CHAPTER – 10: VECTOR ALGEBRA

MARKS WEIGHTAGE – 06 marks

Previous Years Board Exam (Important Questions & Answers)

1. **Write the projection of vector $\hat{i}+\hat{j}+\hat{k}$ along the vector $\hat{j}$.**

   Ans:

   Required projection $=\dfrac{(\hat{i}+\hat{j}+\hat{k}).\hat{j}}{|\hat{j}|}=\dfrac{0+1+0}{\sqrt{0+1+0}}=\dfrac{1}{1}=1$

2. **Find a vector in the direction of vector $2\hat{i}-3\hat{j}+6\hat{k}$ which has magnitude 21 units.**

   Ans:

   Required vector $=21\left(\dfrac{2\hat{i}-3\hat{j}+6\hat{k}}{\sqrt{4+9+36}}\right)=21\left(\dfrac{2\hat{i}-3\hat{j}+6\hat{k}}{\sqrt{49}}\right)$

   $=21\left(\dfrac{2\hat{i}-3\hat{j}+6\hat{k}}{7}\right)=3(2\hat{i}-3\hat{j}+6\hat{k})=6\hat{i}-9\hat{j}+18\hat{k}$

3. **Show that the vectors $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar if $\vec{a},\vec{b},\vec{c}$ are coplanar.**

   Ans:

   Let $\vec{a},\vec{b},\vec{c}$ are coplanar then we have $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$

   $\Rightarrow \vec{a}.(\vec{b}\times\vec{c})=\vec{b}.(\vec{c}\times\vec{a})=\vec{c}.(\vec{a}\times\vec{b})=0$

   Now, $\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right]=(\vec{a}+\vec{b}).\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\}$

   $=(\vec{a}+\vec{b}).\{\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\}$

   $=(\vec{a}+\vec{b}).\{\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}\}$

   $=\vec{a}.(\vec{b}\times\vec{c})+\vec{a}.(\vec{b}\times\vec{a})+\vec{a}.(\vec{c}\times\vec{a})+\vec{b}.(\vec{b}\times\vec{c})+\vec{b}.(\vec{b}\times\vec{a})+\vec{b}.(\vec{c}\times\vec{a})$

   $=\left[\vec{a}\ \vec{b}\ \vec{c}\right]+0+0+0+0+\left[\vec{b}\ \vec{c}\ \vec{a}\right]$

   $=\left[\vec{a}\ \vec{b}\ \vec{c}\right]+\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

   $=2\left[\vec{a}\ \vec{b}\ \vec{c}\right]=2\times 0=0$

   Hence, $\vec{a},\vec{b},\vec{c}$ are coplanar

4. **Show that the vectors $\vec{a},\vec{b},\vec{c}$ are coplanar if $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.**

   Ans:

   Let $\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}$ are coplanar

   $\Rightarrow (\vec{a}+\vec{b}).\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\}=0$

   $\Rightarrow (\vec{a}+\vec{b}).\{\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\}=0$

   $\Rightarrow (\vec{a}+\vec{b}).\{\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}\}=0$

   $\Rightarrow \vec{a}.(\vec{b}\times\vec{c})+\vec{a}.(\vec{b}\times\vec{a})+\vec{a}.(\vec{c}\times\vec{a})+\vec{b}.(\vec{b}\times\vec{c})+\vec{b}.(\vec{b}\times\vec{a})+\vec{b}.(\vec{c}\times\vec{a})=0$

   $\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]+0+0+0+0=0$

   $\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$

   $\Rightarrow \vec{a},\vec{b},\vec{c}$ are coplanar

5. Write a unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1, 3, 0) and (4, 5, 6) respectively.

Ans:

$\overrightarrow{PQ} = (4-1)\hat{i} + (5-3)\hat{j} + (6-0)\hat{k} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$\therefore$ Required unit vector = $\dfrac{3\hat{i}+2\hat{j}+6\hat{k}}{\sqrt{9+4+36}} = \dfrac{3\hat{i}+2\hat{j}+6\hat{k}}{\sqrt{49}} = \dfrac{3\hat{i}+2\hat{j}+6\hat{k}}{7} = \dfrac{3}{7}\hat{i} + \dfrac{2}{7}\hat{j} + \dfrac{6}{7}\hat{k}$

6. Write the value of the following : $\hat{i}\times(\hat{j}+\hat{k}) + \hat{j}\times(\hat{k}+\hat{i}) + \hat{k}\times(\hat{i}+\hat{j})$

Ans:

$\hat{i}\times(\hat{j}+\hat{k}) + \hat{j}\times(\hat{k}+\hat{i}) + \hat{k}\times(\hat{i}+\hat{j})$

$= \hat{i}\times\hat{j} + \hat{i}\times\hat{k} + \hat{j}\times\hat{k} + \hat{j}\times\hat{i} + \hat{k}\times\hat{i} + \hat{k}\times\hat{j}$

$= \hat{k} - \hat{j} + \hat{i} - \hat{k} + \hat{j} - \hat{i} = 0$

7. Find the value of '*p*' for which the vectors $3\hat{i}+2\hat{j}+9\hat{k}$ and $\hat{i}-2p\hat{j}+3\hat{k}$ are parallel.

