

CHAPTER – 2: INVERSE TRIGONOMETRIC FUNCTIONS

MARKS WEIGHTAGE – 05 marks

Previous Years Board Exam (Important Questions & Answers)

1. Evaluate : $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

Ans:

$$\begin{aligned}\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right] &= \sin\left[\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right] \\ &= \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right] = \sin\frac{\pi}{2} = 1\end{aligned}$$

2. Write the value of $\cot(\tan^{-1}a + \cot^{-1}a)$.

Ans:

$$\cot(\tan^{-1}a + \cot^{-1}a) = \cot\left(\frac{\pi}{2} - \cot^{-1}a + \cot^{-1}a\right) = \cot\frac{\pi}{2} = 0$$

3. Find the principal values of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

Ans:

$$\begin{aligned}\cos^{-1}\left(\cos\frac{7\pi}{6}\right) &= \cos^{-1}\left(\cos\left(\pi + \frac{\pi}{6}\right)\right) \\ &= \cos^{-1}\left(-\cos\frac{\pi}{6}\right) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}\end{aligned}$$

4. Find the principal values of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

Ans:

$$\begin{aligned}\tan^{-1}\left(\tan\frac{3\pi}{4}\right) &= \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{4}\right)\right) \\ &= \tan^{-1}\left(-\tan\frac{\pi}{4}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}\end{aligned}$$

5. Prove that: $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$

Ans:

$$\begin{aligned}LHS &= \tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) \\ &= \frac{\tan\frac{\pi}{4} + \tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1 - \tan\frac{\pi}{4}\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)} + \frac{\tan\frac{\pi}{4} - \tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1 + \tan\frac{\pi}{4}\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)} \\ &= \frac{1 + \tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1 - \tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)} + \frac{1 - \tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1 + \tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}\end{aligned}$$

$$\begin{aligned}
&= \frac{\left[1 + \tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)\right]^2 + \left[1 - \tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)\right]^2}{\left[1 - \tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)\right]} \\
&= \frac{2 + 2\tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1 - \tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)} = \frac{2\left(1 + \tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)\right)}{1 - \tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)} \\
&= \frac{2}{\cos 2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)} = \frac{2}{\cos\left(\cos^{-1}\frac{a}{b}\right)} = \frac{2}{\frac{a}{b}} = \frac{2b}{a} = RHS
\end{aligned}$$

6. **Solve:** $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$

Ans:

$$\begin{aligned}
\tan^{-1}(x+1) + \tan^{-1}(x-1) &= \tan^{-1}\frac{8}{31} \\
\Rightarrow \tan^{-1}\left(\frac{(x+1)+(x-1)}{1-(x+1)(x-1)}\right) &= \tan^{-1}\frac{8}{31} \\
\Rightarrow \tan^{-1}\left(\frac{2x}{1-(x^2-1)}\right) &= \tan^{-1}\frac{8}{31} \Rightarrow \tan^{-1}\left(\frac{2x}{2-x^2}\right) = \tan^{-1}\frac{8}{31} \\
\Rightarrow \frac{2x}{2-x^2} &= \frac{8}{31} \Rightarrow 62x = 16 - 8x^2 \Rightarrow 8x^2 + 62x - 16 = 0 \\
\Rightarrow 4x^2 + 31x - 8 &= 0 \Rightarrow (4x-1)(x+8) = 0 \\
\Rightarrow x &= \frac{1}{4} \text{ and } x = -8
\end{aligned}$$

As $x = -8$ does not satisfy the equation

Hence $x = \frac{1}{4}$ is only solution.

7. **Prove that** $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65} = \frac{\pi}{2}$

Ans:

$$\begin{aligned}
\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65} &= \frac{\pi}{2} \\
\Rightarrow \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} &= \frac{\pi}{2} - \sin^{-1}\frac{16}{65} = \cos^{-1}\frac{16}{65}
\end{aligned}$$

Let $\sin^{-1}\frac{4}{5} = x$ and $\sin^{-1}\frac{5}{13} = y$

Therefore $\sin x = \frac{4}{5}$ and $\sin y = \frac{5}{13}$

Now, $\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

and $\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - \left(\frac{5}{13}\right)^2} = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$

We have $\cos(x+y) = \cos x \cos y - \sin x \sin y = \frac{3}{5} \times \frac{12}{13} - \frac{4}{5} \times \frac{5}{13} = \frac{36}{65} - \frac{20}{65} = \frac{16}{65}$

$$\Rightarrow x + y = \cos^{-1} \frac{16}{65}$$

$$\Rightarrow \sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13} = \cos^{-1} \frac{16}{65}$$

