ELECTROSTATICS

MARKS WEIGHTAGE – 15 marks

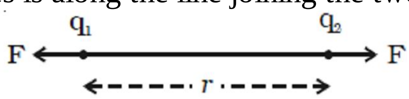
QUICK REVISION (Important Concepts & Formulas)

Charge

- **Quantization:** Charge is always in the form of an integral multiple of electronic charge and never its fraction.
  $q = \pm ne$ where n is an integer and $e \;= 1.6 \times 10^{-19}$ coulomb
  $= 1.6 \times 10^{-19}$ C.
- Charge on an electron/proton is the minimum charge.
  Charge on an electron is –ve. $e = -1.6 \times 10^{-19}$ C.
  Charge on a proton is +ve. $e = +1.6 \times 10^{-19}$ C.
  Total charge = ± ne.
- A particle/body is positively charged because it loses electrons or it has shortage of electrons.
- A particle is negatively charged because it gains electrons or it has excess of electron.
- **Conservation:** The total net charge of an isolated physical system always remains constant. Charge can neither be created nor destroyed. It can be transferred from one body to another.

Coulomb's inverse square law

Coulomb’s law states that the force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The direction of forces is along the line joining the two point charges.



Let q_1 and q_2 be two point charges placed in air or vacuum at a distance r apart (see above Figure). Then, according to Coulomb’s law,

$$F \propto \frac{q_1 q_2}{r^2} \textbf{ or } F = k.\frac{q_1 q_2}{r^2}$$

where k is a constant of proportionality. In air or vacuum, $k = \dfrac{1}{4\pi\varepsilon_0}$

- $F = \dfrac{1}{4\pi\varepsilon_0}.\dfrac{q_1 q_2}{r^2}$ where F denotes the force between two charges q_1 and q_2 separated by a distance r in free space. ε_0 is a constant known as permittivity of free space. Free space is vacuum and may be deemed to be air practically and $\dfrac{1}{4\pi\varepsilon_0} = 9\times10^9 \dfrac{Nm^2}{C^2}$
- ***One Coulomb*** is defined as the quantity of charge, which when placed at a distance of 1 metre in air or vacuum from an equal and similar charge, experiences a repulsive force of 9×10^9 N.
- If free space is replaced by a medium, then ε_0 is replaced by ($\varepsilon_0 K$) or ($\varepsilon_0 \varepsilon_r$) where K is known as **dielectric constant** or relative permittivity or specific inductive capacity (S.I.C.) or dielectric coefficient of the medium/material/matter. Thus
  $$F = \frac{1}{4\pi\varepsilon}.\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0 K}.\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}.\frac{q_1 q_2}{r^2}$$
- $K = \dfrac{\varepsilon}{\varepsilon_0}$ *or* $\varepsilon_r = \dfrac{\varepsilon}{\varepsilon_0}$
  $K = 1$ for vacuum (or air), $K = \infty$ for conductor/metal.
- $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$.

- Vector form of the law (q_1 and q_2 are like charges)

  (i) $\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r_{21}^3}\overrightarrow{r_{21}} = \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{\left|\vec{r_1}-\vec{r_2}\right|^3}\left(\vec{r_1}-\vec{r_2}\right)$

  (ii) $\overrightarrow{F_{21}} = \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r_{12}^3}\overrightarrow{r_{12}} = \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{\left|\vec{r_2}-\vec{r_1}\right|^3}\left(\vec{r_2}-\vec{r_1}\right)$

- If $\hat{r}_{21}$ is a unit vector pointing from q_2 to q_1, then

  $\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r_{21}^3}\hat{r}_{21}$ = force on q_1 by q_2

  When $q_1q_2 > 0$ for like charges.

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  $\overrightarrow{F_{21}} = \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r_{12}^3}\hat{r}_{12}$ = force on q_2 by q_1

  When $q_1q_2 > 0$ for like charges.

Intensity/strength of electric field

- Intensity at a point is numerically equal to the force acting on a unit positive charge placed at the point.
- It is a vector quantity.
- The units of intensity E are NC^{-1}, volt/metre.
- The dimensions of E are $[MLT^{-3}A^{-1}]$.

Intensity due to a charge *q* at distance *r*

(i) $E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2}$

It acts in the direction in which a +ve charge moves.

(ii) $E = \frac{1}{4\pi\varepsilon_0 K}.\frac{q}{r^2}$, if point is in the medium.

Potential (*V*) and intensity (*E*)

(i) $E = -\frac{dV}{dr}$ when potential varies with respect to distance.

(ii) $E = \frac{\text{potential difference}}{\text{distance}} = \frac{V}{r}$, when potential difference is constant.

(iii) Potential at a point distance r from charge q.

$V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$ in free space

(iv) $V = \frac{1}{4\pi\varepsilon_0 K}.\frac{q}{r}$ in medium

(v) Potential is a scalar quantity

$\vec{E}.d\vec{r} = dV$

- From positively charged surface, $\vec{E}$ acts outwards at right angles *i.e.* along outward drawn normal.
- Intensity is equal to flux (number of electric lines of force) crossing unit normal area.

  $\vec{E} = \frac{flux(\Phi)}{area(\vec{s})}$

Electric lines of force

- Electric lines of force start from positive charge and terminate on negative charge.
- From a positively charged conducting surface lines of force are normal to surface in outward direction.
- Electric lines of force about a negative point charge are radial, inwards and about a positive point charge are radial, outwards.
- Electric lines of force are always perpendicular to an equipotential surface.
- These lines of force contract along the length but expand at right angles to their length. There is longitudinal tension and lateral pressure in a line of force. Contraction shows attraction between opposite charges while expansion indicates that similar charges repel.
- The number of electric lines of force (flux) passing through unit normal area at any point indicates electric intensity at that point.
- For a charged sphere these lines are straight and directed along radius.
- These may be open or closed curves. They are not necessarily closed though the magnetic lines of force are closed.
- Two lines of force never intersect or cut each other.
- Lines of force are parallel and equally spaced in a uniform field.
- Tangent to the curve at a point shows direction of field.

Gauss law

- For a closed surface enclosing a net charge q, the net electric flux Φ emerging out is given by

  $$\Phi = \oint_S \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$$

- If a dipole is enclosed by a closed surface, flux Φ is equal to zero.
  Here the algebraic sum of charges ($+q - q = 0$) is zero.

- The flux will come out if +ve charge is enclosed. The flux will enter if negative charge is enclosed.

Flux from a cube

(i) If q is at the centre of cube, total flux (Φ) = $\frac{q}{\varepsilon_0}$.

(ii) From each face of cube, flux = $\frac{q}{6\varepsilon_0}$.

Electric field due to a charged shell

(i) At an external point, $E = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2}$

This is the same as the field due to a point charge placed at the centre.

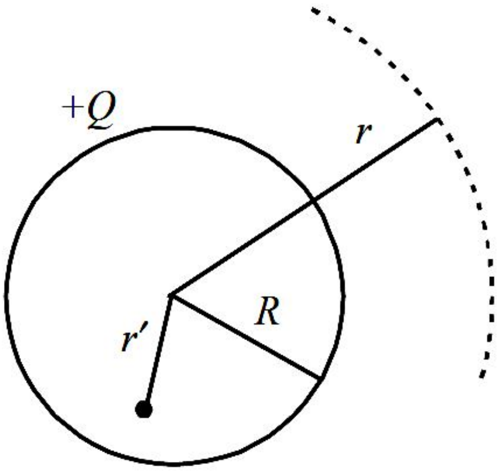
(ii) At a point on surface of shell, this is E_{max} .

$$E = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2}$$

Again the shell behaves like a point charge placed at the centre.

(iii) At an inside point ($r' < R$), $E = 0$.

Thus a charge q placed inside a charged shell does not experience any force due to the shell.



Gaussian surface

(i) For a sphere or spherical shell a concentric sphere.

(ii) For a cylinder or an infinite rod a coaxial cylinder.

(iii) For a plate a cube or a cuboid.

Potential and intensity due to a charged conducting sphere (or shell)
At a point outside the charged sphere

(i) Intensity, $E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2}$ (r > radius of sphere R)

It is a vector quantity.

(ii) Potential, $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$

It is a scalar quantity.

At a point on the surface of charged sphere

(i) Intensity, $E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R^2}$ (r = radius of sphere R)

It is a vector quantity.

(ii) Potential, $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R}$

It is a scalar quantity.

At a point inside the sphere (r < radius of sphere)

(i) Intensity E = zero.

(ii) Potential, $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R}$

Potential is constant inside the sphere. This is same as potential at the surface of sphere.

At the centre of sphere

(i) Intensity E = zero.

(ii) Potential, $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R}$

At infinity

(i) Intensity E = zero.

(ii) Potential V = zero.

Electric field and potential due to charged nonconducting sphere
Outside the sphere when *r* > radius of sphere *R*

(i) Electric intensity, $E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2}$ It is a vector quantity.

(ii) Electric potential, $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$ It is a scalar quantity.

On the surface of the sphere where *r* = *R*

(i) Electric intensity, $E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R^2}$ It is a vector quantity.

(ii) Electric potential, $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R}$ It is a scalar quantity.

Inside the sphere when *r* < *R*

(i) Electric intensity, $E = \frac{1}{4\pi\varepsilon_0}.\frac{qr}{R^3}$

Vectorially, $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^3} \vec{r}$

(ii) Electric potential, $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q(3R^2 - r^2)}{2R^3}$

At the centre of sphere when *r* = 0
(i) Electric intensity E = zero.

