CHAPTER – 10: VECTOR ALGEBRA

MARKS WEIGHTAGE – 06 marks

NCERT Important Questions & Answers

1. **Find the unit vector in the direction of the sum of the vectors,** $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ **and** $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$

   Ans:

   The sum of the given vectors is $\vec{a} + \vec{b}\ \ (= \vec{c}, say) = 4\hat{i} + 3\hat{j} - 2\hat{k}$

   and $|\vec{c}| = \sqrt{4^2 + 3^2 + (-2)^2} = \sqrt{29}$

   Thus, the required unit vector is

   $\hat{c} = \frac{1}{|\vec{c}|}\vec{c} = \frac{1}{\sqrt{29}}(4\hat{i} + 3\hat{j} - 2\hat{k}) = \frac{4}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} - \frac{2}{\sqrt{29}}\hat{k}$

2. **Show that the points are** $A(2\hat{i} - \hat{j} + \hat{k}), B(\hat{i} - 3\hat{j} - 5\hat{k}), C(3\hat{i} - 4\hat{j} - 4\hat{k})$ **the vertices of a right angled triangle.**

   Ans:

   We have $\overrightarrow{AB} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$

   $\overrightarrow{BC} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$

   and $\overrightarrow{CA} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$

   Then $|\overrightarrow{AB}|^2 = 41, |\overrightarrow{BC}|^2 = 6, |\overrightarrow{CA}|^2 = 35$

   $\Rightarrow |\overrightarrow{AB}|^2 = |\overrightarrow{BC}|^2 + |\overrightarrow{CA}|^2$

   Hence, the triangle is a right angled triangle.

3. **Find the direction cosines of the vector joining the points A(1, 2, –3) and B(–1, –2, 1), directed from A to B.**

   Ans:

   The given points are $A(1,2,-3)$ and $B(-1,-2,1)$.

   Then $\overrightarrow{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + (1-(-3))\hat{k} = -2\hat{i} - 4\hat{j} + 4\hat{k}$

   Now, $|\overrightarrow{AB}| = \sqrt{4+16+16} = \sqrt{36} = 6$

   $\therefore$ unit vector along $\overrightarrow{AB} = \frac{1}{|\overrightarrow{AB}|}\overrightarrow{AB} = \frac{1}{6}(-2\hat{i} - 4\hat{j} + 4\hat{k}) = -\frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

   Hence direction cosines are $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

4. **Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are** $\hat{i} + 2\hat{j} - \hat{k}$ **and** $-\hat{i} + \hat{j} + \hat{k}$ **respectively, in the ratio 2 : 1 (i) internally (ii) externally**

   Ans:

   The position vector of a point *R* divided the line segment joining two points *P* and *Q* in the ratio *m*: *n* is given by

   Case I Internally $= \frac{m\vec{b} + n\vec{a}}{m+n}$

   Case II Externally $= \frac{m\vec{b} - n\vec{a}}{m-n}$

   Position vectors of *P* and *Q* are given as $\overrightarrow{OP} = \hat{i} + 2\hat{j} - \hat{k}, \overrightarrow{OQ} = -\hat{i} + \hat{j} + \hat{k}$

(i) Position vector of R [dividing (PQ) in the ratio *2 : 1* internally]

$$=\frac{m\overrightarrow{OQ}+n\overrightarrow{OP}}{m+n}=\frac{2(-\hat{i}+\hat{j}+\hat{k})+1(\hat{i}+2\hat{j}-\hat{k})}{2+1}=\frac{-\hat{i}+4\hat{j}+\hat{k}}{3}-\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$$

(i) Position vector of R [dividing (PQ) in the ratio *2 : 1* externally]

$$=\frac{m\overrightarrow{OQ}-n\overrightarrow{OP}}{m-n}=\frac{2(-\hat{i}+\hat{j}+\hat{k})-1(\hat{i}+2\hat{j}-\hat{k})}{2-1}=\frac{-3\hat{i}+0\hat{j}+3\hat{k}}{1}-3\hat{i}+3\hat{k}$$

5. Find the position vector of the mid point of the vector joining the points P(2, 3, 4) and Q(4, 1, –2).

Ans:

Position vectors of *P* and *Q* are given as $\overrightarrow{OP}=2\hat{i}+3\hat{j}+4\hat{k},\overrightarrow{OQ}=4\hat{i}+\hat{j}-2\hat{k}$