8. Prove that $\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \frac{1}{2} \cos^{-1} \frac{3}{5}$

Ans:

$$LHS = \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9}$$

$$= \tan^{-1} \left(\frac{\frac{1}{4} + \frac{2}{9}}{1 - \frac{1}{4} \cdot \frac{2}{9}} \right) = \tan^{-1} \left(\frac{\frac{9+8}{36}}{1 - \frac{2}{36}} \right) = \tan^{-1} \left(\frac{\frac{17}{36}}{\frac{34}{36}} \right) \quad \left(\because \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right) \right)$$

$$= \tan^{-1} \frac{17}{34} = \tan^{-1} \frac{1}{2} = \frac{1}{2} \left(2 \tan^{-1} \frac{1}{2} \right)$$

$$= \frac{1}{2} \cos^{-1} \left(\frac{1 - \left(\frac{1}{2} \right)^2}{1 + \left(\frac{1}{2} \right)^2} \right) = \frac{1}{2} \cos^{-1} \left(\frac{1 - \frac{1}{4}}{1 + \frac{1}{4}} \right) \quad \left[\because 2 \tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \right]$$

$$= \frac{1}{2} \cos^{-1} \left(\frac{\frac{3}{4}}{\frac{5}{4}} \right) = \frac{1}{2} \cos^{-1} \frac{3}{5} = RHS$$

9. Solve for x: $\cos^{-1} \left(\frac{x^2-1}{x^2+1} \right) + \tan^{-1} \left(\frac{2x}{x^2-1} \right) = \frac{2\pi}{3}$

Ans:

$$\cos^{-1} \left(\frac{x^2-1}{x^2+1} \right) + \tan^{-1} \left(\frac{2x}{x^2-1} \right) = \frac{2\pi}{3}$$

$$\Rightarrow \cos^{-1} \left(\frac{-(1-x^2)}{1+x^2} \right) + \tan^{-1} \left(-\frac{2x}{1-x^2} \right) = \frac{2\pi}{3}$$

$$\Rightarrow \pi - \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) - \tan^{-1} \left(\frac{2x}{1-x^2} \right) = \frac{2\pi}{3}$$

$$\Rightarrow \pi - 2 \tan^{-1} x - 2 \tan^{-1} x = \frac{2\pi}{3} \Rightarrow \pi - 4 \tan^{-1} x = \frac{2\pi}{3}$$

$$\Rightarrow \pi - \frac{2\pi}{3} = 4 \tan^{-1} x \Rightarrow 4 \tan^{-1} x = \frac{\pi}{3} \Rightarrow \tan^{-1} x = \frac{\pi}{12}$$

$$\Rightarrow \tan^{-1} x = \left(\frac{\pi}{4} - \frac{\pi}{6} \right) \Rightarrow x = \tan \left(\frac{\pi}{4} - \frac{\pi}{6} \right) = \frac{\tan \frac{\pi}{4} - \tan \frac{\pi}{6}}{1 - \tan \frac{\pi}{4} \cdot \tan \frac{\pi}{6}}$$

$$\Rightarrow x = \frac{1 - \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\Rightarrow x = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3+1-2\sqrt{3}}{3-1} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

10. Prove that $\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \left(\frac{1-x}{1+x} \right), x \in (0,1)$

Ans:

$$\begin{aligned} LHS &= \tan^{-1} \sqrt{x} = \frac{1}{2} (2 \tan^{-1} \sqrt{x}) \\ &= \frac{1}{2} \cos^{-1} \left(\frac{1-(\sqrt{x})^2}{1+(\sqrt{x})^2} \right) \quad \left[\because 2 \tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \right] \\ &= \frac{1}{2} \cos^{-1} \left(\frac{1-x}{1+x} \right) = RHS \end{aligned}$$

11. Prove that $\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) = \frac{9}{4} \sin^{-1} \left(\frac{2\sqrt{2}}{3} \right)$

Ans:

$$LHS = \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} = \frac{9}{4} \left(\frac{\pi}{2} - \sin^{-1} \frac{1}{3} \right) = \frac{9}{4} \cos^{-1} \frac{1}{3}$$

$$\text{Let } \cos^{-1} \frac{1}{3} = x \Rightarrow \cos x = \frac{1}{3} \Rightarrow \sin x = \sqrt{1 - \cos^2 x}$$

$$\Rightarrow \sin x = \sqrt{1 - \left(\frac{1}{3} \right)^2} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow x = \sin^{-1} \left(\frac{2\sqrt{2}}{3} \right) \Rightarrow \cos^{-1} \frac{1}{3} = \sin^{-1} \left(\frac{2\sqrt{2}}{3} \right)$$

$$\therefore \frac{9}{4} \cos^{-1} \frac{1}{3} = \frac{9}{4} \sin^{-1} \left(\frac{2\sqrt{2}}{3} \right) = RHS$$