(ii) Electric potential, $V = \frac{3}{2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$

Potential at centre = $\frac{3}{2}$ × potential at surface

At infinity
(i) Intensity = zero.
(ii) Potential = zero.

Electric dipole
Two equal and opposite charges (q) each, separated by a small distance (l) constitute an electric dipole. Many of the atoms/molecules are dipoles.

(i) Dipole moment, $\vec{p} = q \times (\vec{l})$

(ii) Dipole moment is a vector quantity.

(iii) The direction of $\vec{p}$ is from negative charge to positive charge.

(iv) Unit of dipole moment = coulombmetre = Cm.

(v) Dimension of dipole moment = [ATL].

Intensity of electric field due to a dipole
(i) Along axis at distance r from centre of dipole

$E = \frac{2p}{r^3} \cdot \frac{1}{4\pi\varepsilon_0}$ Direction of E is along the direction of dipole moment.

(ii) Along equator of dipole at distance r from centre

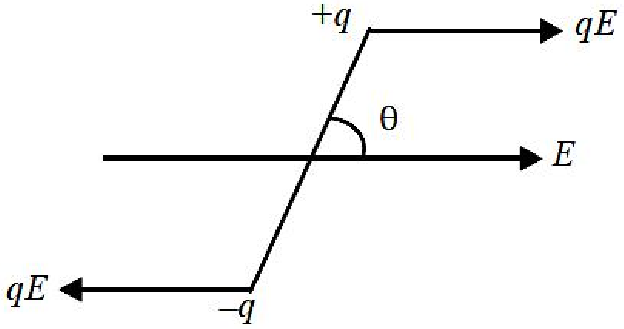
$E = \frac{p}{r^3} \cdot \frac{1}{4\pi\varepsilon_0}$ Direction of E is anti-parallel to direction of p.

(iii) At any point along direction q

$E = \frac{p}{r^3} \sqrt{1 + 3\cos\theta} \cdot \frac{1}{4\pi\varepsilon_0}$

The direction of E makes an angle β with the line joining the point with centre of dipole where $\tan\beta = \frac{1}{2}\tan\theta$.

Torque on a dipole



Two forces [qE and ($-qE$)] equal, opposite and parallel, separated by a distance constitute a couple.

torque ($\vec{\tau}$) = = $\vec{p} \times \vec{E}$

$|\vec{\tau}| = pE\sin\theta$

This direction of $\vec{\tau}$ is perpendicular to the plane containing $\vec{p}$ and $\vec{E}$. The torque tends to align the dipole in the direction of field.
When dipole is parallel to electric field, it is in stable equilibrium. When it is antiparallel to electric field, it is in unstable equilibrium.
Torque is maximum when θ = 90°. Dipole is perpendicular to *E*. Therefore maximum torque = *pE*.

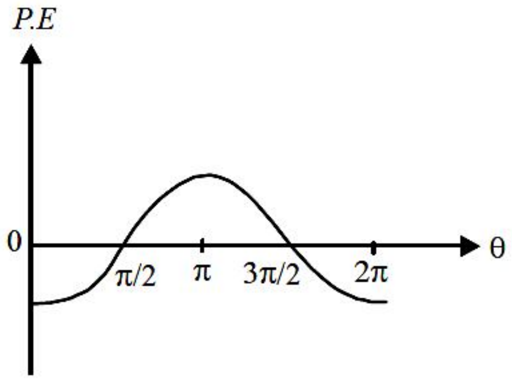
Potential energy of dipole in uniform electric field
Workdone in rotating the dipole from an angle θ_1 to angle θ_2.

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\theta_1}^{\theta_2} pE\sin\theta d\theta = pE\left[-\cos\theta\right]_{\theta_1}^{\theta_2} = -pE(\cos\theta_2 - \cos\theta_1)$$

(i) If θ_1 = 0 and θ_2 = 180°, *W* = 2*pE*.
(ii) If θ_1 = 0 and θ_2 = 90°, *W* = *pE*.

Potential energy of dipole, when it is turned through an angle q from field direction is
$U = -pE\cos\theta$ = - $\vec{p} \times \vec{E}$
(i) If θ = 0, *U* = –*pE*.
The dipole orients itself parallel to field.
(ii) If θ = 90°, *U* = 0.
(iii) If θ = 180°, *U* = *pE*.

Variation of potential energy of dipole with angle θ, between $\vec{E}$ and $\vec{p}$, is shown in the figure.



(i) Potential energy is negative from 0 to π/2 and 3π/2 to 2π. They are regions of stable equilibrium of dipole.
(ii) Potential energy is positive from π/2 to 3π/2. This is the region of unstable equilibrium of the dipole.

Dipole in non-uniform electric field
In non-uniform electric field, the two ends of dipole are acted upon by forces qE_1 and $-qE_2$. They are not equal as $E_1 \neq E_2$ in non-uniform field. Hence a force and a torque both act on the dipole.

Force acting on the dipole can be represented by $\vec{F} = \vec{p} \times \dfrac{d\vec{E}}{dr}$

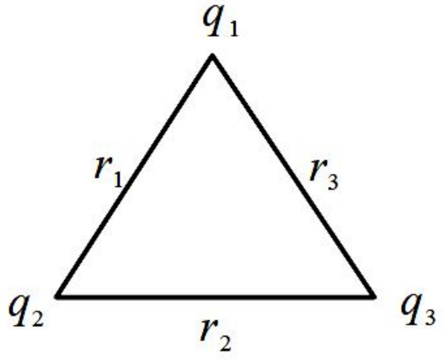
Broadly speaking,
Net force = ($qE_1 - qE_2$) along direction of greater field intensity.
On account of net force upon dipole, it may undergo linear motion.
In a non-uniform electric field, a dipole may, therefore, undergo rotation as well as linear motion.

Potential energy of charge system

For two point charges q_1 and q_2 separated by a distance r, electrostatic potential energy U is given by

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

For three point charges, $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_1} + \frac{q_2 q_3}{r_2} + \frac{q_3 q_1}{r_3}\right]$



- For n charges, consider all pairs with due regard of signs of charges, positive or negative.
- S.I. unit of energy = joule (J)
- Another popular unit is electron volt (eV).
  1 eV = 1.6×10^{-19} joule.

Charged soap bubble

For equilibrium of a charged soap bubble, pressure due to surface tension = $\frac{4T}{r}$ acting inwards.

Electric pressure due to charging = $\frac{\sigma^2}{2\varepsilon_0}$ acting outwards.

At equilibrium, $\frac{4T}{r} = \frac{\sigma^2}{2\varepsilon_0}$ where s = surface density of charge

$$\Rightarrow \frac{4T}{r} = \frac{1}{2\varepsilon_0}\left(\frac{q}{4\pi r^2}\right)^2 \Rightarrow q = 8\pi r\sqrt{2\varepsilon_0 rT}$$

Here air pressures, inside and outside the bubble, are supposed to be same.

Behaviour of a conductor in an electrostatic field

In the case of a charged conductor

(i) Charge resides only on the outer surface of conductor.

(ii) Electric field at any point inside the conductor is zero.

(iii) Electric potential at any point inside the conductor is constant and equal to potential on the surface of the conductor, whatever be the shape and size of the conductor.

(iv) Electric field at any point on the surface of charged conductor is directly proportional to the surface density of charge at that point, but electric potential does not depend upon the surface density of charge.

Capacitance

When a conductor is given a charge, its potential gets raised. The quantity of charge given to a conductor is found to be directly proportional to the potential raised by it. If q is the charge given to conductor and V is potential raised due to it, then $q \propto V$ or $q = CV$, where C is a constant, known as **capacitance** of the conductor.

Capacitance = $\frac{\text{charge}}{\text{potential}}$.

Unit of capacitance is **farad**.
1F = 1 coulomb/volt.
1 Farad = 9 x 10^{11} stat farad.
Dimensions of capacitance are $[M^{-1}L^{-2}T^{4}A^{2}]$.

Capacity of an isolated spherical conductor : Capacitance of an isolated spherical conductor of radius *a* placed in a medium of dielectric constant *K*,

$C = 4\pi\varepsilon_0 Ka$ farad

For vacuum or air, *K* = 1, hence

$C_0 = 4\pi\varepsilon_0 a$ farad.

i.e., capacitance of a spherical conductor µ radius.

Capacitor is a pair of two conductors of any shape which are close to each other and have equal and opposite charges.

A capacitor is an arrangement which can store sufficient quantity of charge.

The quantity of charge that can be given to a capacitor is limited by the fact that every dielectric medium becomes conducting at a certain value of electric field.

Capacitance of a capacitor is
(i) Directly proportional to the area of the plates (*A*).
(ii) Inversely proportional to distance between plates (*d*) –1 .
(iii) Directly proportional to dielectic constant of the medium filled between its plates (*K*).

Parallel plate capacitor : Capacitance of a parallel plate capacitor filled completely with some dielectric medium.

$$C = \frac{K\varepsilon_0 A}{d}$$

For air and vacuum, *K* = 1.

$$C = \frac{\varepsilon_0 A}{d}$$

Capacitance of a parallel plate capacitor filled with dielectric slab of thickness *t* is given by

$$C = \frac{\varepsilon_0 A}{d - t\left[1 - \frac{1}{K}\right]}$$

Capacitance of a parallel plate capacitor filled with a conducting slab of thickness *t* is given by

$$C = \frac{\varepsilon_0 A}{(d - t)}$$

The plates of a parallel plate capacitor attract each other with a force $F = \frac{Q^2}{2A\varepsilon_0}$

Capacitance of a spherical condenser/capacitor, is $C = 4\pi\varepsilon_0 K\left[\frac{ab}{b-a}\right]$

when *a* and *b* are the radii of inner and outer spheres respectively.

