CHAPTER – 13: PROBABILITY

MARKS WEIGHTAGE – 10 marks

Previous Years Board Exam (Important Questions & Answers)

1. **Three cards are drawn at random (without replacement) from a well shuffled pack of 52 playing cards. Find the probability distribution of number of red cards. Hence find the mean of the distribution.**

   Ans:

   Let the number of red card in a sample of 3 cards drawn be random variable *X*. Obviously *X* may have values 0,1,2,3.

   Now $P(X = 0)$= Probability of getting no red card = $\frac{^{26}C_3}{^{52}C_3} = \frac{2600}{22100} = \frac{2}{17}$

   $P(X = 1)$= Probability of getting one red card and two non-red cards = $\frac{^{26}C_1 \times {^{26}C_2}}{^{52}C_3} = \frac{8450}{22100} = \frac{13}{34}$

   $P(X = 2)$= Probability of getting two red card and one non-red card = $\frac{^{26}C_2 \times {^{26}C_1}}{^{52}C_3} = \frac{8450}{22100} = \frac{13}{34}$

   $P(X = 3)$= Probability of getting 3 red cards = $\frac{^{26}C_3}{^{52}C_3} = \frac{2600}{22100} = \frac{2}{17}$

   Hence, the required probability distribution in table as

X	0	1	2	3
***P*(*X*)**	$\frac{2}{17}$	$\frac{13}{34}$	$\frac{13}{34}$	$\frac{2}{17}$

   $\therefore$ Required mean = $E(X)$ = $\sum p_i x_i = 0\times\frac{2}{17} + 1\times\frac{13}{34} + 2\times\frac{13}{34} + 3\times\frac{2}{17}$

   $= \frac{13}{34} + \frac{26}{34} + \frac{6}{17} = \frac{13+26+12}{34} = \frac{51}{34} = \frac{3}{2}$

2. **There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two-headed coin?**

   Ans:

   Let E_1 ,E_2 ,E_3 and A be events defined as

   E_1 = selection of two-headed coin

   E_2 = selection of biased coin that comes up head 75% of the times.

   E_3 = selection of biased coin that comes up tail 40% of the times.

   A = getting head.

   $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

   $P(A/E_1) = 1$, $P(A/E_2) = \frac{3}{4}$ and $P(A/E_3) = \frac{3}{5}$

   By using Baye's theorem, we have

   $$P(E_1 / A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$=\frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{3}{5}}=\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}}=\frac{\frac{1}{3}}{\frac{20+15+12}{20}}=\frac{1}{3}\times\frac{60}{47}=\frac{20}{47}$$

3. **Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean of the distribution.**

Ans:

First six positive integers are 1, 2, 3, 4, 5, 6

If two numbers are selected at random from above six numbers then sample space *S* is given by

$$S=\{(1,2)\,(1,3),(1,4),(1,5),(1,6),(2,1),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,4),(3,5),$$
$$(3,6),(4,1),(4,2),(4,3),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,6),(6,1),(6,2),(6,3)$$
$$(6,4)\,(6,5)\}$$

$n(s)$ = 30.

Here, X is random variable, which may have value 2, 3, 4, 5 or 6.

Therefore, required probability distribution is given as

$P(X = 2)$ = Probability of event getting (1, 2), (2, 1) = $\frac{2}{30}$

$P(X = 3)$ = Probability of event getting (1, 3), (2, 3), (3, 1), (3, 2) = $\frac{4}{30}$

$P(X = 4)$ = Probability of event getting (1, 4), (2, 4), (3, 4), (4, 1), (4, 2), (4, 3) = $\frac{6}{30}$

$P(X = 5)$ = Probability of event getting (1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4) = $\frac{8}{30}$

$P(X = 6)$ = Probability of event getting (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5) = $\frac{10}{30}$

It is represented in tabular form as

X	2	3	4	5	6
***P*(*X*)**	$\frac{2}{30}$	$\frac{4}{30}$	$\frac{6}{30}$	$\frac{8}{30}$	$\frac{10}{30}$

Required mean = $E(x)$ = $\sum p_i x_i = 2\times\frac{2}{30}+3\times\frac{4}{30}+4\times\frac{6}{30}+5\times\frac{8}{30}+6\times\frac{10}{30}$

$$=\frac{4+12+24+40+60}{30}=\frac{140}{30}=\frac{14}{3}=4\frac{2}{3}$$

4. **An experiment succeeds thrice as often as it fails. Find the probability that in the next five trials, there will be at least 3 successes.**

Ans:

An experiment succeeds thrice as often as it fails.