12. Find the principal value of $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$

Ans:

$$\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$$

$$= \tan^{-1} \left(\tan \frac{\pi}{3} \right) - \sec^{-1} \left(-\sec \frac{\pi}{3} \right)$$

$$= \frac{\pi}{3} - \sec^{-1} \left(\sec \left(\pi - \frac{\pi}{3} \right) \right) = \frac{\pi}{3} - \sec^{-1} \left(\sec \frac{2\pi}{3} \right)$$

$$= \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$$

13. Prove that : $\cos \left(\sin^{-1} \frac{3}{5} + \cot^{-1} \frac{3}{2} \right) = \frac{6}{5\sqrt{13}}$

Ans:

$$\text{Let } \sin^{-1} \frac{3}{5} = x \text{ and } \cot^{-1} \frac{3}{2} = y$$

$$\text{Then } \sin x = \frac{3}{5} \text{ and } \cot y = \frac{3}{2}$$

$$\text{Now } \cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\text{and } \sin y = \frac{1}{\sqrt{1 + \cot^2 y}} = \frac{1}{\sqrt{1 + \left(\frac{3}{2}\right)^2}} = \frac{1}{\sqrt{1 + \frac{9}{4}}} = \frac{1}{\sqrt{\frac{13}{4}}} = \frac{2}{\sqrt{13}}$$

$$\Rightarrow \cos y = \frac{3}{\sqrt{13}}$$

$$\begin{aligned} LHS &= \cos(x + y) = \cos x \cos y - \sin x \sin y = \frac{4}{5} \times \frac{3}{\sqrt{13}} - \frac{3}{5} \times \frac{2}{\sqrt{13}} \\ &= \frac{12}{5\sqrt{13}} - \frac{6}{5\sqrt{13}} = \frac{6}{5\sqrt{13}} = RHS \end{aligned}$$

Write the value of $\tan\left(2 \tan^{-1} \frac{1}{5}\right)$

Ans:

$$\text{Let } 2 \tan^{-1} \frac{1}{5} = x \Rightarrow \tan^{-1} \frac{1}{5} = \frac{x}{2} \Rightarrow \tan \frac{x}{2} = \frac{1}{5}$$

$$\tan\left(2 \tan^{-1} \frac{1}{5}\right) = \tan x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}} = \frac{2 \times \frac{1}{5}}{1 - \left(\frac{1}{5}\right)^2} = \frac{2}{5} \times \frac{25}{24} = \frac{5}{12}$$

14. Find the value of the following: $\tan \frac{1}{2} \left[\sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-y^2}{1+y^2} \right]$, $|x| < 1$, $y > 0$ and $xy < 1$

Ans:

$$\text{Let } x = \tan \alpha \text{ and } y = \tan \beta \Rightarrow \alpha = \tan^{-1} x, \beta = \tan^{-1} y$$

$$\tan \frac{1}{2} \left[\sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-y^2}{1+y^2} \right]$$

$$= \tan \frac{1}{2} \left[\sin^{-1} \frac{2 \tan \alpha}{1 + \tan^2 \alpha} + \cos^{-1} \frac{1 - \tan^2 \beta}{1 + \tan^2 \beta} \right]$$

$$= \tan \frac{1}{2} \left[\sin^{-1}(\sin 2\alpha) + \cos^{-1}(\cos 2\beta) \right] \quad \left[\because \sin 2\alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha} \text{ and } \cos 2\beta = \frac{1 - \tan^2 \beta}{1 + \tan^2 \beta} \right]$$

$$= \tan \frac{1}{2} [2\alpha + 2\beta] = \tan[\alpha + \beta] = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} = \frac{x + y}{1 - xy}$$

15. Write the value of $\tan^{-1} \left[2 \sin \left(2 \cos^{-1} \frac{\sqrt{3}}{2} \right) \right]$

Ans:

$$\tan^{-1} \left[2 \sin \left(2 \cos^{-1} \frac{\sqrt{3}}{2} \right) \right]$$

$$= \tan^{-1} \left[2 \sin \left(2 \cdot \frac{\pi}{6} \right) \right] = \tan^{-1} \left[2 \sin \left(\frac{\pi}{3} \right) \right] = \tan^{-1} \left[2 \times \frac{\sqrt{3}}{2} \right]$$

$$= \tan^{-1} \sqrt{3} = \frac{\pi}{3}$$

16. Prove that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$

Ans:

$$\text{Let } \sin^{-1}\frac{3}{4} = \alpha \Rightarrow \sin \alpha = \frac{3}{4}$$

$$\Rightarrow \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} = \frac{3}{4} \quad \left[\because \sin 2\alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha} \right]$$