Dielectrics are of two types : Nonpolar and polar. The nonpolar dielectrics (like N_2, O_2, benzene, methane) etc. are made up of nonpolar atoms / molecules, in which the centre of mass of negative coincides with the centre of mass of negative charge of the atom / molecule.

The polar dielectrics (like H_2O, CO_2 , NH_3 , HCl) etc. are made up of polar atoms / molecules, in which the centre of mass of positive charge does not coincide with the centre of mass of negative charge of the atom / molecule.

A non-polar dielectric can be polarized by applying an external electric field on the dielectric.

The effective electric field $\vec{E}$ in a polarised dielectric is given by $\vec{E} = \vec{E}_0 - \vec{E}_P$ where $\vec{E}_0$ is strength of external field applied and $\vec{E}_P$ is intensity of induced electric field set up due to polarization. It is equal to surface density of induced charge.
The ratio $E_0 / E = K$, dielectric constant.

When a dielectric slab is placed between the plates of a parallel plate capacitor, the charge induced on its sides due to polarization of dielectric is

$$q_i = q\frac{(K-1)}{K}$$

Capacitors in series: Equivalent capacitance of a series combination of capacitors is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

In series combination of capacitors, charge is same on each capacitor and is equal to charge supplied by source $CV = C_1V_1 = C_2V_2 = ...$

Capacitors in parallel : Equivalent capacitance of a parallel combination of capacitors is $Cp = C_1 + C_2 + C_3 + ...$

In parallel combination of capacitors, potential difference is same across each capacitor and is equal to applied potential difference $\frac{q}{C} = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3} = ...$

Electric potential energy stored in a charged conductor or capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

The electric potential energy of capacitor resides in the dielectric medium between the plates of the condenser.

When two charged conductors are connected together, the redistributed charges on them are in the ratio of their capacitance.

When two charged conductors having charges q_1 and q_2 and capacitances C_1 and C_2 are connected together, then after redistribution of charges, the common potential is $V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ where V_1 and V_2 are the initial potentials of the charged conductors.

In case of charged capacitors, when plates of same polarity are connected together, common potential

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

But when plates of opposite polarity are connected together, then common potential is $V = \frac{C_1V_1 \sim C_2V_2}{C_1 + C_2}$

Total energy stored in any grouping of capacitors is equal to sum of the energies stored in individual capacitors.

If n charged drops, each of capacity C, charged to potential V with charge q, surface density σ and potential energy U coalesce to form a single drop, then for such a drop,
total charge = nq
total capacity = $n^{1/3}C$
potential = $n^{2/3}V$
Surface density of charge = $n^{1/3}\sigma$,
and total potential energy = $n^{2/3}U$.

Sharing of charges
(i) ***Common potential*** : When two capacitors at different potentials V_1 and V_2 are connected, charged q_1 ($= C_1V_1$) and q_2 ($= C_2V_2$) are redistributed till a common potential V is reached. Then

$$V = \frac{\text{total charge}}{\text{total capacity}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

(ii) ***Loss of energy*** : During sharing of charges, energy is lost; mostly as heat, partly as cracking noise and partly as sparking light.

$$\text{Loss of energy} = \frac{1}{2}\frac{C_1C_2(V_1 - V_2)^2}{C_1 + C_2}$$

Van de Graaff generator
Van de Graaff designed this electrostatic machine in 1931 to build up high potential difference of the order of few million volt.

The generator is based on the following points:
(i) The action of sharp points *i.e.* the phenomenon of corona dischage.
(ii) The property that the charge resides on the outer surface of a conductor. Charge given to a hollow conductor is transferred to outer surface and is distributed uniformly over it.

The high potential generated is used to accelerate charged particles like electrons, protons, ions etc. The particles hit the target with the huge energy acquired and carry out the artificial transmutation etc.
Since, Capacity of a spherical shell, $C = 4\pi\varepsilon_0 R$

$$\therefore \text{Potential generated } V = \frac{\text{Charge } Q}{\text{Capacity}} \quad \text{or} \quad V = \frac{Q}{4\pi\varepsilon_0 R}$$

The positive charge goes on accumulating on the large spherical conducting shell.
$\therefore$ Energy of the ions to be accelerated = qV where q denotes charge on the ion or particle accelerated to hit the required target.

ELECTROSTATICS

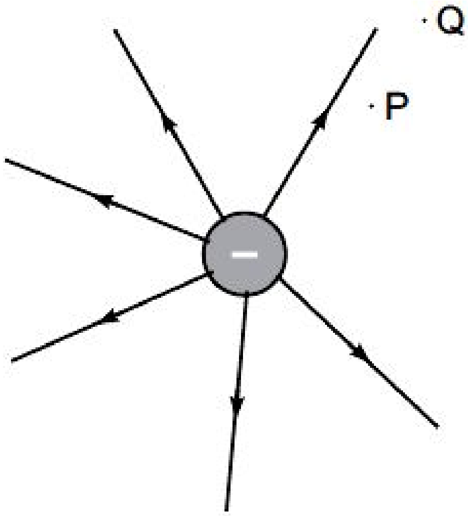
MARKS WEIGHTAGE – 15 marks

Important Questions & Answers

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

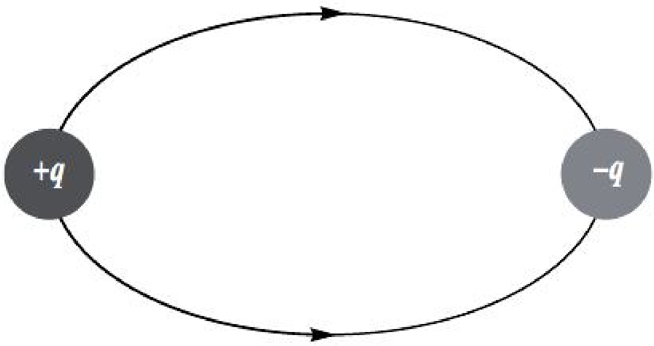
1. **Why must electrostatic field at the surface of a charged conductor be normal to the surface at every point? Give reason.**
   Ans. The work done in moving a charge from one point to another on an equipotential surface is zero. If electric field is not normal, it will have a non-zero component along the surface which would cause work to be done in moving a charge on an equipotential surface.

2. **Figure shows the field lines due to a positive point charge. Give the sign of potential energy difference of a small negative charge between the points *Q* and *P*.**



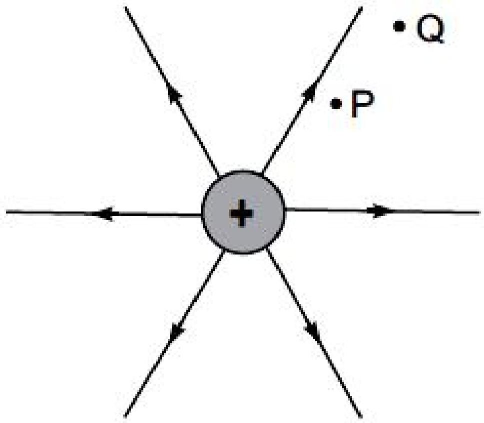
   Ans. $U = \frac{1}{4\pi\varepsilon_0} . \frac{q_1 q_2}{r'}$
   Here, $U_Q < U_P$
   Therefore, $U_Q - U_P$ is negative

3. **Why do the electrostatic field lines not form closed loops?**
   Ans. The electrostatic field lines start from positive charge and end on negative charge.



4. **Why do the electric field lines never cross each other?**
   Ans. If the field lines cross each other, then at the point of intersection, there will be two directions for the same electric field which is not possible.

5. **Figure shows the field lines on a positive charge. Is the work done by the field in moving a small positive charge from *Q* to *P* positive or negative? Give reason.**



Ans. The work done by the field is negative. This is because the charge is moved against the force exerted by the field.

6. **At what position is the electric dipole in uniform electric field in its most stable equilibrium position? [*AI 2008*]**
   Ans. When θ = 0° between $\vec{P}$ and $\vec{E}$

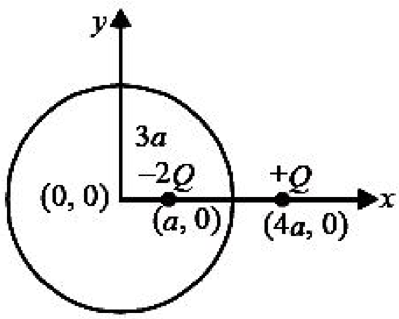
7. **If the radius of the Gaussian surface enclosing a charge is halved, how does the electric flux through the Gaussian surface change? [*AI 2008*]**
   Ans. The electric flux remains the same, as the charge enclosed remains the same.

8. **Define the term electric dipole moment of a dipole. State its S.I. unit. [*AI 2008*]**
   Ans. Strength of an electric dipole is measured by its electric dipole moment, whose magnitude is equal to product of magnitude of either charge and separation between the two charges *i.e.*, $\vec{p} = q.2\vec{a}$ and is directed from negative to positive charge, along the line joining the two charges. Its SI unit is Cm.