The position vector of the mid point of the vector joining the points P(2, 3, 4) and Q(4, 1, –2) is given by

Position Vector of the mid-point of (*PQ*) = $\frac{1}{2}\left(\overrightarrow{OQ}+\overrightarrow{OP}\right)=\frac{1}{2}\left(4\hat{i}+\hat{j}-2\hat{k}+2\hat{i}+3\hat{j}+4\hat{k}\right)$

$$=\frac{1}{2}\left(6\hat{i}+4\hat{j}+2\hat{k}\right)=3\hat{i}+2\hat{j}+\hat{k}$$

6. Show that the points A, B and C with position vectors, $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$ respectively form the vertices of a right angled triangle.

Ans:

Position vectors of points *A*, *B* and *C* are respectively given as

$\vec{a}=3\hat{i}-4\hat{j}-4\hat{k},\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ *and* $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$

Now, $\overrightarrow{AB}=\vec{b}-\vec{a}=2\hat{i}-\hat{j}+\hat{k}-3\hat{i}+4\hat{j}+4\hat{k}=-\hat{i}+3\hat{j}+5\hat{k}$

$\Rightarrow|\overrightarrow{AB}|^2=1+9+25=35$

$\overrightarrow{BC}=\vec{c}-\vec{b}=\hat{i}-3\hat{j}-5\hat{k}-2\hat{i}+\hat{j}-\hat{k}=-\hat{i}-2\hat{j}-6\hat{k}$

$\Rightarrow|\overrightarrow{BC}|^2=1+4+36=41$

$\overrightarrow{CA}=\vec{a}-\vec{c}=3\hat{i}-4\hat{j}-4\hat{k}-\hat{i}+3\hat{j}+5\hat{k}=2\hat{i}-\hat{j}+\hat{k}$

$\Rightarrow|\overrightarrow{CA}|^2=4+1+1=6$

$\Rightarrow|\overrightarrow{BC}|^2=|\overrightarrow{AB}|^2+|\overrightarrow{CA}|^2$

Hence it form the vertices of a right angled triangle.

7. Find angle 'θ' between the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$

Ans:

The angle θ between two vectors $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

Now, $\vec{a}.\vec{b}=(\hat{i}+\hat{j}-\hat{k}).(\hat{i}-\hat{j}+\hat{k})=1-1-1=-1$

Therefore, we have $\cos\theta=\frac{-1}{3}\Rightarrow\theta=\cos^{-1}\left(\frac{-1}{3}\right)$

8. If $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-5\hat{k}$, then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular.

Ans:

We know that two nonzero vectors are perpendicular if their scalar product is zero.

Here, $\vec{a}+\vec{b}=5\hat{i}-\hat{j}-3\hat{k}+\hat{i}+3\hat{j}-5\hat{k}=6\hat{i}+2\hat{j}-8\hat{k}$

and $\vec{a}-\vec{b}=5\hat{i}-\hat{j}-3\hat{k}-\hat{i}-3\hat{j}+5\hat{k}=4\hat{i}-4\hat{j}+2\hat{k}$

Now, $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = (6\hat{i}+2\hat{j}-8\hat{k}).(4\hat{i}-4\hat{j}+2\hat{k}) = 24-8-16=0$

Hence $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular.

9. Find $|\vec{a}-\vec{b}|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=2$, $|\vec{b}|=3$ and $\vec{a}.\vec{b}$ = 4.

Ans:

We have

$|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b}).(\vec{a}-\vec{b}) = \vec{a}.\vec{a}-\vec{a}.\vec{b}-\vec{b}.\vec{a}+\vec{b}.\vec{b}$

$=|\vec{a}|^2 - 2(\vec{a}.\vec{b}) + |\vec{b}|^2 = 2^2 - 2(4) + 3^2 = 4-8+9=5$

$\Rightarrow |\vec{a}-\vec{b}| = \sqrt{5}$

10. Show that the points $A(-2\hat{i}+3\hat{j}+5\hat{k}), B(\hat{i}+2\hat{j}+3\hat{k}), C(7\hat{i}-\hat{k})$ are collinear.

