CHAPTER – 1: RELATIONS AND FUNCTIONS

MARKS WEIGHTAGE – 06 marks

Previous Years Board Exam (Important Questions & Answers)

1. If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in R$, find $(fog)(7)$

Ans:

Given $f(x) = x + 7$ and $g(x) = x - 7$, $x \in R$

$fog(x) = f(g(x)) = g(x) + 7 = (x - 7) + 7 = x$

$\Rightarrow (fog)(7) = 7$.

2. If $f(x)$ is an invertible function, find the inverse of $f(x) = \frac{3x-2}{5}$

Ans:

Given $f(x) = \frac{3x-2}{5}$

$Let\ \ y = \frac{3x-2}{5}$

$\Rightarrow 3x - 2 = 5y \Rightarrow x = \frac{5y+2}{3}$

$\Rightarrow f^{-1}(x) = \frac{5x+2}{3}$

3. Let T be the set of all triangles in a plane with R as relation in T given by $R = \{(T_1, T_2) : T_1 \cong T_2\}$. Show that R is an equivalence relation.

Ans:

(*i*) Reflexive

R is reflexive if $T_1\ R\ T_1$

Since $T_1 \cong T_1$

$\therefore$ R is reflexive.

(*ii*) Symmetric

R is symmetric if $T_1\ R\ T_2 \Rightarrow T_2\ R\ T_1$

Since $T_1 \cong T_2 \Rightarrow T_2 \cong T_1$

$\therefore$ R is symmetric.

(*iii*) Transitive

R is transitive if $T_1\ R\ T_2$ and $T_2\ R\ T_3 \Rightarrow T_1\ R\ T_3$

Since $T_1 \cong T_2$ and $T_2 \cong T_3 \Rightarrow T_1 \cong T_3$

$\therefore$ R is transitive

From (*i*), (*ii*) and (*iii*), we get R is an equivalence relation.

4. If the binary operation * on the set of integers Z, is defined by $a * b = a + 3b^2$, then find the value of 2 * 4.

Ans:

Given $a * b = a + 3b^2 \quad \forall a, b \in z$

$\therefore$ $2*4 = 2 + 3 \times 4^2 = 2 + 48 = 50$

5. Let * be a binary operation on N given by $a * b$ = HCF (a, b) $a, b \in N$. Write the value of 22 * 4.

Ans:

Given $a * b$ = HCF (a, b), $a, b \in N$

$\Rightarrow$ 22 * 4 = HCF (22, 4) = 2

6. Let $f : N \to N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, if \ \ n \ \ is \ \ odd \\ \frac{n}{2}, if \ \ n \ \ is \ \ even \end{cases}$ $for \ \ all \ \ n \in N$. Find whether the function f is bijective.

Ans:

Given that $f : N \to N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, if \ \ n \ \ is \ \ odd \\ \frac{n}{2}, if \ \ n \ \ is \ \ even \end{cases}$ $for \ \ all \ \ n \in N$.

Let $x, y \in N$ and let they are odd then

$f(x) = f(y) \Rightarrow \frac{x+1}{2} = \frac{y+1}{2} \Rightarrow x = y$

If $x, y \in N$ are both even then also

$f(x) = f(y) \Rightarrow \frac{x}{2} = \frac{y}{2} \Rightarrow x = y$

If $x, y \in N$ are such that x is even and y is odd then

$f(x) = \frac{x+1}{2} \ \ and \ \ f(y) = \frac{y}{2}$

Thus, $x \neq y$ for $f(x) = f(y)$

Let $x = 6$ and $y = 5$

We get $f(6) = \frac{6}{2} = 3, f(5) = \frac{5+1}{2} = 3$

$\therefore$ $f(x) = f(y)$ but $x \neq y$ ...(i)

So, $f(x)$ is not one-one.

Hence, $f(x)$ is not bijective.

7. If the binary operation *, defined on Q, is defined as $a * b = 2a + b - ab$, for all $a, b \in Q$, find the value of 3 * 4 .

