honesty amounts to ` 11,000. The award money given for Honesty and Hardwork together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hardwork, suggest one more value which the school must include for awards.**

Ans:

Let x, y and z be the awarded money for honesty, Regularity and hardwork.

From the statement

$$x + y + z = 6000 \ldots(i)$$
$$x + 3z = 11000 \ldots(ii)$$
$$x + z = 2y \Rightarrow x - 2y + z = 0 \ldots(iii)$$

The above system of three equations may be written in matrix form as $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 0 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad and \quad B = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$Here, |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 0 & -2 & 1 \end{vmatrix} = 1(0+6) - 1(1-3) + 1(-2-0) = 6 + 2 - 2 = 6 \neq 0$$

Hence A^{-1} exist

If Aij is co-factor of aij then

$A_{11} = 0 + 6 = 6$

$A_{12} = -(1 - 3) = 2$;

$A_{13} = (-2 - 0) = -2$

$A_{21} = -(1 + 2) = -3$

$A_{22} = 0$

$A_{23} = (-2 - 1) = -3$

$A_{31} = 3 - 0 = 3$

$A_{32} = -(3 - 1) = -2$;

$A_{33} = 0 - 1 = -1$

$$adj(A) = \begin{bmatrix} 6 & 2 & -2 \\ -3 & 0 & 3 \\ 3 & -2 & -1 \end{bmatrix}^T = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 36000-33000+0 \\ 12000+0+0 \\ -12000+33000+0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 3000 \\ 12000 \\ 21000 \end{bmatrix} = \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$$

$\Rightarrow x = 500, y = 2000, z = 3500$

Except above three values, school must include discipline for award as discipline has great importance in student's life.

17. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$**, then write the value of** x**.**

Ans:

Given that $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

$\Rightarrow (x+1)(x+2) - (x-1)(x-3) = 12 + 1$

$\Rightarrow x^2 + 2x + x + 2 - x^2 + 3x + x - 3 = 13$

$\Rightarrow 7x - 1 = 13$

$\Rightarrow 7x = 14$

$\Rightarrow x = 2$

18. Using properties of determinants, prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

Ans:

$$LHS = \begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$$

$$= \begin{vmatrix} a & a & a+b+c \\ 2a & 3a & 4a+3b+2c \\ 3a & 6a & 10a+6b+3c \end{vmatrix} + \begin{vmatrix} a & b & a+b+c \\ 2a & 2b & 4a+3b+2c \\ 3a & 3b & 10a+6b+3c \end{vmatrix}$$

$$= a^2\begin{vmatrix} 1 & 1 & a+b+c \\ 2 & 3 & 4a+3b+2c \\ 3 & 6 & 10a+6b+3c \end{vmatrix} + ab\begin{vmatrix} 1 & 1 & a+b+c \\ 2 & 2 & 4a+3b+2c \\ 3 & 3 & 10a+6b+3c \end{vmatrix}$$

$$= a^2\begin{vmatrix} 1 & 1 & a+b+c \\ 2 & 3 & 4a+3b+2c \\ 3 & 6 & 10a+6b+3c \end{vmatrix} + ab.0$$ [since $C_1 = C_2$ in second determinant]

$$= a^2\begin{vmatrix} 1 & 1 & a+b+c \\ 2 & 3 & 4a+3b+2c \\ 3 & 6 & 10a+6b+3c \end{vmatrix}$$

$$= a^2\begin{vmatrix} 1 & 1 & a \\ 2 & 3 & 4a \\ 3 & 6 & 10a \end{vmatrix} + \begin{vmatrix} 1 & 1 & b \\ 2 & 3 & 3b \\ 3 & 6 & 6b \end{vmatrix} + \begin{vmatrix} 1 & 1 & c \\ 2 & 3 & 2c \\ 3 & 6 & 3c \end{vmatrix}$$

$$= a^2.a\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 10 \end{vmatrix} + b\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 3 & 6 & 6 \end{vmatrix} + c\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 6 & 3 \end{vmatrix}$$

$$= a^2.a\begin{vmatrix}1 & 1 & 1\\2 & 3 & 4\\3 & 6 & 10\end{vmatrix} + b.0 + c.0$$ [since $C_2 = C_3$ in second determinant and $C_1 = C_3$ in third determinant]

$$= a^3\begin{vmatrix}1 & 1 & 1\\2 & 3 & 4\\3 & 6 & 10\end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$ we get

$$= a^3\begin{vmatrix}1 & 0 & 0\\2 & 1 & 2\\3 & 3 & 7\end{vmatrix}$$

Expanding along R_1 we get

$= a^3 .1(7 – 6) – 0 + 0$

$= a^3$.

19. Using matrices, solve the following system of equations:

$x – y + z = 4; 2x + y – 3z = 0; x + y + z = 2$

Ans:

Given equations

$x – y + z = 4$

$2x + y – 3z = 0$

$x + y + z = 2$

We can write this system of equations as AX = B where

$$A = \begin{bmatrix}1 & -1 & 1\\2 & 1 & -3\\1 & 1 & 1\end{bmatrix}, X = \begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B = \begin{bmatrix}4\\0\\2\end{bmatrix}$$

$$Here, |A| = \begin{vmatrix}1 & -1 & 1\\2 & 1 & -3\\1 & 1 & 1\end{vmatrix}$$

= 1 (1 + 3) - (- 1) (2 + 3) + 1 (2 - 1) = 4 + 5 + 1 = 10

$\therefore$ A^{-1} exists.

$A_{11} = 4, A_{12} = –5, A_{13} = 1$

$A_{21} = 2, A_{22} = 0, A_{23} = –2$

$A_{31} = 2, A_{32} = 5, A_{33} = 3$

$$adj(A) = \begin{bmatrix}4 & -5 & 1\\2 & 0 & -2\\2 & 5 & 3\end{bmatrix}^T = \begin{bmatrix}4 & 2 & 2\\-5 & 0 & 5\\1 & -2 & 3\end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{10}\begin{bmatrix}4 & 2 & 2\\-5 & 0 & 5\\1 & -2 & 3\end{bmatrix}$$

$$\text{Now, } X = A^{-1}B \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{10}\begin{bmatrix}4 & 2 & 2\\-5 & 0 & 5\\1 & -2 & 3\end{bmatrix}\begin{bmatrix}4\\0\\2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4-0+6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

The required solution is

$\therefore$ $x = 2$, $y = -1$, $z = 1$

20. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

Ans:

For B^{-1}

$$|B| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix} = 1(3-0) - 2(-1-0) - 2(2-0) = 3 + 2 - 4 = 1 \neq 0$$

i.e., B is invertible matrix

$\Rightarrow B^{-1}$ exist.

$A_{11} = 3$, $A_{12} = 1$, $A_{13} = 2$

$A_{21} = 2$, $A_{22} = 1$, $A_{23} = 2$

$A_{31} = 6$, $A_{32} = 2$, $A_{33} = 5$

$$adj(B) = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}^T = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{1}{|B|}(adjb) = \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now $(AB)^{-1} = B^{-1}. A^{-1}$

$$\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$