$\Rightarrow p$ = P(getting success) =

and q = P(getting failure) =

Here, number of trials = n = 5

By binomial distribution, we have

P(X = x) = ${}^nC_x q^{n-x} p^x$, x = 0, 1, 2,...,n

$$P(X=r)={}^5C_r\left(\frac{3}{4}\right)^r\left(\frac{1}{4}\right)^{5-r}$$

Now , P(getting at least 3 success) = $P(X = 3) + P(X = 4) + P(X = 5)$

$= {}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + {}^5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^1 + {}^5C_5\left(\frac{3}{4}\right)^5$

$= \left(\frac{3}{4}\right)^3\left[10\times\frac{1}{16}+15\times\frac{1}{16}+9\times\frac{1}{16}\right] = \frac{27}{64}\times\frac{34}{16} = \frac{459}{512}$

5. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.

Ans:

Let E_1, E_2 , E_3 ,E_4 and A be event defined as

E_1 = the lost card is a spade card.

E_2 = the lost card is a heart card.

E_3 = the lost card is a club card.

E_4 = the lost card is diamond card.

and A = Drawing three spade cards from the remaining cards.

$P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$

P(A/E$_1$) = $\frac{{}^{12}C_3}{{}^{51}C_3} = \frac{220}{20825}$, P(A/E$_2$) = $\frac{{}^{13}C_3}{{}^{51}C_3} = \frac{286}{20825}$

P(A/E$_3$) = $\frac{{}^{13}C_3}{{}^{51}C_3} = \frac{286}{20825}$ P(A/E$_4$) = $\frac{{}^{13}C_3}{{}^{51}C_3} = \frac{286}{20825}$

By using Baye's theorem, we have

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)+P(E_4)P(A/E_4)}$$

$$= \frac{\frac{1}{4}\times\frac{220}{20825}}{\frac{1}{4}\times\frac{220}{20825}+\frac{1}{4}\times\frac{286}{20825}+\frac{1}{4}\times\frac{286}{20825}+\frac{1}{4}\times\frac{286}{20825}} = \frac{220}{220+286+286+286} = \frac{220}{1078} = \frac{10}{49}$$

6. From a lot of 15 bulbs which include 5 defectives, a sample of 4 bulbs is drawn one by one with replacement. Find the probability distribution of number of defective bulbs. Hence find the mean of the distribution.

Ans:

Let the number of defective bulbs be represented by a random variable X. X may have value 0, 1, 2, 3, 4.

If p is the probability of getting defective bulb in a single draw then $p = \frac{5}{15} = \frac{1}{3}$

$\therefore$ q = Probability of getting non defective bulb = 1 – p = $1-\frac{1}{3} = \frac{2}{3}$

Since each trial in this problem is Bernoulli trials, therefore we can apply binomial distribution as

P(X = x) = ${}^nC_x q^{n-x} p^x$, x = 0, 1, 2,...,n

$P(X = r) = {}^4C_r\left(\frac{1}{3}\right)^r\left(\frac{2}{3}\right)^{4-r}$

Now, P(X = 1) = ${}^4C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^3 = 4\times\frac{1}{3}\times\frac{8}{27} = \frac{32}{81}$

P(X = 2) = ${}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 = 6\times\frac{1}{9}\times\frac{4}{9} = \frac{24}{81}$

$P(X = 3) = {}^4C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1 = 4\times\frac{1}{27}\times\frac{2}{3} = \frac{8}{81}$

$P(X = 4) = {}^4C_4\left(\frac{1}{3}\right)^4 = \frac{1}{81}$

Now probability distribution table is

X	0	1	2	3	4
***P*(*X*)**	$\frac{16}{81}$	$\frac{32}{81}$	$\frac{24}{81}$	$\frac{8}{81}$	$\frac{1}{81}$

Now mean E(*X*) = $\sum p_i x_i = 0\times\frac{16}{81}+1\times\frac{32}{81}+2\times\frac{24}{81}+3\times\frac{8}{81}+4\times\frac{1}{81}$

$= \frac{32+48+24+4}{81} = \frac{106}{81} = \frac{4}{3}$

7. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (*i*) the youngest is a girl? (*ii*) atleast one is a girl?

Ans:

A family has 2 children, then Sample space = *S* = {*BB*, *BG*,*GB*,*GG*}, where *B* stands for Boy and *G* for Girl.