Ans:

Since given two vectors are parallel.

$\Rightarrow \dfrac{3}{1} = \dfrac{2}{-2p} = \dfrac{9}{3} \Rightarrow \dfrac{3}{1} = \dfrac{2}{-2p}$

$\Rightarrow -6p = 2 \Rightarrow p = -\dfrac{1}{3}$

8. Find $\vec{a}.(\vec{b}\times\vec{c})$, if $\vec{a} = 2\hat{i}+\hat{j}+3\hat{k}, \vec{b} = -\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}+2\hat{k}$.

Ans:

Given that $\vec{a} = 2\hat{i}+\hat{j}+3\hat{k}, \vec{b} = -\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}+2\hat{k}$

$$\therefore \vec{a}.(\vec{b}\times\vec{c}) = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 2(4-1) - 1(-2-3) + 3(-1-6)$$

$= 6 + 5 - 21 = -10$

9. Show that the four points *A, B, C* and *D* with position vectors $4\hat{i}+5\hat{j}+\hat{k}, -\hat{j}-\hat{k}, 3\hat{i}+9\hat{j}+4\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$ are coplanar.

Ans:

Position vectors of *A*, *B*, *C* and *D* are

Position vector of *A* = $4\hat{i}+5\hat{j}+\hat{k}$

Position vector of *B* = $-\hat{j}-\hat{k}$

Position vector of *C* = $3\hat{i}+9\hat{j}+4\hat{k}$

Position vector of *D* = $4(-\hat{i}+\hat{j}+\hat{k}) = -4\hat{i}+4\hat{j}+4\hat{k}$

$\therefore \overrightarrow{AB} = -4\hat{i}-6\hat{j}-2\hat{k}, \overrightarrow{AC} = -\hat{i}+4\hat{j}+3\hat{k}, \overrightarrow{AD} = -8\hat{i}-\hat{j}+3\hat{k}$

$$\text{Now, } \overrightarrow{AB}.(\overrightarrow{AC}\times\overrightarrow{AD}) = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = -4(12+3) + 6(-3+24) - 2(1+32)$$

$= -60 + 126 - 66 = 0$

$\Rightarrow \overrightarrow{AB}.(\overrightarrow{AC}\times\overrightarrow{AD}) = 0$

Hence $\overrightarrow{AB}$, $\overrightarrow{AC}$ *and* $\overrightarrow{AD}$ are coplanar i.e. A, B, C and D are coplanar.

10. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$, making an angle of $\frac{\pi}{4}$ with *x*-axis., $\frac{\pi}{2}$ with y-axis and an acute angle θ with *z*-axis.

Ans:

Direction cosines of required vector $\vec{a}$ are

$l = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, $m = \cos\frac{\pi}{2} = 0$ and $n = \cos\theta$

$\because l^2 + m^2 + n^2 = 1$

$\therefore \left(\frac{1}{\sqrt{2}}\right)^2 + 0 + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \frac{1}{2} = \frac{1}{2}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow n = \frac{1}{\sqrt{2}}$

$\therefore$ Unit vector in the direction of $\vec{a} = \frac{1}{\sqrt{2}}\hat{i} + 0\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$

$\therefore \vec{a} = 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\hat{i} + 0\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right) = 5\hat{i} + 5\hat{k}$

11. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a}+\vec{b}|$ = 13 and $|\vec{a}|$ = 5 find the value of $|\vec{b}|$.

Ans:

Given $|\vec{a}+\vec{b}|$ = 13

$|\vec{a}+\vec{b}|^2 = 169 \Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = 169$

$\Rightarrow |\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = 169$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 = 169 \quad \left[\because \vec{a} \perp \vec{b} \Rightarrow \vec{a}.\vec{b} = 0\right]$

$\Rightarrow |\vec{b}|^2 = 169 - |\vec{a}|^2 = 169 - 25 = 144$

$\Rightarrow |\vec{b}| = 12$

12. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$.