Ans:

We have

$\overrightarrow{AB} = (1+2)\hat{i}+(2-3)\hat{j}+(3-5)\hat{k} = 3\hat{i}-\hat{j}-2\hat{k}$

$\overrightarrow{BC} = (7-1)\hat{i}+(0-2)\hat{j}+(-1-3)\hat{k} = 6\hat{i}-2\hat{j}-4\hat{k}$

$\overrightarrow{CA} = (7+2)\hat{i}+(0-3)\hat{j}+(-1-5)\hat{k} = 9\hat{i}-3\hat{j}-6\hat{k}$

Now, $|\overrightarrow{AB}|^2 = 14, |\overrightarrow{BC}|^2 = 56, |\overrightarrow{CA}|^2 = 126$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{14}, |\overrightarrow{BC}| = 2\sqrt{14}, |\overrightarrow{CA}| = 3\sqrt{14}$

$\Rightarrow |\overrightarrow{CA}| = |\overrightarrow{AB}| + |\overrightarrow{BC}|$

Hence the points A, B and C are collinear.

11. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

Ans:

Given that $|\vec{a}|=1, |\vec{b}|=1, |\vec{c}|=1, \vec{a}+\vec{b}+\vec{c}=0$

$(\vec{a}+\vec{b}+\vec{c})^2 = (\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c}) = 0$

$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{b}+\vec{b}.\vec{c}+\vec{b}.\vec{a}+\vec{c}.\vec{a}+\vec{c}.\vec{b}+\vec{c}.\vec{c}=0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$

$\Rightarrow 1+1+1+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$

$\Rightarrow 2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = -3$

$\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a} = \frac{-3}{2}$

12. If the vertices A, B, C of a triangle ABC are (1, 2, 3), (–1, 0, 0), (0, 1, 2), respectively, then find $\angle$ ABC.

Ans:

We are given the points $A(1, 2, 3)$, $B(-1, 0, 0)$ and $C(0, 1, 2)$.

Also, it is given that $\angle ABC$ is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$

Now, $\overrightarrow{BA} = (\hat{i}+2\hat{j}+3\hat{k})-(-\hat{i}+0\hat{j}+0\hat{k}) = 2\hat{i}+2\hat{j}+3\hat{k}$

$\Rightarrow |\overrightarrow{BA}| = \sqrt{4+4+9} = \sqrt{17}$

and $\overrightarrow{BC} = (0\hat{i}+\hat{j}+2\hat{k})-(-\hat{i}+0\hat{j}+0\hat{k}) = \hat{i}+\hat{j}+2\hat{k}$

$\Rightarrow |\overrightarrow{BC}| = \sqrt{1+1+4} = \sqrt{6}$

$\overrightarrow{BA}.\overrightarrow{BC} = (2\hat{i}+2\hat{j}+3\hat{k}).(\hat{i}+\hat{j}+2\hat{k}) = 2+2+6=10$

$\cos\theta = \frac{\overrightarrow{BA}.\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|} \Rightarrow \cos\angle ABC == \frac{10}{(\sqrt{17})(\sqrt{6})} = \frac{10}{\sqrt{102}}$

$\Rightarrow \angle ABC = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$

13. Show that the points A(1, 2, 7), B(2, 6, 3) and C(3, 10, –1) are collinear.

Ans:

The given points are $A(1, 2, 7)$, B (2, 6, 3) and $C(3, 10, -1)$.

$\overrightarrow{AB} = (2\hat{i}+6\hat{j}+3\hat{k})-(\hat{i}+2\hat{j}+7\hat{k}) = \hat{i}+4\hat{j}-4\hat{k}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{1+16+16} = \sqrt{33}$

$\overrightarrow{BC} = (3\hat{i}+10\hat{j}-\hat{k})-(2\hat{i}+6\hat{j}+3\hat{k}) = \hat{i}+4\hat{j}-4\hat{k}$

$\Rightarrow |\overrightarrow{BC}| = \sqrt{1+16+16} = \sqrt{33}$

and $\overrightarrow{AC} = (3\hat{i}+10\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+7\hat{k}) = 2\hat{i}+8\hat{j}-8\hat{k}$

$\Rightarrow |\overrightarrow{AC}| = \sqrt{4+64+64} = \sqrt{132} = 2\sqrt{33}$

$\therefore |\overrightarrow{AC}| = |\overrightarrow{AB}| + |\overrightarrow{BC}|$

Hence, the given points A, B and C are collinear.