9. **A charge '*q*' is placed at the centre of a cube of side *l*. What is the electric flux passing through each face of the cube? [*AI 2012*]**
   Ans. Flux through whole of the cube, $\phi = \frac{q}{\varepsilon_0}$

   Flux through each face of the cube, $\phi' = \frac{\phi}{6} = \frac{q}{6\varepsilon_0}$

10. **Two charges of magnitudes –2*Q* and +*Q* are located at points (*a*, 0) and (4*a*, 0) respectively. What is the electric flux due to these charges through a sphere of radius '3*a*' with its centre at the origin? [*AI 2013*]**



   Ans. *Electric flux* $\phi = \frac{q_{inside}}{\varepsilon_0} = \frac{-2Q}{\varepsilon_0}$

11. **What is the electrostatic potential due to an electric dipole at an equatorial point? [*AI 2009*]**
    Ans. Zero

12. **Name the physical quantity whose S.I. unit is J C^{-1} . Is it a scalar or a vector quantity? [*AI 2010*]**
    Ans. J C^{-1} is the S.I. unit of electrostatic potential. It is a scalar quantity.

13. **What is the value of the angle and between the vectors $\vec{p}$ and $\vec{E}$ for which the potential energy of an electric dipole of dipole moment $\vec{p}$, kept in an external electric field $\vec{E}$, has the maximum value.**

Ans. Potential energy = $-\vec{p}.\vec{E} = -pE\cos\theta$

Therefore, Potential energy is the maximum when $\cos\theta = -1$ *i.e.* $\theta = \pi$ *or* 180^0

14. A point charge *Q* is placed at point *O* as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero, if *Q* is (i) positive (ii) negative? [*AI 2011*]

Ans. (i) If Q is positive, $V_A - V_B$ is positive.
(ii) If Q is negative, $V_A - V_B$ is negative.

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

15. Define electric flux. Write its S.I. unit.

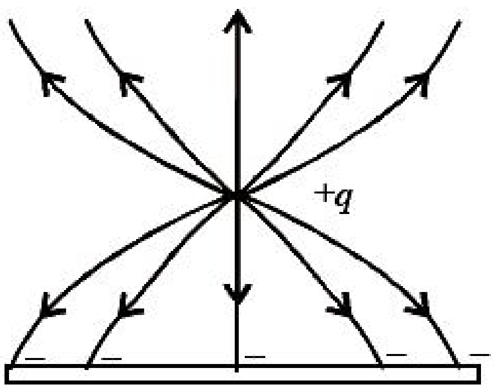
A charge q is enclosed by a spherical surface of radius R. If the radius is reduced to half, how would the electric flux through the surface change? [*AI 2009*]

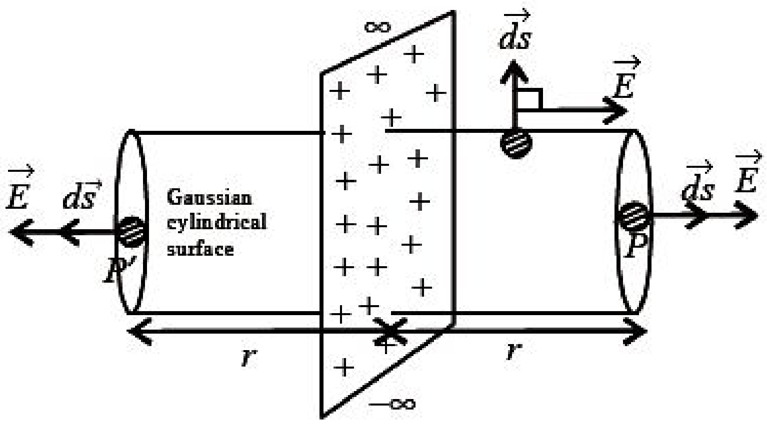
Ans. Electric flux linked with a surface is the number of electric lines of force cutting through the surface normally. It's SI unit is Nm^2C^{-1} or Vm on decreasing the radius of spherical surface to half there will be no effect on the electric flux.

16. A positive point charge (+*q*) is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines originating from the point on to the surface of the plate. Derive the expression for the electric field at the surface of a charged conductor. [*AI 2009*]

Ans. Let us consider an infinite plane sheet of charge of uniform charge density where q is charge in area A on sheet of charge.

$$\sigma = \frac{q}{A}$$





Let P be any point on the one side of sheet and P' on the other side of sheet, at same distance r from it. We draw a Gaussian cylindrical surface S of cross section area A cutting through the plane sheet of charge, such that points P and P' lie on its plane faces. Then electric flux linked with cylindrical surface S is

$$\phi = \oint \vec{E}\cdot d\vec{s}$$

$$\text{or } \phi = \int_{\text{lpf}} \vec{E}\cdot d\vec{s} + \int_{\text{cs}} \vec{E}\cdot d\vec{s} + \int_{\text{rpf}} \vec{E}\cdot d\vec{s}$$

$$\text{or } \phi = \int_{\text{lpt}} Eds\cos 0 + \int_{\text{cs}} Eds\cos 90 + \int_{\text{rpf}} Eds\cos 0$$

$$\text{or } \phi = E\int_{\text{lpf}} ds + 0 + E\int_{\text{rpf}} ds = EA + EA$$

or $\phi = 2EA$ ...(ii)

But by Gauss's theorem

$$\phi = \frac{q}{\varepsilon_0} \quad ...(iii)$$

where q is the charge in area A of sheet, enclosed by cylindrical surface S.

By equations (ii) and (iii), we get

$$2EA = \frac{q}{\varepsilon_0} \quad \text{or} \quad E = \frac{q}{2A\varepsilon_0}$$

$$\text{or} \quad E = \frac{\sigma}{2\varepsilon_0}$$

This gives the electric field intensity at any point near or on the surface of the infinite thin plane sheet of charge.

17. A parallel plate capacitor is charged by a battery. After some time the battery is disconnected and a dielectric slab of dielectric constant *K* is inserted between the plates. How would (i) the capacitance, (ii) the electric field between the plates and (iii) the energy stored in the capacitor, be affected? Justify your answer. [*AI 2009*]

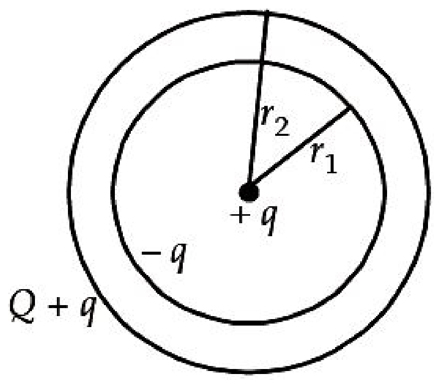
Ans. (i) On filling the dielectric constant of K in the space between the plates, capacitance of parallel plate capacitor becomes K times *i.e.* $C = KC_0$

(ii) As the battery was disconnected, so the charge on the capacitor remains the same *i.e.* $Q = Q_0$. So, the electric field in the space between the plates becomes $E = \frac{E_0}{KA\varepsilon_0} = \frac{Q_0}{KA\varepsilon_0}$ *or* $E = \frac{E_0}{K}$ *i.e.* electric field becomes $\frac{1}{K}$ times.

(iii) Energy stored in capacitor becomes $U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q_0^2}{KC}$ *or* $U = \frac{1}{2}U_0$ i.e. becomes $\frac{1}{K}$ times.

18. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge *Q*. A charge *q* is placed at the centre of the shell.
(a) What is the surface charge density on the (i) inner surface, (ii) outer surface of the shell?
(b) Write the expression for the electric field at a point *x* > *r* 2 from the centre of the shell. [*AI 2010*]

Ans. (a) (i) Surface charge density on the inner surface of shell is $\sigma_{in} = \frac{-q}{4\pi r_1^2}$



(ii) Surface charge density on the outer surface of shell is $\sigma_{out} = \frac{Q+q}{4\pi r_2^2}$

(b) Using, Gauss's law, $E(x) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q+q}{x^2}$

19. Explain the meaning of the statement 'electric charge of a body is quantized'.

Ans. The electric charge of a body is quantized means that the charge on a body can occur in some particular values only. Charge on any body is the integral multiple of charge on an electron because the charge of an electron is the elementary charge in nature. The charge on any body can be expressed by the formula

$q = \pm ne$, where, n = number of electrons transferred and e = charge on one electron.

The cause of quantization is that only integral number of electrons can be transferred from one body to other.

20. Why can one ignore quantization of electric charge when dealing with macroscopic, *i.e.*, large scale charges?

Ans. We can ignore the quantization of electric charge when dealing with macroscopic charges because the charge on one electron is 1.6 x 10^{-19} C in magnitude, which is very small as compared to the large scale change.

21. An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?

Ans. An electrostatic field line represents the actual path travelled by a unit positive charge in an electric field. If the line have sudden breaks it means the unit positive test charge jumps from one place to another which is not possible. It also means that electric field becomes zero suddenly at the breaks which is not possible. So, the field line cannot have any sudden breaks.

22. Explain why two field lines never cross each other at any point?

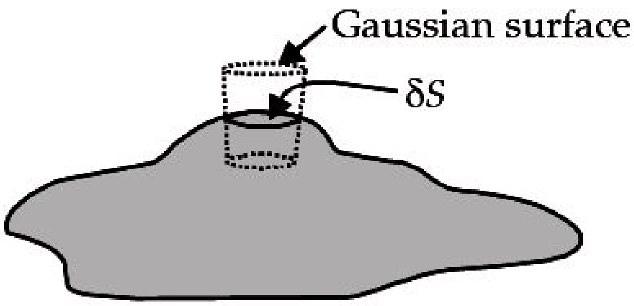
Ans. If two field lines cross each other, then we can draw two tangents at the point of intersection which indicates that (as tangent drawn at any point on electric line of force gives the direction of electric field at that point) there are two directions of electric field at a particular point, which is not possible at the same instant. Thus, two field lines never cross each other at any point.