Ans:

Let $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Projection of the vector $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{(\hat{i}+3\hat{j}+7\hat{k}).(2\hat{i}-3\hat{j}+6\hat{k})}{|2\hat{i}-3\hat{j}+6\hat{k}|}$

$= \frac{2-9+42}{\sqrt{4+9+36}} = \frac{35}{\sqrt{49}} = \frac{35}{7} = 5$

13. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+\vec{b}$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

Ans:

Given that $\vec{a}+\vec{b}$ is also a unit vector

$\therefore |\vec{a}+\vec{b}| = 1$

$|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b}).(\vec{a}+\vec{b})$

$\Rightarrow |\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = 1^2 = 1$

$\Rightarrow 1 + 2\vec{a}.\vec{b} + 1 = 1 \quad \left[\because |\vec{a}| = 1, |\vec{b}| = 1\right]$

$\Rightarrow 2\vec{a}.\vec{b} = -1 \Rightarrow \vec{a}.\vec{b} = -\frac{1}{2} \Rightarrow |\vec{a}||\vec{b}|\cos\theta = -\frac{1}{2}$

$\Rightarrow 1\times 1\times \cos\theta = -\frac{1}{2} \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \cos\theta = \cos\frac{2\pi}{3}$

$\Rightarrow \theta = \frac{2\pi}{3}$

14. Prove that, for any three vectors $\vec{a},\ \vec{b},\ \vec{c}$

$\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right] = 2\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]$

Ans:

$\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right] = (\vec{a}+\vec{b}).\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\}$

$= (\vec{a}+\vec{b}).\{\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\}$

$= (\vec{a}+\vec{b}).\{\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}\}$

$= \vec{a}.(\vec{b}\times\vec{c})+\vec{a}.(\vec{b}\times\vec{a})+\vec{a}.(\vec{c}\times\vec{a})+\vec{b}.(\vec{b}\times\vec{c})+\vec{b}.(\vec{b}\times\vec{a})+\vec{b}.(\vec{c}\times\vec{a})$

$= \left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]+0+0+0+0+\left[\vec{b}\ \ \vec{c}\ \ \vec{a}\right]$

$= \left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]+\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]$

$= 2\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]$

15. Vectors $\vec{a},\ \vec{b},\ \vec{c}$ are such that $\vec{a}+\vec{b}+\vec{c}$ = 0 and $|\vec{a}|=3,\ |\vec{b}|=5\ \ and\ \ |\vec{c}|=7$. Find the angle between $\vec{a}$ and $\vec{b}$.

Ans:

$\vec{a}+\vec{b}+\vec{c}=0 \Rightarrow \vec{a}+\vec{b}=-\vec{c}$

$\Rightarrow (\vec{a}+\vec{b})^2 = (-\vec{c})^2$

$|\Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = \vec{c}.\vec{c}$

$\Rightarrow |\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = |\vec{c}|^2 \Rightarrow 9 + 2\vec{a}.\vec{b} + 25 = 49$

$\Rightarrow 2\vec{a}.\vec{b} = 49-25-9 = 15$

$\Rightarrow \vec{a}.\vec{b} = \frac{15}{2} \Rightarrow |\vec{a}||\vec{b}|\cos\theta = \frac{15}{2}$

$\Rightarrow 3\times 5\times\cos\theta = \frac{15}{2} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \cos\theta = \cos\frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{3}$

16. If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a})(\vec{x}+\vec{a}) = 24$, then write the value of $|\vec{x}|$.

Ans:

Given that $(\vec{x}-\vec{a})(\vec{x}+\vec{a}) = 24$

$\Rightarrow \vec{x}.\vec{x}+\vec{x}.\vec{a}-\vec{a}.\vec{x}-\vec{a}.\vec{a} = 24$

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 24 \qquad \left[\because \vec{x}.\vec{a} = \vec{a}.\vec{x}\right]$

$\Rightarrow |\vec{x}|^2 - 1 = 24 \Rightarrow |\vec{x}|^2 = 25 \Rightarrow |\vec{x}| = 5$

17. For any three vectors $\vec{a},\ \vec{b}\ \ and\ \ \vec{c}$, write the value of the following:

$\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})$

Ans:

$\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})$

$= \vec{a}\times\vec{b}+\vec{a}\times\vec{c}+\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}+\vec{c}\times\vec{b}$

$= \vec{a}\times\vec{b}+\vec{a}\times\vec{c}+\vec{b}\times\vec{c}-\vec{a}\times\vec{b}-\vec{a}\times\vec{c}-\vec{b}\times\vec{c} = 0$

18. The magnitude of the vector product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to $\sqrt{2}$. Find the value of λ.

Ans:

Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = 2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c} = \lambda\hat{i}+2\hat{j}+3\hat{k}$

Now, $\vec{b}+\vec{c} = 2\hat{i}+4\hat{j}-5\hat{k}+\lambda\hat{i}+2\hat{j}+3\hat{k} = (2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

$\Rightarrow |\vec{b}+\vec{c}| = \sqrt{(2+\lambda)^2+36+4} = \sqrt{4+\lambda^2+4\lambda+40} = \sqrt{\lambda^2+4\lambda+44}$

The vector product of $\hat{i}+\hat{j}+\hat{k}$ with this unit vector is $\sqrt{2}$.

