CHAPTER – 3: MATRICES

MARKS WEIGHTAGE – 03 marks

Previous Years Board Exam (Important Questions & Answers)

1. Use elementary column operation $C_2 \to C_2 - 2C_1$ in the matrix equation
$$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

Ans:

Given that $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Applying $C_2 \to C_2 - 2C_1$, we get
$$\begin{bmatrix} 4 & -6 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ 1 & -1 \end{bmatrix}$$

2. If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$ write the value of $a - 2b$.

Ans:

Give that $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$

On equating, we get

$a + 4 = 2a + 2$, $3b = b + 2$, $a - 8b = -6$

$\Rightarrow a = 2, b = 1$

Now the value of $a - 2b = 2 - (2 \times 1) = 2 - 2 = 0$

3. If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is an identity matrix.

Ans:

$7A - (I + A)^3 = 7A - \{I^3 + 3I^2A + 3I.A^2 + A^3\}$

$= 7A - \{I + 3A + 3A + A^2A\}$ $\quad [\because I^3 = I^2 = I , A^2 = A]$

$= 7A - \{I + 6A + A^2\} = 7A - \{I + 6A + A\}$

$= 7A - \{I + 7A\} = 7A - I - 7A = -I$

4. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x + y$.

Ans:

Given that $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$

Equating, we get

$x - y = -1 \ldots (i)$

$2x - y = 0 \ldots (ii)$

$z = 4, w = 5$

$(ii) - (i) \Rightarrow 2x - y - x + y = 0 + 1$

$\Rightarrow x = 1$ and $y = 2$

$\therefore x + y = 2 + 1 = 3.$

5. Solve the following matrix equation for x : $\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$

Ans:

Given that $\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} x-2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$

$\Rightarrow x - 2 = 0$

$\Rightarrow x = 2$

6. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$**, find (*x* – *y*).**

Ans:

Given that $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

Equating we get $8 + y = 0$ and $2x + 1 = 5$

$\Rightarrow y = -8$ and $x = 2$

$\Rightarrow x - y = 2 + 8 = 10$

7. For what value of *x*, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ **a skew-symmetric matrix?**

Ans:

A will be skew symmetric matrix if

$A = -A'$

$\Rightarrow \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -x \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$

Equating, we get $x = 2$

8. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ **and $A^2 = kA$, then write the value of k.**

Ans:

Given $A^2 = kA$

$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow k = 2$

9. Find the value of *a* if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Ans:

Given that $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Equating the corresponding elements we get.

$a - b = -1 \ldots(i)$

$2a + c = 5 \ldots(ii)$

$2a - b = 0 \ldots(iii)$

$3c + d = 13 \ldots(iv)$

From (iii) $2a = b$

$\Rightarrow a = \frac{b}{2}$

Putting in (i) we get $\frac{b}{2} - b = -1$

$\Rightarrow -\frac{b}{2} = -1 \Rightarrow b = 2$

$\therefore a = 1$

(ii) $c = 5 - 2 \times 1 = 5 - 2 = 3$

(iv) $d = 13 - 3 \times (3) = 13 - 9 = 4$

i.e. $a = 1, b = 2, c = 3, d = 4$

10. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A.

Ans:

Given that $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$

11. If A is a square matrix such that $A^2 = A$, then write the value of $(I + A)^2 - 3A$.

Ans:

$(I + A)^2 - 3A = I^2 + A^2 + 2A - 3A$

$= I^2 + A^2 - A$

$= I^2 + A - A$ [$\because A^2 = A$]

$= I^2 = I \cdot I = I$

12. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, write the value of x.

Ans:

Given that $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

Equating the corresponding elements we get.

$2x - y = 10 \ldots(i)$

$3x + y = 5 \ldots(ii)$

Adding (i) and (ii), we get $2x - y + 3x + y = 10 + 5$

$\Rightarrow 5x = 15 \Rightarrow x = 3.$

13. Find the value of $x + y$ from the following equation: $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Ans:

Given that $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Equating the corresponding element we get

$2x + 3 = 7$ and $2y - 4 = 14$

$\Rightarrow x = \frac{7-3}{2} \ \ and \ \ y = \frac{14+4}{2}$

$\Rightarrow x = 2$ and $y = 9$

$\therefore\ x + y = 2 + 9 = 11$

14. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ ***and*** $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$**, then find $A^T - B^T$.**

Ans:

Given that $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

Now, $A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

15. If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$**, write the value of x**

Ans:

Given that $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2-6 & -6+12 \\ 5-14 & -15+28 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

Equating the corresponding elements, we get

$x = 13$

16. Simplify: $\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

Ans:

$\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos^2\theta+\sin^2\theta & 0 \\ 0 & \cos^2\theta+\sin^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

17. Write the values of $x - y + z$ from the following equation: $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Ans:

By definition of equality of matrices, we have

$x + y + z = 9$ ... (i)

$x + z = 5$ .... (ii)

$y + z = 7$ ... (iii)

$(i) - (ii) \Rightarrow x + y + z - x - z = 9 - 5$

$\Rightarrow y = 4$ ... (iv)

$(ii) - (iv) \Rightarrow x - y + z = 5 - 4$

$\Rightarrow x - y + z = 1$

18. If $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$**, then find the value of y.**

Ans:

Given that $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$

By definition of equality of matrices, we have

$y + 2x = 7$

$-x = -2 \Rightarrow x = 2$

$\therefore y + 2(2) = 7 \Rightarrow y = 3$