(*i*) Let *A* and *B* be two event such that

A = Both are girls = {*GG*}

B = the youngest is a girl = {*BG, GG*}

Now, $P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$

(*ii*) Let *C* be event such that

C = at least one is a girl = {*BG*,*GB*,*GG*}

Now, $P(A/C) = \frac{P(A\cap C)}{P(C)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

8. In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid. Find the mean of the distribution also. Write one more value which is expected from a well trained scout.

Ans:

Let *X* be no. of selected scouts who are well trained in first aid. Here random variable *X* may have value 0, 1, 2.

Now, $P(X = 0) = \frac{{}^{20}C_2}{{}^{50}C_2} = \frac{20\times19}{50\times49} = \frac{38}{245}$

$P(X = 1) = \frac{{}^{20}C_1\times{}^{30}C_1}{{}^{50}C_2} = \frac{20\times30\times2}{50\times49} = \frac{120}{245}$

$P(X = 2) = \frac{{}^{30}C_2}{{}^{50}C_2} = \frac{30\times29}{50\times49} = \frac{87}{245}$

Now probability distribution table is

X	0	1	2
$P(x)$	$\frac{38}{245}$	$\frac{120}{245}$	$\frac{87}{245}$

Now mean $E(X) = \sum p_i x_i = 0\times\frac{38}{245}+1\times\frac{120}{245}+2\times\frac{87}{245}=\frac{120+174}{245}=\frac{294}{245}$

A well trained scout should be disciplined

9. Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is actually a six. Find the probability that it is actually a six. Do you also agree that the value of truthfulness leads to more respect in the society?

Ans:

Let E_1, E_2 and E be three events such that

E_1 = six occurs

E_2 = six does not occurs

E = man reports that six occurs in the throwing of the dice.

Now $P(E_1) = \frac{1}{6}$, $P(E_2) = \frac{5}{6}$

$P(E/E_1) = \frac{4}{5}$, $P(E/E_2) = 1-\frac{4}{5}=\frac{1}{5}$

By using Baye's theorem, we obtain

$$P(E_1/E)=\frac{P(E_1)P(E/E_1)}{P(E_1)P(E/E_1)+P(E_2)P(E/E_2)}$$

$$=\frac{\frac{1}{6}\times\frac{4}{5}}{\frac{1}{6}\times\frac{4}{5}+\frac{5}{6}\times\frac{1}{5}}=\frac{4}{4+5}=\frac{4}{9}$$

10. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time. Write at least one advantage of coming to school in time.

Ans:

Let E_1 and E_2 be two events such that

E_1 = A coming to the school in time.

E_2 = B coming to the school in time.

Here $P(E_1) = \frac{3}{7}$ and $P(E_2) = \frac{5}{7}$

$\Rightarrow P(\overline{E_1})=\frac{4}{7}, P(\overline{E_2})=\frac{2}{7}$

P (only one of them coming to the school in time) = $P(E_1)P(\overline{E_2})+P(\overline{E_1})P(E_2)$

$=\frac{3}{7}\times\frac{2}{7}+\frac{5}{7}\times\frac{4}{7}=\frac{6+20}{49}=\frac{26}{49}$

Coming to school in time i.e., punctuality is a part of discipline which is very essential for development of an individual.

11. In a hockey match, both teams *A* and *B* scored same number of goals up to the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternately

and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, find their respective probabilities of winning the match and state whether the decision of the referee was fair or not.

Ans:

Let E_1, E_2 be two events such that

E_1 = the captain of team '*A*' gets a six.

E_2 = the captain of team '*B*' gets a six.

Here $P(E_1) = \frac{1}{6}$, $P(E_2) = \frac{1}{6}$

$$P(E_1') = 1 - \frac{1}{6} = \frac{5}{6},\ P(E_2') = 1 - \frac{1}{6} = \frac{5}{6}$$

Now P (winning the match by team A) = $\frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + ...$

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + ...$$

$$= \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11}$$

$\therefore$ P (winning the match by team B) = $1 - \frac{6}{11} = \frac{5}{11}$

The decision of refree was not fair because the probability of winning match is more for that team who start to throw dice.