Ans:

Given binary operation is $a*b = 2a + b - ab$

$\therefore$ $3 * 4 = 2 \times 3 + 4 - 3 \times 4$

$\Rightarrow$ $3 * 4 = -2$

8. What is the range of the function $f(x) = \frac{|x-1|}{(x-1)}$?

Ans:

We have given $f(x) = \frac{|x-1|}{(x-1)}$

$|x-1| = \begin{cases} (x-1), \ if \ \ x-1 > 0 \ \ or \ \ x > 1 \\ -(x-1), if \ \ x-1 < 0 \ \ or \ \ x < 1 \end{cases}$

(i) For $x > 1$, $f(x) = \frac{(x-1)}{(x-1)} = 1$

(ii) For $x < 1$, $f(x) = \frac{-(x-1)}{(x-1)} = -1$

$\therefore$ Range of $f(x) = \frac{|x-1|}{(x-1)}$ is $\{-1, 1\}$.

9. Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a, b) ; a, b \in Z$, and $(a - b)$ is divisible by 5.$\}$ Prove that R is an equivalence relation.

Ans:

We have provided $R = \{(a, b) : a, b \in Z$, and$(a - b)$ is divisible by 5$\}$

(i) As $(a - a) = 0$ is divisible by 5.

$\therefore (a, a) \in R \ \forall \ a \in R$

Hence, R is reflexive.

(ii) Let $(a, b) \in R$

$\Rightarrow (a - b)$ is divisible by 5.

$\Rightarrow -(b - a)$ is divisible by 5.

$\Rightarrow (b - a)$ is divisible by 5.

$\therefore (b, a) \in R$

Hence, R is symmetric.

(iii) Let $(a, b) \in R$ and $(b, c) \in Z$

Then, $(a - b)$ is divisible by 5 and $(b - c)$ is divisible by 5.

$(a - b) + (b - c)$ is divisible by 5.

$(a - c)$ is divisible by 5.

$\therefore (a, c) \in R$

$\Rightarrow R$ is transitive.

Hence, R is an equivalence relation.

10. Let * be a binary operation on Q defined by $a * b = \frac{3ab}{5}$. Show that * is commutative as well as associative. Also find its identity element, if it exists.

Ans:

For commutativity, condition that should be fulfilled is $a * b = b * a$

Consider $a * b = \frac{3ab}{5} = \frac{3ba}{5} = b * a$

$\therefore a * b = b * a$

Hence, * is commutative.

For associativity, condition is $(a * b) * c = a * (b * c)$

Consider $(a * b) * c = \left(\frac{3ab}{5}\right) * c = \frac{9ab}{25}$

and $a * (b * c) = a * \left(\frac{3bc}{5}\right) = \frac{9ab}{25}$

Hence, $(a * b) * c = a * (b * c)$

$\therefore$ * is associative.

Let $e \in Q$ be the identity element,

Then $a * e = e * a = a$

$\Rightarrow \frac{3ae}{5} = \frac{3ea}{5} = a \Rightarrow e = \frac{5}{3}$

11. If $f : R \rightarrow R$ be defined by $f(x) = (3 - x^3)^{1/3}$, then find $fof(x)$.

Ans:

If $f : R \rightarrow R$ be defined by $f(x) = (3 - x^3)^{1/3}$ then $(fof)\, x = f(f(x)) = f\,[(3 - x^3)^{1/3}]$

$= [3 - \{(3 - x^3)^{1/3}\}^3]^{1/3} = [3 - (3 - x^3)]^{1/3} = (x^3)^{1/3} = x$

12. Let $A = N ´ N$ and * be a binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that * is commutative and associative. Also, find the identity element for * on A, if any.

Ans:

Given $A = N \times N$

* is a binary operation on A defined by

$(a, b) * (c, d) = (a + c, b + d)$

(i) Commutativity: Let $(a, b), (c, d) \in N \times N$

Then $(a, b) * (c, d) = (a + c, b + d) = (c + a, d + b)$

($\because a, b, c, d \in \text{N}, a + c = c + a$ and $b + d = d + c$)

$= (c, d) * b$

Hence, $(a, b) * (c, d) = (c, d) * (a, b)$

$\therefore$ * is commutative.