23. Show that the electric field at the surface of a charged conductor is given by $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$ where σ is the surface charge density and is a unit vector normal to the surface in the outward direction. [*AI 2010*]

Ans. Consider an elementary area δS on the surface of the charged conductor. Enclose this area element with a cylindrical gaussian surface as shown in figure.

Now electric field inside a charged conductor is zero. Therefore, direction of field, just out side δS will be normally outward *i.e.* in direction of $\hat{n}$.

According to Gauss's theorem, total electric flux coming out is



$\vec{E}.\delta\vec{S} = \frac{\sigma\delta S}{\varepsilon_0}$ [$\vec{E}$ is electric field at the surface]

$$\Rightarrow E\delta S\cos 0^0 = \frac{\sigma\delta S}{\varepsilon_0}$$

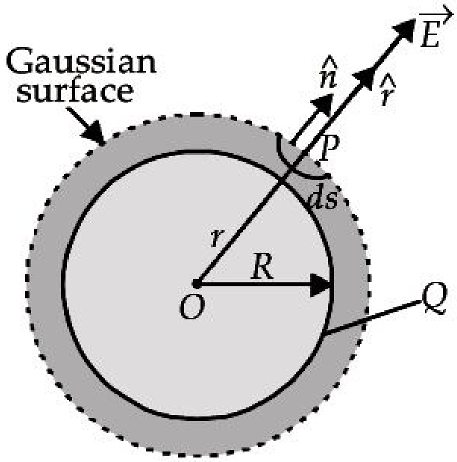
$$\Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

24. Using Gauss's law obtain the expression for the electric field due to a uniformly charged thin spherical shell of radius *R* at a point outside the shell. Draw a graph showing the variation of electric field with *r*, for *r* > *R* and *r* < *R*. [*AI 2011*]

Ans. Consider a thin spherical shell of radius *R* carrying charge *Q*. To find the electric field outside the shell, we consider a spherical Gaussian surface of radius *r* (> *R*), concentric with given shell.

The electric field $\vec{E}$ is same at every point of Gaussian surface and directed radially outwards (as is unit vector $\hat{n}$ so that q = 0°)

According to Gauss's theorem, $\oint_s \vec{E}.d\vec{s} = \oint_s \vec{E}.\hat{n}d\vec{s} = \frac{Q}{\varepsilon_0}$

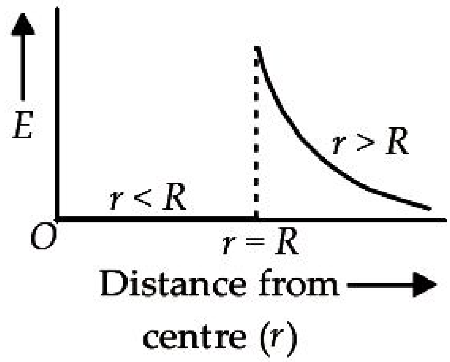


$$or \;\; E\oint_s ds = \frac{Q}{\varepsilon_0}$$

$$\therefore E(4\pi r^2) = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

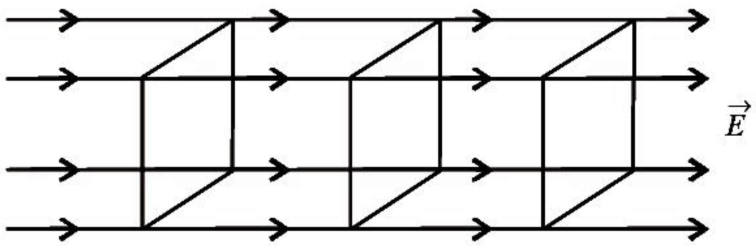
Hence, electric field outside a charged thin spherical shell is the same as if the whole charge *Q* is concentrated at the centre.

The variation of electric field $\vec{E}$ with distance from centre of a uniformly charged spherical shell is shown in figure.

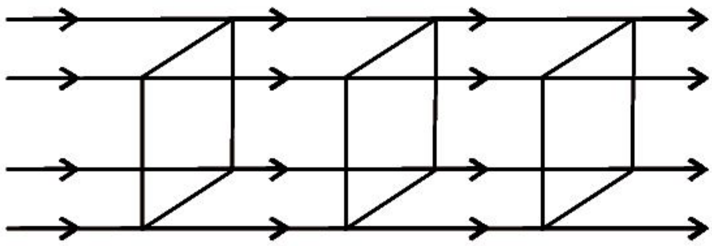


25. Draw 3 equipotential surfaces corresponding to a field that uniformly increases in magnitude but remains constant along *Z* direction. How are these surfaces different from that of a constant electric field along *Z* direction? [*AI 2009*]

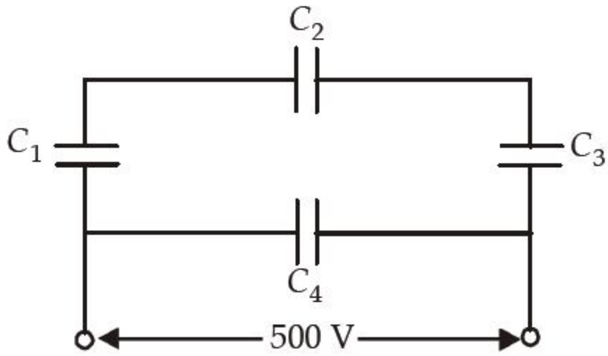
Ans. For constant electric field, equipotential surfaces are equidistant for same potential difference between these surfaces. For increasing electric field, separation between equipotential surfaces decreases, in the direction of increasing field, for the same potential difference between them.



For increasing electric field


For constamt electric field

26. A network of four capacitors each of 12 μF capacitance is connected to a 500 V supply as shown in the figure. Determine (a) equivalent capacitance of the network and (b) charge on each capacitor.



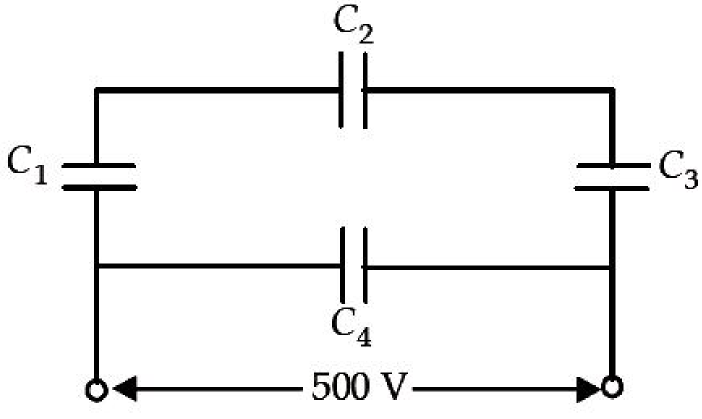
Ans. Here C_1 , C_2 and C_3 are in series, hence their equivalent capacitance is C' given by

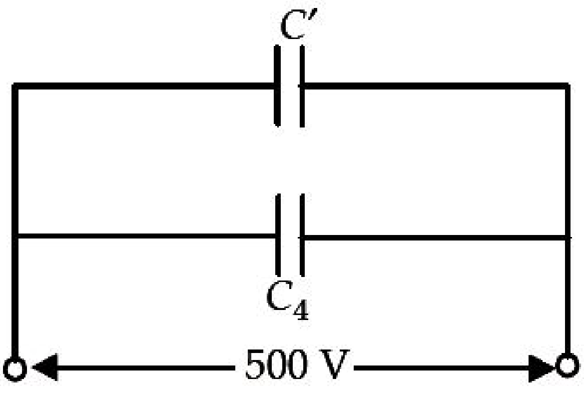
$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$C' = \frac{12}{3}\mu F \Rightarrow C' = 4\mu F$$

The circuit can be redrawn as shown, above. Since C' and C_4 are in parallel

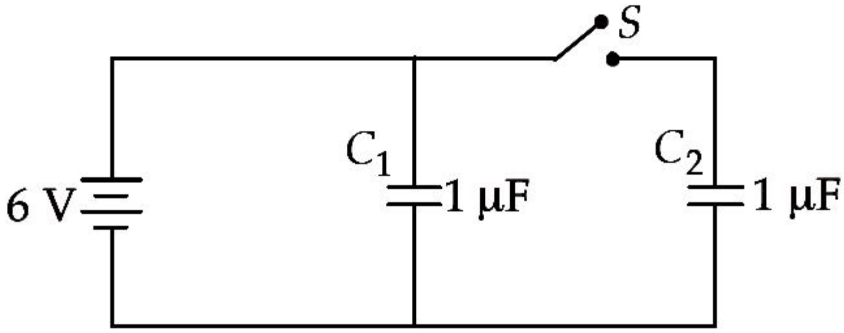
$$\therefore C_{net} = C' + C_4 = 4\mu F + 12\mu F = 16\mu F$$



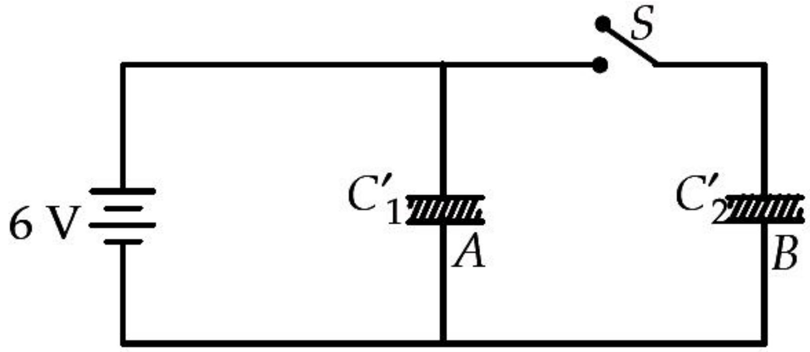


(b) Since C' and C_4 are in parallel, potential difference across both of them is 500 V.
$\therefore$ Charge across C_4 is $Q_4 = C_4 \times 500$ C
$= 12 \times 10^{-6} \times 500$ C = 6 mC
and Charge across C', $Q' = C' \times 500$ C
$= 4 \times 10^{-6} \times 500$ C = 2 mC
$\because C_1, C_2, C_3$ are in series, charge across them is same, which is $Q' = 2$ mC

27. Figure shows two identical capacitors C_1 and C_2, each of 1 μF capacitance connected to a battery of 6 V. Initially switch *S* is closed. After sometime *S* is left open and dielectric slabs of dielectric constant $K = 3$ are inserted to fill completely the space between the plates of the two capacitors. How will the (i) charge and (ii) potential difference between the plates of the capacitors be affected after the slabs are inserted?