14. Show that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ form the vertices of a right angled triangle.

Ans:

Let A= $2\hat{i}-\hat{j}+\hat{k}$ B = $\hat{i}-3\hat{j}-5\hat{k}$ and C = $3\hat{i}-4\hat{j}-4\hat{k}$

$\overrightarrow{AB} = (\hat{i}-3\hat{j}-5\hat{k})-(2\hat{i}-\hat{j}+\hat{k}) = -\hat{i}-2\hat{j}-6\hat{k}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{1+4+36} = \sqrt{41}$

$\overrightarrow{BC} = (3\hat{i}-4\hat{j}-4\hat{k})-(\hat{i}-3\hat{j}-5\hat{k}) = 2\hat{i}-\hat{j}+\hat{k}$

$\Rightarrow |\overrightarrow{BC}| = \sqrt{4+1+1} = \sqrt{6}$

and $\overrightarrow{AC} = (3\hat{i}-4\hat{j}-4\hat{k})-(2\hat{i}-\hat{j}+\hat{k}) = \hat{i}-3\hat{j}-5\hat{k}$

$\Rightarrow |\overrightarrow{AC}| = \sqrt{1+9+25} = \sqrt{35}$

$\therefore |\overrightarrow{AB}|^2 = |\overrightarrow{AC}|^2 + |\overrightarrow{BC}|^2$

Hence, ABC is a right angled triangle.

15. Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a} = \hat{i}+\hat{j}+\hat{k}, \vec{b} = \hat{i}+2\hat{j}+3\hat{k}$.

Ans:

We have $\vec{a}+\vec{b} = 2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{a}-\vec{b} = -\hat{j}-2\hat{k}$

A vector which is perpendicular to both $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ is given by

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = -2\hat{i}+4\hat{j}-2\hat{k} (= \vec{c}, say)$$

Now, $|\vec{c}| = \sqrt{4+16+4} = \sqrt{24} = 2\sqrt{6}$

Therefore, the required unit vector is

$$\hat{c} = \frac{1}{|\vec{c}|}\vec{c} = \frac{1}{2\sqrt{6}}(-2\hat{i}+4\hat{j}-2\hat{k}) = \frac{-1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{2}{\sqrt{6}}\hat{k}$$

16. Find the area of a triangle having the points A(1, 1, 1), B(1, 2, 3) and C(2, 3, 1) as its vertices.

Ans:

We have $\overrightarrow{AB} = \hat{j}+2\hat{k}$ and $\overrightarrow{AC} = \hat{i}+2\hat{j}$.

The area of the given triangle is $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

Now, $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 2\\ 1 & 2 & 0\end{vmatrix}=-4\hat{i}+2\hat{j}-\hat{k}$

Therefore, $|\overrightarrow{AB}\times\overrightarrow{AC}|=\sqrt{16+4+1}=\sqrt{21}$

Thus, the required area is $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{1}{2}\sqrt{21}$

17. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a}=3\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$.

Ans:

The area of a parallelogram with $\vec{a}$ and $\vec{b}$ as its adjacent sides is given by $|\vec{a}\times\vec{b}|$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 4\\ 1 & -1 & 1\end{vmatrix}=5\hat{i}+\hat{j}-4\hat{k}$

Therefore, $|\vec{a}\times\vec{b}|=\sqrt{25+1+16}=\sqrt{42}$

and hence, the required area is $\sqrt{42}$.

18. Find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

Ans:

$\overrightarrow{AB}=(2\hat{i}+3\hat{j}+5\hat{k})-(\hat{i}+\hat{j}+2\hat{k})=\hat{i}+2\hat{j}+3\hat{k}$

$\overrightarrow{AC}=(\hat{i}+5\hat{j}+5\hat{k})-(\hat{i}+\hat{j}+2\hat{k})=4\hat{j}+3\hat{k}$

Now, $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 0 & 4 & 3\end{vmatrix}=-6\hat{i}-3\hat{j}+4\hat{k}$

$\Rightarrow|\overrightarrow{AB}\times\overrightarrow{AC}|=\sqrt{36+9+16}=\sqrt{61}$

Area of triangle ABC = $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{\sqrt{61}}{2}$ sq. units.

19. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$.

Ans:

Adjacent sides of parallelogram are given by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$.

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 3\\ 2 & -7 & 1\end{vmatrix}=20\hat{i}+5\hat{j}-5\hat{k}$

$\Rightarrow|\vec{a}\times\vec{b}|=\sqrt{400+25+25}=\sqrt{450}=15\sqrt{2}$

Hence, the area of the given parallelogram is $15\sqrt{2}$ sq. units.

20. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$, then $\vec{a}\times\vec{b}$ is a unit vector, find the angle between $\vec{a}$ and $\vec{b}$.

Ans:

Given that vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$.

Also, $\vec{a}\times\vec{b}$ is a unit vector $\Rightarrow |\vec{a}\times\vec{b}|=1$

$\Rightarrow |\vec{a}|.|\vec{b}|\sin\theta=1 \Rightarrow 3\times\frac{\sqrt{2}}{3}\sin\theta=1$

$\Rightarrow \sin\theta=\frac{1}{\sqrt{2}} \Rightarrow \theta=\frac{\pi}{4}$

21. If $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ are the position vectors of points A, B, C and D respectively, then find the angle between $\overline{AB}$ and $\overline{CD}$. Deduce that $\overline{AB}$ and $\overline{CD}$ are collinear.

Ans:

Note that if θ is the angle between $\overline{AB}$ and $\overline{CD}$, then θ is also the angle between $\overline{AB}$ and $\overline{CD}$.

Now $\overline{AB}$ = Position vector of B – Position vector of A

$= (2\hat{i}+5\hat{j})-(\hat{i}+\hat{j}+\hat{k})=\hat{i}+4\hat{j}-\hat{k}$

Therefore, $|\overrightarrow{AB}|=\sqrt{1+16+1}=\sqrt{18}=3\sqrt{2}$

Similarly, $\overline{CD}=-2\hat{i}-8\hat{j}+2\hat{k} \Rightarrow |\overrightarrow{CD}|=\sqrt{4+64+4}=\sqrt{72}=6\sqrt{2}$

Thus, $\cos\theta=\frac{\overrightarrow{AB}.\overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|}=\frac{1(-2)+4(-8)+(-1)(2)}{(3\sqrt{2})(6\sqrt{2})}=-1$

Since 0 ≤ θ ≤ π, it follows that θ = π. This shows that $\overline{AB}$ and $\overline{CD}$ are collinear.

22. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=3, |\vec{b}|=4, |\vec{c}|=5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a}+\vec{b}+\vec{c}|$.

Ans:

Given that each one of them being perpendicular to the sum of the other two.

Therefore, $\vec{a}.(\vec{b}+\vec{c})=0, \vec{b}.(\vec{c}+\vec{a})=0, \vec{c}.(\vec{a}+\vec{b})=0$

Now, $|\vec{a}+\vec{b}+\vec{c}|^2=(\vec{a}+\vec{b}+\vec{c})^2=(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})$

$=\vec{a}.\vec{a}+\vec{a}.(\vec{b}+\vec{c})+\vec{b}.\vec{b}+\vec{b}.(\vec{c}+\vec{a})+\vec{c}.(\vec{a}+\vec{b})+\vec{c}.\vec{c}$

$=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2$

$=9+16+25=50$

Therefore, $|\vec{a}+\vec{b}+\vec{c}|=\sqrt{50}=5\sqrt{2}$

23. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$.

Ans:

Given vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$.

Let $\vec{c}$ be the resultant vector $\vec{a}$ and $\vec{b}$ then

$\vec{c}=(2\hat{i}+3\hat{j}-\hat{k})+(\hat{i}-2\hat{j}+\hat{k})=3\hat{i}+\hat{j}+0\hat{k}$

$\Rightarrow |\vec{c}|=\sqrt{9+1+0}=\sqrt{10}$

$\therefore$ Unit vector in the direction of $\vec{c}$ = $\hat{c}=\frac{1}{|\vec{c}|}\vec{c}=\frac{1}{\sqrt{10}}(3\hat{i}+\hat{j})$

Hence, the vector of magnitude 5 units and parallel to the resultant of vectors $\vec{a}$ and $\vec{b}$ is

$\pm5\hat{c}=\pm5\frac{1}{\sqrt{10}}(3\hat{i}+\hat{j})=\pm\frac{3\sqrt{10}}{2}\hat{i}\pm\frac{\sqrt{10}}{2}\hat{j}$

24. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

Ans:

Two adjacent sides of a parallelogram are given by $\vec{a}=2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}-3\hat{k}$