(ii) Associativity: let $(a, b), (b, c), (c, d)$

Then $[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f) = ((a + c) + e, (b + d) + f)$

$= \{a + (c + e), b + (d + f)]$ ($\because$ set N is associative)

$= (a, b) * (c + e, d + f) = (a, b) * \{(c, d) * (e, f)\}$

Hence, $[(a, b) * (c, d)] * (e, f) = (a, b) * \{(c, d) * (e, f)\}$

$\therefore$ * is associative.

(iii) Let (x, y) be identity element for $\forall$ on A,

Then $(a, b) * (x, y) = (a, b)$

$\Rightarrow (a + x, b + y) = (a, b)$

$\Rightarrow a + x = a, b + y = b$

$\Rightarrow x = 0, y = 0$

But $(0, 0) \notin A$

$\therefore$ For *, there is no identity element.

13. If $f : R \to R$ and $g : R \to R$ are given by $f(x) = \sin x$ and $g(x) = 5x^2$, find $gof(x)$.

Ans:

Given $f : R \to R$ and $g : R \to R$ defined by $f(x) = \sin x$ and $g(x) = 5x^2$

$\therefore gof(x) = g[f(x)] = g(\sin x) = 5(\sin x)^2 = 5\sin^2 x$

14. Consider the binary operation* on the set {1, 2, 3, 4, 5} defined by a * b = min. {a, b}. Write the operation table of the operation *.

Ans:

Required operation table of the operation * is given as

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

15. If $f : R \to R$ is defined by $f(x) = 3x + 2$, define $f[f(x)]$.

Ans:

$f(f(x)) = f(3x + 2)$

$= 3.(3x + 2) + 2 = 9x + 6 + 2$

$= 9x + 8$

16. Write fog, if $f : R \to R$ and $g : R \to R$ are given by $f(x) = 8x^3$ and $g(x) = x^{1/3}$.

Ans:

$fog\,(x) = f\,(g(x))$

$= f\,(x^{1/3})$

$= 8(x^{1/3})^3$

$= 8x$

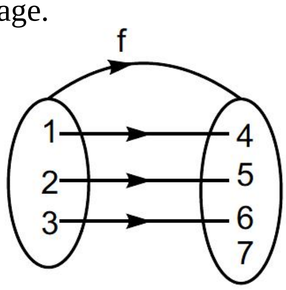
17. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2,5), (3, 6)} be a function from A to B. State whether f is one-one or not.

Ans:

f is one-one because

$f(1) = 4$; $f(2) = 5$; $f(3) = 6$

No two elements of A have same f image.



18. Let $f : R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g : R \to R$ such that gof = fog = I_R.

Ans:

$\because gof = fog = IR$

$\Rightarrow fog = IR$

$\Rightarrow fog(x) = I\,(x)$

$\Rightarrow f\,(g(x)) = x$ [$\because I(x) = x$ being identity function]

$\Rightarrow 10(g(x)) + 7 = x$ [$\because f(x) = 10x + 7$]

$\Rightarrow g(x) = \dfrac{x-7}{10}$

i.e., $g : R \to R$ is a function defined as $g(x) = \dfrac{x-7}{10}$

19. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \left(\dfrac{x-2}{x-3}\right)$.

Show that f is one-one and onto and hence find f^{-1}.

Ans:

Let $x_1, x_2 \in A$.

Now, $f(x_1) = f(x_2) \Rightarrow \dfrac{x_1-2}{x_1-3} = \dfrac{x_2-2}{x_2-3}$

$\Rightarrow (x_1-2)(x_2-3) = (x_1-3)(x_2-2)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

Hence f is one-one function.