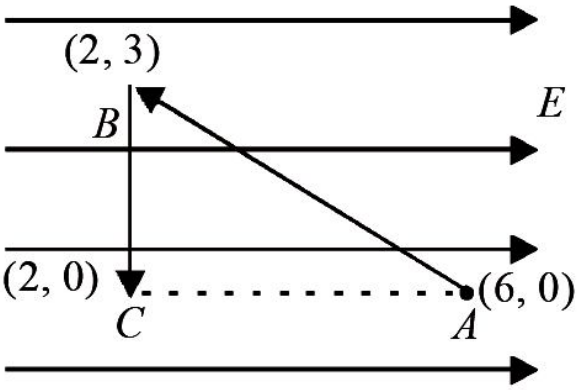


Ans. When the switch *S* is closed, the two capacitors in parallel will be charged by the same potential difference *V*.
So, charge on capacitor C_1
$q_1 = C_1V = 1 \times 6 = 6$ mC
and charge on capacitor C_2
$q_2 = C_2 V = 1 \times 6 = 6$ mC
$q = q_1 + q_2 = 6 + 6 = 12$ mC.
When switch *S* is opened and dielectric is introduced. Then



Capacity of both the capacitors becomes *K* times
i.e., $C'_1 = C'_2 = KC = 3 \times 1 = 3$ mF
Capacitor *A* remains connected to battery
$\because V'_1 = V = 6$ V
$q'_1 = Kq = 3 \times 6$ mC = 18 m C
Capacitor *B* becomes isolated
$\because q'_2 = q_2$ or $C'_2 V'_2 = C_2V_2$ or $(KC)V'_2 = CV$
$V'_2 = \frac{V}{K} = \frac{6}{3} = 2V$

28. A test charge '*q*' is moved without acceleration from *A* to *C* along the path from *A* to *B* and then from *B* to *C* in electric field *E* as shown in the figure. (i) Calculate the potential difference between *A* and *C*'. (ii) At which point (of the two) is the electric potential more and why? [*AI 2012*]



Ans. In the relation

$$E = \frac{-dV}{dr} \Rightarrow E = -\left[\frac{V_C - V_A}{(2-6)}\right]$$

(i)$V_C - V_A = 4E$ (ii) Hence $V_C > V_A$

Also electric field is directed from points of high potential to low potential.

29. Deduce the expression for the electrostatic energy stored in a capacitor of capacitance '*C*' and having charge '*Q*'. How will the (i) energy stored and (ii) the electric field inside the capacitor be affected when it is completely filled with a dielectric material of dielectric constant '*K*'? [*AI 2012*]

Ans. Potential difference between the plates of capacitor $V = \frac{q}{C}$

Work done to add additional charge dq on the capacitor

$dW = V \times dq = \frac{q}{C} \times dq$

$\because$ Total energy stored in the capacitor

$$U = \int dW = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{2}$$

When battery is disconnected

(i) Energy stored will be decreased or energy stored = $\frac{1}{K}$ times the initial energy.

(ii) Electric field would decrease

or $E' = \frac{E}{K}$

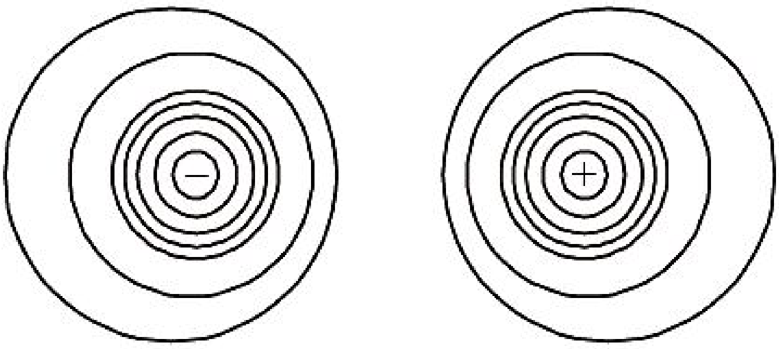
Alternatively, if a student attempts to answer by keeping the battery connected, then

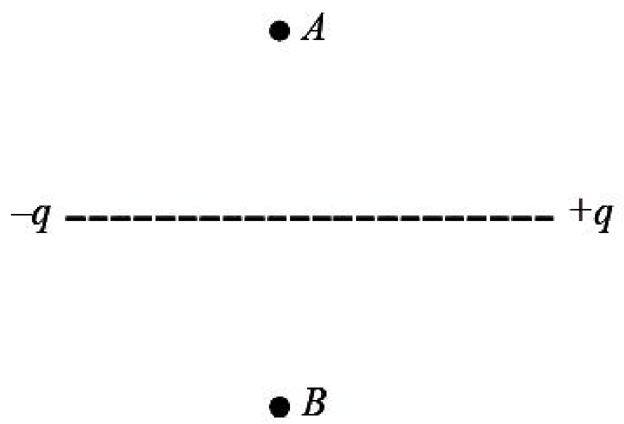
(i) energy stored will increase or become K times the initial energy.

(ii) electric field will not change.

30. Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero.

Ans.

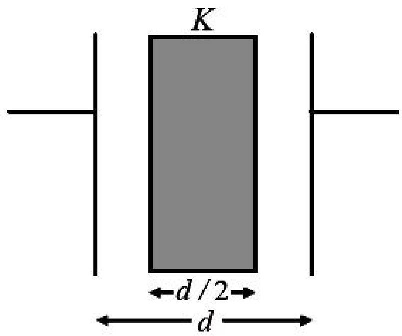




Alternatively Any point on the equatorial plane (*AB*) of the dipole.

31. A slab of material of dielectric constant *K* has the same area as that of the plates of a parallel plate capacitor but has the thickness *d*/2, where *d* is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor. [*AI 2013*]

Ans.

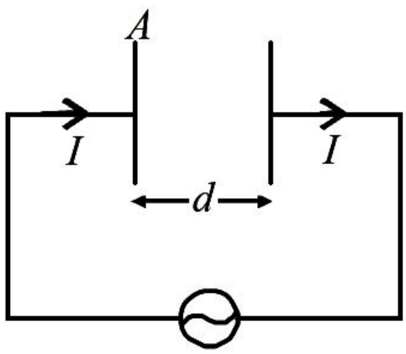


Capacitance of a capacitor partially filled with a dielectric

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

$$\Rightarrow C = \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{2\varepsilon_0 AK}{d(K+1)}$$

32. A capacitor, made of two parallel plates each of plate area *A* and separation *d*, is being charged by an external ac source. Show that the displacement current inside the capacitor is the same as the current charging the capacitor. [*AI 2013*]



Ans. The displacement current within capacitor plates

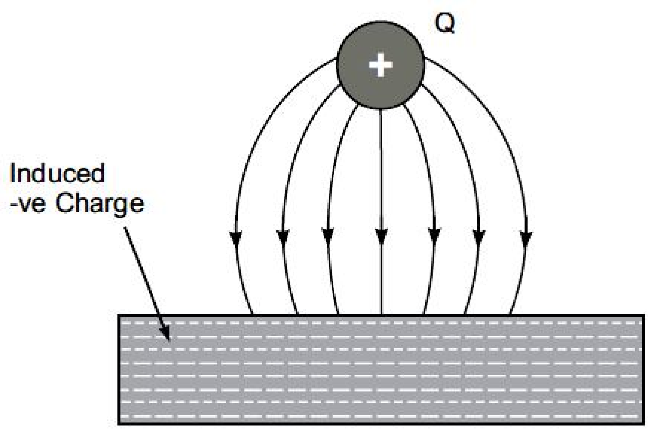
$$I_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

$$where\ \ \phi_E = EA = \frac{q}{A\varepsilon_0} A = \frac{q}{\varepsilon_0}$$

$$I_d = \frac{\varepsilon_0}{\varepsilon_0} \frac{dq}{dt} \Rightarrow I_d = I$$

33. A point charge (+*Q*) is kept in the vicinity of uncharged conducting plate. Sketch electric field lines between the charge and the plate.

Ans.



The lines of force start from + Q and terminate at metal place inducing negative charge on it.
The lines of force will be perpendicular to the metal surface.

34. Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole.