$\therefore \left|\vec{a}\times\frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|}\right| = \sqrt{2} \Rightarrow \left|\frac{\vec{a}\times(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|}\right| = \sqrt{2}$

Now, $\vec{a}\times(\vec{b}+\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2+\lambda & 6 & -2 \end{vmatrix} = (-2-6)\hat{i} - (-2-2-\lambda)\hat{j} - (6-2-\lambda)\hat{k}$

$= -8\hat{i}+(4+\lambda)\hat{j}+(4-\lambda)\hat{k}$

$\therefore \left|\frac{\vec{a}\times(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|}\right| = \sqrt{2} \Rightarrow \left|\frac{-8\hat{i}+(4+\lambda)\hat{j}+(4-\lambda)\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}\right| = \sqrt{2}$

$\Rightarrow \frac{\sqrt{64+(4+\lambda)^2+(4-\lambda)^2}}{\sqrt{\lambda^2+4\lambda+44}} = \sqrt{2}$

$\frac{64+(4+\lambda)^2+(4-\lambda)^2}{\lambda^2+4\lambda+44} = 2 \Rightarrow \frac{64+16+\lambda^2+8\lambda+16+\lambda^2-8\lambda}{\lambda^2+4\lambda+44} = 2 \Rightarrow \frac{96+2\lambda^2}{\lambda^2+4\lambda+44} = 2$

$\Rightarrow 96+2\lambda^2 = 2(\lambda^2+4\lambda+44) \Rightarrow 96+2\lambda^2 = 2\lambda^2+8\lambda+88$

$\Rightarrow 8\lambda = 8 \Rightarrow \lambda = 1$

19. Find a unit vector perpendicular to each of the vectors $\vec{a}+2\vec{b}$ and $2\vec{a}+\vec{b}$, where $\vec{a} = 3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}-2\hat{k}$.

Ans.

Given that $\vec{a} = 3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}-2\hat{k}$

$\therefore \vec{a}+2\vec{b} = 3\hat{i}+2\hat{j}+2\hat{k}+2(\hat{i}+2\hat{j}-2\hat{k})$

$= 3\hat{i}+2\hat{j}+2\hat{k}+2\hat{i}+4\hat{j}-4\hat{k} = 5\hat{i}+6\hat{j}-2\hat{k}$

and $2\vec{a}+\vec{b} = 2(3\hat{i}+2\hat{j}+2\hat{k})+\hat{i}+2\hat{j}-2\hat{k}$

$= 6\hat{i}+4\hat{j}+4\hat{k}+\hat{i}+2\hat{j}-2\hat{k} = 7\hat{i}+6\hat{j}+2\hat{k}$

Now, perpendicular vector of $\vec{a}+2\vec{b}$ and $2\vec{a}+\vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -2 \\ 7 & 6 & 2 \end{vmatrix} = (12+12)\hat{i} - (10+14)\hat{j} + (30-42)\hat{k}$

$= 24\hat{i}-24\hat{j}-12\hat{k} = 12(2\hat{i}-2\hat{j}-\hat{k})$

$\therefore$ Required unit vector $= \pm\frac{12(2\hat{i}-2\hat{j}-\hat{k})}{12\sqrt{4+4+1}} = \pm\frac{2\hat{i}-2\hat{j}-\hat{k}}{\sqrt{9}}$

$= \pm\frac{2\hat{i}-2\hat{j}-\hat{k}}{3} = \pm\left(\frac{2}{3}\hat{i}-\frac{2}{3}\hat{j}-\frac{1}{3}\hat{k}\right)$

20. If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ, so that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors.

Ans:

Given that $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$

$\therefore\ \vec{a} + \vec{b} = \hat{i} - \hat{j} + 7\hat{k} + 5\hat{i} - \hat{j} + \lambda\hat{k} = 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}$

and $\vec{a} - \vec{b} = \hat{i} - \hat{j} + 7\hat{k} - 5\hat{i} + \hat{j} - \lambda\hat{k} = -4\hat{i} + (7-\lambda)\hat{k}$

Now, $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors

$\Rightarrow (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 0$

$\Rightarrow (6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}).(-4\hat{i} + (7-\lambda)\hat{k} = 0$

$\Rightarrow -24 + 0 + (7+\lambda)(7-\lambda) = 0$

$\Rightarrow -24 + 49 - \lambda^2 = 0 \Rightarrow \lambda^2 = 25 \Rightarrow \lambda = \pm 5$