For Onto

Let $y = \dfrac{x-2}{x-3} \Rightarrow xy - 3y = x - 2$

$\Rightarrow xy - x = 3y - 2 \Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = \dfrac{3y-2}{y-1}$ - - - - - (i)

From above it is obvious that $\forall$ y except 1, *i.e.*, $\forall y \in B = R - \{1\}$ $\exists x \in A$
Hence f is onto function.
Thus f is one-one onto function.
It f^{-1} is inverse function of f then $f^{-1}(y) = \dfrac{3y-2}{y-1}$ [from (i)]

20. The binary operation * : $R \times R \rightarrow R$ is defined as $a * b = 2a + b$. Find (2 * 3) * 4
Ans:
(2 * 3) * 4 = (2 × 2 +3) * 4
= 7 * 4
= 2 × 7 + 4 = 18

21. Show that f : N → N, given by $f(x) = \begin{cases} x+1, if \quad x \; is \; odd \\ x-1, if \quad x \; is \; even \end{cases}$ **is both one-one and onto.**

Ans:
For one-one
Case I : When x_1, x_2 are odd natural number.
$\therefore$ $f(x_1) = f(x_2) \Rightarrow x_1+1 = x_2 +1 \; \forall \; x_1 , x_2 \in N$
$\Rightarrow x_1 = x_2$
i.e., f is one-one.

Case II : When x_1, x_2 are even natural number
$\therefore f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1$
$\Rightarrow x_1 = x_2$
i.e., f is one-one.

Case III : When x_1 is odd and x_2 is even natural number
$f(x_1) = f(x_2) \Rightarrow x_1+1 = x_2 - 1$
$\Rightarrow x_2 - x_1 = 2$ which is never possible as the difference of odd and even number is always odd number.
Hence in this case $f(x_1) \neq f(x_2)$
i.e., f is one-one.

Case IV: When x_1 is even and x_2 is odd natural number
Similar as case III, We can prove f is one-one
For onto:
$\therefore f(x) = x +1$ if x is odd
$= x - 1$ if x is even
$\Rightarrow$ For every even number 'y' of codomain $\exists$ odd number y - 1 in domain and for every odd number y of codomain $\exists$ even number y +1 in Domain.
i.e. f is onto function.
Hence f is one-one onto function.

22. Consider the binary operations * : $R \times R \rightarrow$ R and o : $R \times R \rightarrow R$ defined as $a * b = |a - b|$ and $aob = a$ for all $a, b \in R$. Show that '*' is commutative but not associative, 'o' is associative but not commutative.
Ans:
For operation '*'
'*' : R ´ $R \rightarrow R$ such that
$a*b = |a - b| \;\; \forall a, b \in R$
Commutativity
$a*b = |a - b| = |b - a| = b * a$
i.e., '*' is commutative

Associativity
$\forall\ a, b, c \in R$
$(a * b) * c = |a - b| * c = ||a - b| - c|$
$a * (b * c) = a * |b - c| = |a - |b - c||$
But $||a - b| - c| \neq |a - |b - c||$
$\Rightarrow (a*b)* c \neq a*(b * c)$ " $a, b, c \in R$
$\Rightarrow$ * is not associative.
Hence, ‘*’ is commutative but not associative.

For Operation ‘o’
$o : R \times R \to R$ such that $aob = a$
Commutativity $\forall\ a, b \in R$
$aob = a$ and $boa = b$
$\because a \neq b \Rightarrow aob \neq boa$
$\Rightarrow$ ‘o’ is not commutative.
Associativity: " $a, b, c \in R$
$(aob)\ oc = aoc = a$
$ao(boc) = aob = a$
$\Rightarrow (aob)\ oc = ao\ (boc)$
$\Rightarrow$ ‘o’ is associative
Hence ‘o’ is not commutative but associative.