Ans. It is the product of magnitude of either charge and the distance between the two equal and opposite charges. Alternatively,

$\vec{p} = q2a\hat{a}$

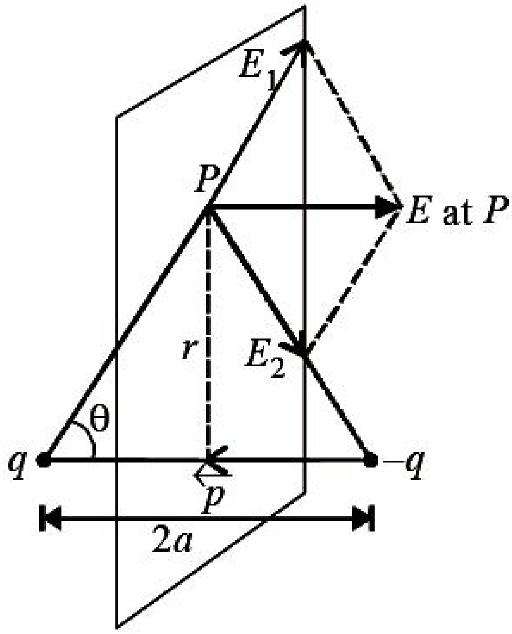
It is a vector quantity.

$E = E_1 \cos\theta + E_1 \cos\theta = 2E_1 \cos\theta$

$$E = \frac{2}{4\pi\varepsilon_0} \frac{q}{(r^2 + a^2)} \cdot \frac{a}{(r^2 + a^2)^{1/2}}$$

$$E = \frac{2}{4\pi\varepsilon_0} \frac{qa}{(r^2 + a^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0} \frac{2qa}{(r^2 + a^2)^{3/2}}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{p}{(r^2 + a^2)^{3/2}} \quad where \quad p = 2qa$$



35. Using Gauss' law deduce the expression for the electric field due to a uniformly charged spherical conducting shell of radius *R* at a point (i) outside and (ii) inside the shell. Plot a graph showing variation of electric field as a function of *r* > *R* and *r* < *R*. (*r* being the distance from the centre of the shell)

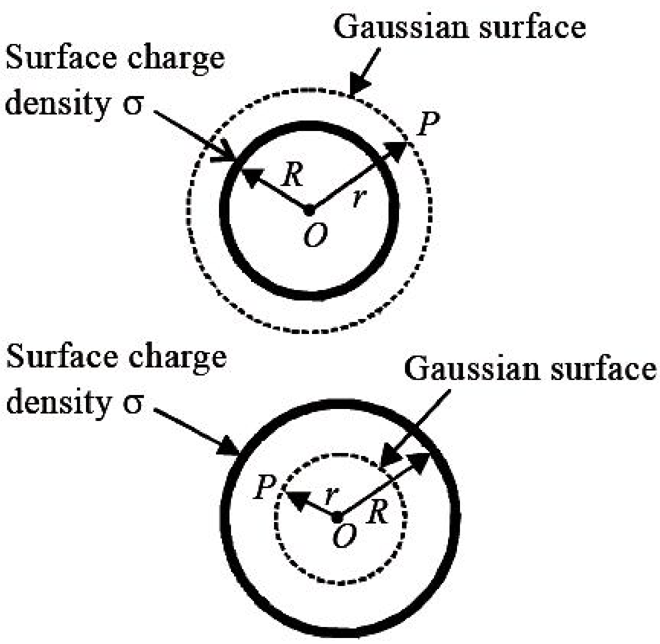
Ans.

By Gauss Law, $r > R$ (outside)

$$\oint \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E\oint d\vec{S} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$



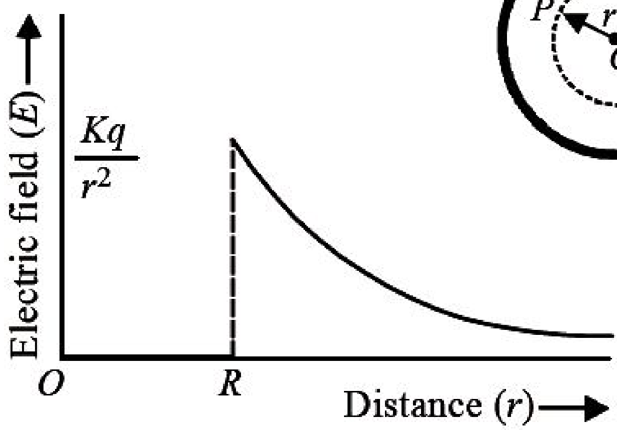
Similarly, $r < R$ (inside)

$$E\oint d\vec{S} = \frac{q}{\varepsilon_0}$$

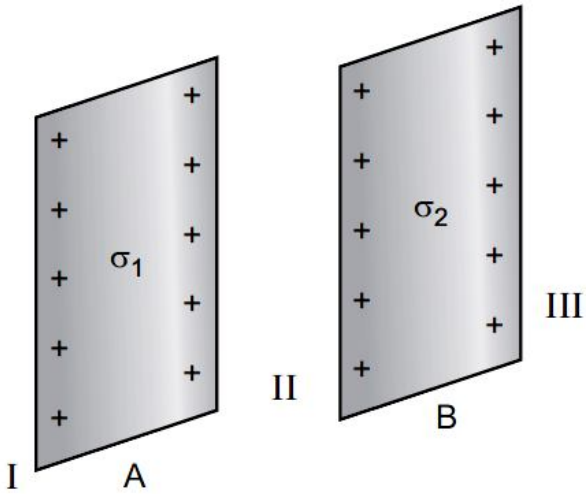
As inside the shell $q = 0$

$\therefore E4\pi R^2 = 0$

$As \quad R \neq 0, \quad E = 0$



36. Two infinitely large plane thin parallel sheets having surface charge densities σ_1 and σ_2 ($\sigma_1 > \sigma_2$) are shown in the figure. Write the magnitudes and directions of the net fields in the regions marked II and III.



Ans. (*i*) Net electric field in region II = $\frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2)$

Direction of electric field in from sheet *A* to sheet *B*.

(*ii*) Net electric field in region III = $\frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2)$

Direction is away from the two sheets i.e. towards right side.

37. In a parallel plate capacitor with air between the plates, each plate has an area of 6 × 10^{-3} m^2 and the separation between the plates is 3 mm.
(*i*) Calculate the capacitance of the capacitor.
(*ii*) If this capacitor is connected to 100 V supply, what would be the charge on each plate?
(*iii*) How would charge on the plates be affected, if a 3 mm thick mica sheet of *K* = 6 is inserted between the plates while the voltage supply remains connected?

Ans. Here, $A = 6 \times 10^{-3}$ m^2, $d = 3$ mm $= 3 \times 10^{-3}$ m

(*i*) Capacitance, $C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}} = 17.7\times10^{-12} F$

(*ii*) Charge, $Q = CV = 17.7\times10^{-12}\times100C = 17.7\times10^{-10}C$

(*iii*) New charge, Q' = KQ = $= 6\times17.7\times10^{-10}C = 106.2\times10^{-10}C$

38. In a parallel plate capacitor with air between the plates, each plate has an area of 5 × 10^{-3} m^2 and the separation between the plates is 2.5 mm.
(*i*) Calculate the capacitance of the capacitor.
(*ii*) If this capacitor is connected to 100 V supply, what would be the charge on each plate?
(*iii*) How would charge on the plates be affected, if a 2.5 mm thick mica sheet of *K* = 8 is inserted between the plates while the voltage supply remains connected?

Ans. Here, $A = 5 \times 10^{-3}$ m^2, $d = 2.5$ mm $= 2.5 \times 10^{-3}$ m

(*i*) Capacitance, $C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times5\times10^{-3}}{2.5\times10^{-3}} = 17.7\times10^{-12} F$

(*ii*) Charge, $Q = CV = 17.7\times10^{-12}\times100C = 17.7\times10^{-10}C$

(*iii*) New charge, Q' = KQ = $= 8\times17.7\times10^{-10}C = 141.6\times10^{-10}C$

39. Two equal balls having equal positive charge '*q*' coulombs are suspended by two insulating strings of equal length. What would be the effect on the force when a plastic sheet is inserted between the two?

Ans. Force will decrease.

Reason: Force between two charges each '*q*' in vacuum is

$$F_0 = \frac{1}{4\pi\varepsilon_0}.\frac{q^2}{r^2}$$

On inserting a plastic sheet (a dielectric $K > 1$)

Then $F = \frac{1}{4\pi\varepsilon_0 K}.\frac{q^2}{r^2}$ i.e. Force $F = \frac{F_0}{K}$

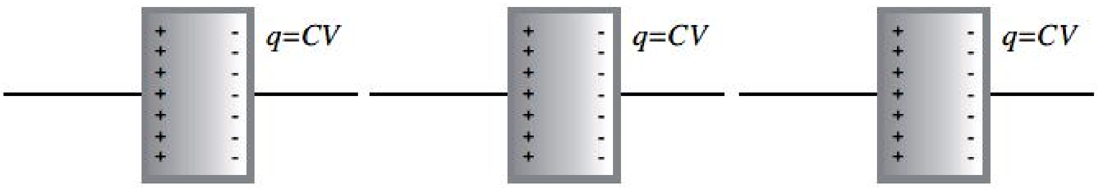
The force between charged balls will decrease.

40. A parallel plate capacitor of capacitance *C* is charged to a potential *V*. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor.

Ans. The charge on the capacitor $q = CV$ and initial energy stored in the capacitor

$$U_1 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 \qquad \text{----------- (i)}$$

(*a*) If another uncharged capacitor is connected in series then the same amount of the charge will transfer as shown in figure.