12. A speaks truth in 60% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A?

Ans:

Let E_1 be the event that A speaks truth and E_2 be the event that B speaks truth. Then E and f are independent events such that

$$P(E_1) = \frac{60}{100} = \frac{3}{5}, P(E_1) = \frac{90}{100} = \frac{9}{10}$$

$$\Rightarrow P(\overline{E_1}) = \frac{2}{5}, P(\overline{E_2}) = \frac{1}{10}$$

P (A and B contradict each other) = $P(E_1)P(\overline{E_2}) + P(\overline{E_1})P(E_2)$

$$= \frac{3}{5} \times \frac{1}{10} + \frac{2}{5} \times \frac{9}{10} = \frac{3+18}{50} = \frac{21}{50}$$

Yes, the statement of *B* will carry more weight as the probability of B to speak truth is more than that of *A*.

13. Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. Interpret the result and state which of the above stated methods is more beneficial for the patient.

Ans:

Let E_1, E_2 , A be events defined as

E_1 = treatment of heart attack with Yoga and meditation

E_2 = treatment of heart attack with certain drugs.
A = Person getting heart attack.

$P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$

P(A/E$_1$) = $40\% - \left(40 \times \frac{30}{100}\right)\% = 40\% - 12\% = 28\% = \frac{28}{100}$

P(A/E$_2$) = $40\% - \left(40 \times \frac{25}{100}\right)\% = 40\% - 10\% = 30\% = \frac{30}{100}$

By using Baye's theorem, we have

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{28}{100}}{\frac{1}{2} \times \frac{28}{100} + \frac{1}{2} \times \frac{30}{100}} = \frac{28}{28+30} = \frac{28}{58} = \frac{14}{29}$$

The problem emphasises the importance of Yoga and meditation.
Treatment with Yoga and meditation is more beneficial for the heart patient.

14. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes and hence find its mean.

Ans:

Since each trial in this problem is Bernoulli trials, therefore we can apply binomial distribution as

P(X = x) = ${}^nC_x q^{n-x} p^x$, x = 0, 1, 2,...,n

$$P(X = r) = {}^4C_r \left(\frac{1}{6}\right)^r \left(\frac{5}{6}\right)^{4-r}$$

Now, P(X = 0) = ${}^4C_0 \left(\frac{5}{6}\right)^4 = 1 \times \frac{625}{1296} = \frac{625}{1296}$

P(X = 1) = ${}^4C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^3 = 4 \times \frac{1}{6} \times \frac{125}{216} = \frac{500}{1296}$

P(X = 2) = ${}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = 4 \times \frac{1}{36} \times \frac{25}{36} = \frac{150}{1296}$

P(X = 3) = ${}^4C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^1 = 4 \times \frac{1}{216} \times \frac{5}{6} = \frac{20}{1296}$

P(X = 4) = ${}^4C_4 \left(\frac{1}{6}\right)^4 = 1 \times \frac{1}{1296} = \frac{1}{1296}$

Now probability distribution table is

X or x_i	0	1	2	3	4
P(X) or p_i	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$

Now mean E(X) = $\sum p_i x_i = 0 \times \frac{625}{1296} + 1 \times \frac{500}{1296} + 2 \times \frac{150}{1296} + 3 \times \frac{20}{1296} + 4 \times \frac{1}{1296}$

$$= \frac{500+300+60+4}{1296} = \frac{864}{1296} = \frac{2}{3}$$

15. In a certain college, 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 metres. Find the probability that the selected student is a girl.

Ans:

Let E_1, E_2, A be events such that

E_1 = student selected is girl

E_2 = student selected is Boy

A = student selected is taller than 1.75 metres.

$P(E_1) = \frac{60}{100} = \frac{3}{5}$, $P(E_2) = \frac{40}{100} = \frac{2}{5}$

$P(A/E_1) = \frac{1}{100}$, $P(A/E_2) = \frac{4}{100}$

By using Baye's theorem, we have

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{3}{5}\times\frac{1}{100}}{\frac{3}{5}\times\frac{1}{100} + \frac{2}{5}\times\frac{4}{100}} = \frac{3}{3+8} = \frac{3}{11}$$

16. Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

Ans:

Total no. of cards in the deck = 52

Number of red cards = 26

No. of cards drawn = 2 simultaneously

$\therefore$ X = value of random variable = 0, 1, 2

X or x_i	$P(X)$	$x_i P(X)$	$x_i^2 P(X)$
0	$\frac{^{26}C_0 \times ^{26}C_2}{^{52}C_2} = \frac{25}{102}$	0	0
1	$\frac{^{26}C_1 \times ^{26}C_1}{^{52}C_2} = \frac{52}{102}$	$\frac{52}{102}$	$\frac{52}{102}$
2	$\frac{^{26}C_0 \times ^{26}C_2}{^{52}C_2} = \frac{25}{102}$	$\frac{50}{102}$	$\frac{100}{102}$
		$\Sigma x_i P(X) = 1$	$\Sigma x_i^2 P(X) = \frac{152}{102}$

$$\text{Mean} = \mu = \Sigma x_i P(X) = 1$$

$$\text{Variance} = \sigma^2 = \Sigma x_i^2 P(X) - \mu^2$$

$$= \frac{152}{102} - 1 = \frac{50}{102} = \frac{25}{51} = 0.49$$

17. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin 3 times and notes the number of heads. If she gets 1,2,3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1,2,3, or 4 with the die?

Ans:

Consider the following events:

E_1 = Getting 5 or 6 in a single throw of a die.

E_2 = Getting 1, 2, 3, or 4 in a single throw of a die.

A = Getting exactly one head.

$P(E_1) = \frac{2}{6} = \frac{1}{3}$, $P(E_2) = \frac{4}{6} = \frac{2}{3}$

P(A/E$_1$) = Probability of getting exactly one head when a coin is tossed three times

$= {}^3C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2 = 3 \times \frac{1}{2} \times \frac{1}{4} = \frac{3}{8}$

P(A/E$_2$) = Probability of getting exactly one head when a coin is tossed once only = $\frac{1}{2}$

By using Baye's theorem, we have

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{8} + \frac{1}{3}} = \frac{1}{3} \times \frac{24}{11} = \frac{8}{11}$$

18. How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%?

Ans:

Let no. of times of tossing a coin be n.

Here, Probability of getting a head in a chance = $p = \frac{1}{2}$

Probability of getting no head in a chance = $q = 1 - \frac{1}{2} = \frac{1}{2}$

Now, P (having at least one head) = $P(X \geq 1)$

$= 1 - P(X = 0) = 1 - {}^nC_0 p^0 q^{n-0} = 1 - 1.1.\left(\frac{1}{2}\right)^n = 1 - \left(\frac{1}{2}\right)^n$

From question

$$1 - \left(\frac{1}{2}\right)^n > \frac{80}{100}$$

$$\Rightarrow 1 - \left(\frac{1}{2}\right)^n > \frac{8}{10} \Rightarrow 1 - \frac{8}{10} > \frac{1}{2^n}$$

$$\Rightarrow \frac{1}{5} > \frac{1}{2^n} \Rightarrow 2^n > 5 \Rightarrow n \geq 3$$

A man must have to toss a fair coin 3 times.

19. Of the students in a college, it is known that 60% reside in hostel and 40% day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain 'A' grade and 20% of day scholars attain 'A' grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an 'A' grade, what is the probability that the student is a hosteler?

Ans:

Let E_1, E_2 and A be events such that

E_1 = student is a hosteler

E_2 = student is a day scholar

A = getting A grade.

$P(E_1) = \frac{60}{100} = \frac{6}{10}$, $P(E_2) = \frac{40}{100} = \frac{4}{10}$

P(A/E$_1$) = $\frac{30}{100} = \frac{3}{10}$, P(A/E$_2$) = $\frac{20}{100} = \frac{2}{10}$

By using Baye's theorem, we have

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{6}{10} \times \frac{3}{10}}{\frac{6}{10} \times \frac{3}{10} + \frac{4}{10} \times \frac{2}{10}} = \frac{18}{18+8} = \frac{18}{26} = \frac{9}{13}$$

20. Find the mean number of heads in three tosses of a fair coin.

Ans:

The sample space of given experiment is S ={(HHH), (HHT), (HTT), (TTT), (TTH), (THH), (HTH), (THT)}

Let X denotes the no. of heads in three tosses of a fair coin Here, X is random which may have values 0, 1, 2, 3.

Now, $P(X = 0) = \frac{1}{8}$, $P(X = 1) = \frac{3}{8}$

$P(X = 2) = \frac{3}{8}$, $P(X = 3) = \frac{1}{8}$

Therefore, Probability distribution is

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

∴ Mean number ($E(x)$) = $\sum p_i x_i = 0\times\frac{1}{8} + 1\times\frac{3}{8} + 2\times\frac{3}{8} + 3\times\frac{1}{8}$

$$= \frac{3+6+3}{8} = \frac{12}{8} = \frac{3}{2} = 1.5$$