23. If the binary operation * on the set Z of integers is defined by $a * b = a + b - 5$, then write the identity element for the operation * in Z.
Ans:
Let e $\in Z$ be required identity
$\therefore\ a* e = a\ \forall\ a \in Z$
$\Rightarrow a + e - 5 = a$
$\Rightarrow e = a - a + 5$
$\Rightarrow e = 5$

24. If the binary operation * on set R of real numbers is defined as $a*b = \frac{3ab}{7}$, write the identity element in R for *.
Ans:
Let $e \in R$ be identity element.
$\therefore\ a * e = a\ \forall\ a \in R$
$\Rightarrow \frac{3ae}{7} = a \Rightarrow e = \frac{7a}{3a}$
$\Rightarrow e = \frac{7}{3}$

25. Prove that the relation R in the set $A = \{5, 6, 7, 8, 9\}$ given by $R = \{(a, b) : |a - b|$, is divisible by 2}, is an equivalence relation. Find all elements related to the element 6.
Ans:
Here R is a relation defined as $R = \{(a, b) : |a - b|$ is divisible by 2}
Reflexivity
Here $(a, a) \in R$ as $|a - a| = 0 = 0$ divisible by 2 *i.e.*, R is reflexive.
Symmetry
Let $(a, b) \in R$
$(a, b) \in R \Rightarrow |a - b|$ is divisible by 2
$\Rightarrow a - b = \pm 2m \Rightarrow b - a = \mp 2m$
$\Rightarrow |b - a|$ is divisible by 2 $\Rightarrow (b, a) \in R$

Hence R is symmetric

Transitivity Let $(a, b), (b, c) \in R$

Now, $(a, b), (b, c) \in R \Rightarrow |a - b|, |b - c|$ are divisible by 2

$\Rightarrow a - b = \pm 2m$ and $b - c = \pm 2n$

$\Rightarrow a - b + b - c = \pm 2(m + n)$

$\Rightarrow (a - c) = \pm 2k \quad [\because k = m + n]$

$\Rightarrow (a - c) = 2k$

$\Rightarrow (a - c)$ is divisible by 2 $\Rightarrow (a, c) \in R$.

Hence R is transitive.

Therefore, R is an equivalence relation.

The elements related to 6 are 6, 8.

26. Let * *be* a binary operation, on the set of all non-zero real numbers, given by $a * b = \frac{ab}{5}$ for all $a, b \in R - \{0\}$. Find the value of *x*, given that 2 * (*x* * 5) = 10.

Ans:

Given 2 * (x * 5) = 10

$\Rightarrow 2 * \frac{x \times 5}{5} = 10 \Rightarrow 2 * x = 10$

$\Rightarrow \frac{2 \times x}{5} = 10 \Rightarrow x = \frac{10 \times 5}{5} \Rightarrow x = 25$

27. Let *A* = {1, 2, 3, …, 9} and *R* be the relation in *A* × *A* defined by (*a, b*) *R* (*c, d*) if *a* + *d* = *b* + *c* for (*a, b*), (*c, d*) in *A* × *A*. Prove that *R* is an equivalence relation. Also obtain the equivalence class [(2, 5)].

Ans:

Given, R is a relation in $A \times A$ defined by $(a, b)R(c, d) \Rightarrow a + d = b + c$

(i) **Reflexivity:** $\forall a, b \in A$

Q $a + b = b + a \Rightarrow (a, b)R(a, b)$

So, R in reflexive.

(ii) **Symmetry:** Let $(a, b)\ R\ (c, d)$

Q $(a, b)R(c, d) \Rightarrow a + d = b + c$

$\Rightarrow b + c = d + a$ [Q $a, b, c, d \in N$ and N is commutative under addition[

$\Rightarrow c + b = d + a$

$\Rightarrow (c, d)R(a, b)$

So, R is symmetric.

(iii) **Transitivity:** Let $(a, b)R(c, d)$ and $(c, d)R(e, f)$

Now, $(a, b)R(c, d)$ and $(c, d)R(e, f) \Rightarrow a + d = b + c$ and $c + f = d + e$

$\Rightarrow a + d + c + f = b + c + d + e$

$\Rightarrow a + f = b + e$

$\Rightarrow (a, b)R(e, f)$.

R is transitive.

Hence, R is an equivalence relation.

2nd Part: Equivalence class: $[(2, 5)] = \{(a, b) \in A \times A: (a, b)R(2, 5)\}$

$= \{(a, b) \in A \times A: a + 5 = b + 2\}$

$= \{(a, b) \in A \times A: b - a = 3\}$

$= \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$