$$\Rightarrow 3 + 3 \tan^2 \frac{\alpha}{2} = 8 \tan \frac{\alpha}{2} \Rightarrow 3 \tan^2 \frac{\alpha}{2} - 8 \tan \frac{\alpha}{2} + 3 = 0$$

$$\Rightarrow \tan \frac{\alpha}{2} = \frac{8 \pm \sqrt{64 - 36}}{6} = \frac{8 \pm \sqrt{28}}{6}$$

$$\Rightarrow \tan \frac{\alpha}{2} = \frac{8 \pm 2\sqrt{7}}{6} \Rightarrow \tan \frac{\alpha}{2} = \frac{4 \pm \sqrt{7}}{3}$$

$$\Rightarrow \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$$

17. If $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$, **then prove that** $\sin y = \tan^2 \frac{x}{2}$

Ans:

$$y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$$

$$\Rightarrow y = \frac{\pi}{2} - \tan^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x}) = \frac{\pi}{2} - 2 \tan^{-1}(\sqrt{\cos x})$$

$$\Rightarrow y = \frac{\pi}{2} - \cos^{-1}\left(\frac{1 - \cos x}{1 + \cos x}\right) \quad \left[\because 2 \tan^{-1} x = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right) \right]$$

$$\Rightarrow y = \sin^{-1}\left(\frac{1 - \cos x}{1 + \cos x}\right)$$

$$\Rightarrow \sin y = \frac{1 - \cos x}{1 + \cos x} = \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} = \tan^2 \frac{x}{2}$$

18. If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1$, **then find the value of x.**

Ans:

$$\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1 \Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1} x = \sin^{-1} 1$$

$$\Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1} x = \frac{\pi}{2} \Rightarrow \sin^{-1}\frac{1}{5} = \frac{\pi}{2} - \cos^{-1} x = \sin^{-1} x$$

$$\Rightarrow x = \frac{1}{5}$$

19. Prove that $2 \tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2 \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

Ans:

$$LHS = 2 \tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2 \tan^{-1}\frac{1}{8}$$

$$\begin{aligned}
&= 2 \tan^{-1} \frac{1}{5} + 2 \tan^{-1} \frac{1}{8} + \sec^{-1} \frac{5\sqrt{2}}{7} = 2 \left(\tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{8} \right) + \sec^{-1} \frac{5\sqrt{2}}{7} \\
&= 2 \tan^{-1} \left(\frac{\frac{1}{5} + \frac{1}{8}}{1 - \frac{1}{5} \cdot \frac{1}{8}} \right) + \tan^{-1} \sqrt{\left(\frac{5\sqrt{2}}{7} \right)^2 - 1} \\
&= 2 \tan^{-1} \left(\frac{\frac{8+5}{40}}{1 - \frac{1}{40}} \right) + \tan^{-1} \sqrt{\frac{50}{49} - 1} \\
&= 2 \tan^{-1} \left(\frac{\frac{13}{40}}{\frac{39}{40}} \right) + \tan^{-1} \sqrt{\frac{1}{49}} = 2 \tan^{-1} \left(\frac{13}{39} \right) + \tan^{-1} \frac{1}{7} \\
&= 2 \tan^{-1} \left(\frac{1}{3} \right) + \tan^{-1} \frac{1}{7} = \tan^{-1} \left(\frac{2 \left(\frac{1}{3} \right)}{1 - \left(\frac{1}{3} \right)^2} \right) + \tan^{-1} \frac{1}{7} \quad \left[\because 2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right) \right] \\
&= \tan^{-1} \left(\frac{\frac{2}{3}}{1 - \frac{1}{9}} \right) + \tan^{-1} \frac{1}{7} = \tan^{-1} \left(\frac{\frac{2}{3}}{\frac{8}{9}} \right) + \tan^{-1} \frac{1}{7} = \tan^{-1} \left(\frac{2}{3} \times \frac{9}{8} \right) + \tan^{-1} \frac{1}{7} \\
&= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7} = \tan^{-1} \left(\frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \cdot \frac{1}{7}} \right) \quad \left(\because \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-x.y} \right) \right) \\
&= \tan^{-1} \left(\frac{\frac{21+4}{28}}{1 - \frac{3}{28}} \right) = \tan^{-1} \left(\frac{\frac{25}{28}}{\frac{25}{28}} \right) = \tan^{-1} 1 = \frac{\pi}{4}
\end{aligned}$$

20. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$ then write the value of $x + y + xy$.

Ans:

$$\begin{aligned}
&\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4} \\
&\Rightarrow \tan^{-1} \left(\frac{x+y}{1-x.y} \right) = \frac{\pi}{4} \Rightarrow \frac{x+y}{1-x.y} = \tan \frac{\pi}{4} = 1 \\
&\Rightarrow x+y = 1-xy \Rightarrow x+y+xy = 1
\end{aligned}$$