Then the diagonal of a parallelogram is given by $\vec{c}=\vec{a}+\vec{b}$

$\therefore\ \vec{c}=\vec{a}+\vec{b}=2\hat{i}-4\hat{j}+5\hat{k}+\hat{i}-2\hat{j}-3\hat{k}=3\hat{i}-6\hat{j}+2\hat{k}$

$\Rightarrow |\vec{c}|=\sqrt{9+36+4}=\sqrt{49}=7$

Unit vector parallel to its diagonal = $\hat{c}=\frac{1}{|\vec{c}|}\vec{c}=\frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k})=\frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 5\\ 1 & -2 & -3\end{vmatrix}=22\hat{i}+11\hat{j}+0\hat{k}$

Then the area of a parallelogram = $|\vec{a}\times\vec{b}|=\sqrt{484+121+0}=\sqrt{605}=11\sqrt{5}$ sq. units.

25. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}, \vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d}=15$.

Ans:

The vector which is perpendicular to both $\vec{a}$ and $\vec{b}$ must be parallel to $\vec{a}\times\vec{b}$.

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 4 & 2\\ 3 & -2 & 7\end{vmatrix}=32\hat{i}-\hat{j}-14\hat{k}$

Let $\vec{d}=\lambda(\vec{a}\times\vec{b})=\lambda(32\hat{i}-\hat{j}-14\hat{k})$

Also $\vec{c}.\vec{d}=15\Rightarrow(2\hat{i}-\hat{j}+4\hat{k}).\lambda(32\hat{i}-\hat{j}-14\hat{k})=15$

$\Rightarrow 64\lambda+\lambda-56\lambda=15\Rightarrow 9\lambda=15\Rightarrow\lambda=\frac{15}{9}=\frac{5}{3}$

$\therefore$ Required vector $\vec{d}=\frac{5}{3}(32\hat{i}-\hat{j}-14\hat{k})$

26. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

Ans: Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$

Now, $\vec{b}+\vec{c}=2\hat{i}+4\hat{j}-5\hat{k}+\lambda\hat{i}+2\hat{j}+3\hat{k}=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

$\Rightarrow|\vec{b}+\vec{c}|=\sqrt{(2+\lambda)^2+36+4}=\sqrt{4+\lambda^2+4\lambda+40}=\sqrt{\lambda^2+4\lambda+44}$

Unit vector along $\vec{b}+\vec{c}$ is $\frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|}=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$

The scalar product of $\hat{i}+\hat{j}+\hat{k}$ with this unit vector is 1.

$\therefore(\hat{i}+\hat{j}+\hat{k}).\frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|}=1\Rightarrow(\hat{i}+\hat{j}+\hat{k}).\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}=1$

$\Rightarrow\frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}}=1\Rightarrow\frac{\lambda+6}{\sqrt{\lambda^2+4\lambda+44}}=1$

$\Rightarrow\lambda+6=\sqrt{\lambda^2+4\lambda+44}\Rightarrow(\lambda+6)^2=\lambda^2+4\lambda+44$

$\Rightarrow\lambda^2+12\lambda+36=\lambda^2+4\lambda+44\Rightarrow 8\lambda=8\Rightarrow\lambda=1$

27. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Ans:

Given that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors.

$\therefore\ \vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$

It is also given that $|\vec{a}|=|\vec{b}|=|\vec{c}|$

Let vector $\vec{a}+\vec{b}+\vec{c}$ be inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$ at angles α, β and γ respectively.

$$\cos\alpha=\frac{(\vec{a}+\vec{b}+\vec{c}).\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{\vec{a}.\vec{a}+\vec{b}.\vec{a}+\vec{c}.\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{|\vec{a}|^2+0+0}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$$=\frac{|\vec{a}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\cos\beta=\frac{(\vec{a}+\vec{b}+\vec{c}).\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|}=\frac{\vec{a}.\vec{b}+\vec{b}.\vec{b}+\vec{c}.\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{0+|\vec{b}|^2+0}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$$=\frac{|\vec{b}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\cos\gamma=\frac{(\vec{a}+\vec{b}+\vec{c}).\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}=\frac{\vec{a}.\vec{c}+\vec{b}.\vec{c}+\vec{c}.\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{0+0+|\vec{c}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$$=\frac{|\vec{c}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

Now as $|\vec{a}|=|\vec{b}|=|\vec{c}|$, therefore, cosα = cosβ = cosγ

$\therefore\ \alpha=\beta=\gamma$

Hence, the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

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