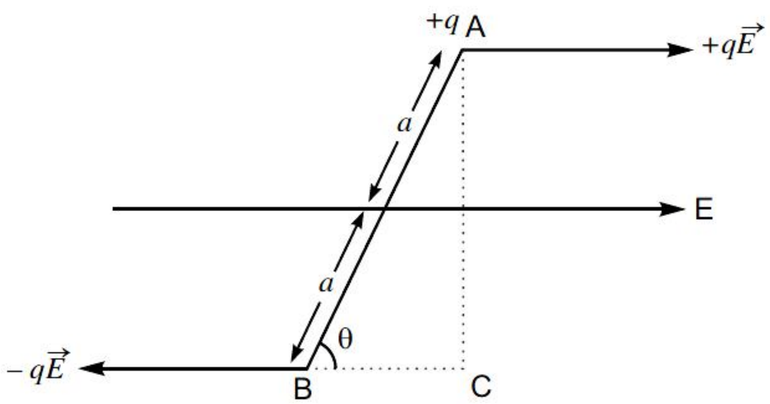
Keeping charge constant, and final voltage *v*□□2*v*

$$U_f = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}\frac{q^2}{C} = \frac{q^2}{C}$$

$$U_f : U_i = \frac{q^2}{C} : \frac{q^2}{2C} = 2:1$$

41. Deduce the expression for the torque acting on a dipole of dipole moment $\vec{p}$ in the presence of a uniform electric field $\vec{E}$.

Ans. Expression for torque



An electric dipole having charges ± *q*, and of size 2*a* is placed in uniform electric field $\vec{E}$ as shown in figure. The forces, acting on the charges are $+q\vec{E}$ and $-q\vec{E}$.

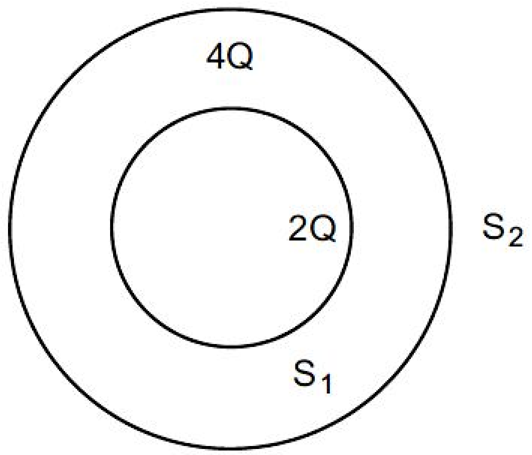
The net force on the dipole is $\vec{F} = +q\vec{E} + (-q\vec{E}) = 0$

Both forces provides an equivalent torque with magnitude

t =| *qE* | x Perpendicular distance (AC)

$= q\,|E|\,.2a\sin\theta$
$= |P|\,|E|\sin\theta$
The direction of the torque can be given by $\vec{\tau} = \vec{p} \times \vec{E}$

42. Consider two hollow concentric spheres, S_1 and S_2 , enclosing charges 2Q and 4Q respectively as shown in the figure. (*i*) Find out the ratio of the electric flux through them. (*ii*) How will the electric flux through the sphere S_1 change if a medium of dielectric constant 'ε_r ' is introduced in the space inside S_1 in place of air? Deduce the necessary expression.



Ans. Using Gauss's Theorem $\oint \vec{E}.\vec{ds} = \dfrac{q(T)}{\varepsilon_0}$

Electric flux through sphere S_1 = $\phi_1 = \dfrac{2(q)}{\varepsilon_0}$

Electric flux through sphere S_2 = $\phi = \dfrac{(2Q+4Q)}{\varepsilon_0} = \dfrac{6Q}{\varepsilon_0}$

Ratio = $\dfrac{\phi_1}{\phi} = \dfrac{\frac{2Q}{\varepsilon_0}}{\frac{6Q}{\varepsilon_0}} = \dfrac{1}{3}$

If a medium of dielectric constant $K(=\varepsilon_r)$ is filled in the sphere S_1, electric flux through sphere, $\phi'_1 = \dfrac{2Q}{\varepsilon_r \varepsilon_0} = \dfrac{2Q}{K\varepsilon_0}$

43. "For any charge configuration, equipotential surface through a point is normal to the electric field." Justify.
Ans. The work done in moving a charge from one point to another on an equipotential surface is zero. If electric field is not normal to the equipotential surface, it would have non-zero component along the surface. In that case work would be done in moving a charge on an equipotential surface.

44. An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $4\sqrt{3}$ Nm. Calculate the potential energy of the dipole, if it has charge ±8 *nC*.
Ans. Torque, t = *pE* sin q
4$\sqrt{3}$ = *pE* sin 60°
$4\sqrt{3} = pE \times \dfrac{\sqrt{3}}{2} \Rightarrow pE = 8$

Now, potential energy, $U = -pE\cos\theta$ = – 8 cos 60° = -8 ´ $\frac{1}{2}$ = -4 J

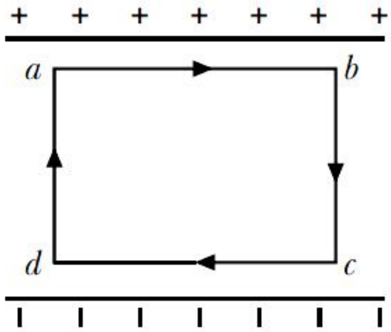
45. Given a uniform electric field $\vec{E}$ = 5 × 10^3 $\hat{i}$ N/C, find the flux of this field through a square of 10 cm on a side whose plane is parallel to the *y-z* plane. What would be the flux through the same square if the plane makes a 30° angle with the *x*-axis?

Ans. Here, $\vec{E}$ = 5 × 10^3 $\hat{i}$ N/C, *i.e.* field is along positive direction of *x*-axis.
Surface area, A = 10 cm × 10 cm = 0.10 m × 0.10 m = 10^{-2} m-

(*i*) When plane parallel to *y - z* plane, the normal to plane is along *x* axis. Hence
θ = 0°
$\phi = EA\cos\theta = 5 \times 10^3 \times 10^{-2}\cos 0° = 50 NC^{-1}m^2$

(*ii*) When the plane makes a 30° angle with the *x*-axis, the normal to its plane makes 60° angle with *x*-axis. Hence
θ = 60°
$\phi = EA\cos\theta = 5 \times 10^3 \times 10^{-2}\cos 60° = 25\ NC^{-1}m^2$

46. The electric field inside a parallel plate capacitor is *E*. Find the amount of work done in moving a charge *q* over a closed rectangular loop *abcda*.



Ans. Work done in moving a charge *q* from *a* to *b* = 0
Work done in moving a charge *q* from *c* to *d* = 0
This is because the electric field is perpendicular to the displacement.
Now, work done from *b* to *c* = – work done from *d* to *a*
Therefore, total work done in moving a charge *q* over a closed loop = 0.

47. Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.

Ans. **When a capacitor is charged by a battery, work is done by the charging battery at the expense of its chemical energy. This work is stored in the capacitor in the form of electrostatic potential energy.**

Consider a capacitor of capacitance *C*. Initial charge on capacitor is zero. Initial potential difference between capacitor plates = zero. Let a charge *Q* be given to it in small steps. When charge is given to capacitor, the potential difference between its plates increases. Let at any instant when charge on capacitor be *q*, the potential difference between its plates $V = \frac{q}{C}$

Now work done in giving an additional infinitesimal charge *dq* to capacitor

$$dW = Vdq = V = \frac{q}{C}dq$$

The total work done in giving charge from 0 to *Q* will be equal to the sum of all such infinitesimal works, which may be obtained by integration. Therefore total work

$$W = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{C}\left(\frac{Q^2}{2} - 0\right) = \frac{Q^2}{2C}$$

If V is the final potential difference between capacitor plates, then $Q = CV$

$$W = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

This work is stored as electrostatic potential energy of capacitor *i.e.*,

Electrostatic potential energy, $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$

Energy density: Consider a parallel plate capacitor consisting of plates, each of area A, separated by a distance d. If space between the plates is filled with a medium of dielectric constant K, then

Capacitance of capacitor, $C = \frac{K\varepsilon_0 A}{d}$

If σ is the surface charge density of plates, then electric field strength between the plates

$$E = \frac{\sigma}{K\varepsilon_0} \Rightarrow \sigma = K\varepsilon_0 E$$

Charge on each plate of capacitor $Q = \sigma A = K\varepsilon_0 EA$

$\therefore$ Energy stored by capacitor, $U = \frac{Q^2}{2C} = \frac{(K\varepsilon_0 EA)^2}{2\left(\frac{K\varepsilon_0 A}{d}\right)} = \frac{1}{2}K\varepsilon_0 E^2 Ad$

But Ad = volume of space between capacitor plates

$\therefore$ Energy stored, $U = \frac{1}{2}K\varepsilon_0 E^2 Ad$

Electrostatic Energy stored per unit volume, $u_e = \frac{U}{Ad} = \frac{1}{2}K\varepsilon_0 E^2$

This is expression for electrostatic energy density in medium of dielectric constant K.

In air or free space ($K = 1$), therefore energy density, $u_e = \frac{1}{2}\varepsilon_0 E^2$

48. Two charged spherical conductors of radii R_1 and R_2 when connected by a conducting wire acquire charges q_1 and q_2 respectively. Find the ratio of their surface charge densities in terms of their radii.

Ans. When two charged spherical conductors are connected by a conducting wire, they acquire the same potential.

$$\frac{kq_1}{R_1} = \frac{kq_2}{R_2}$$

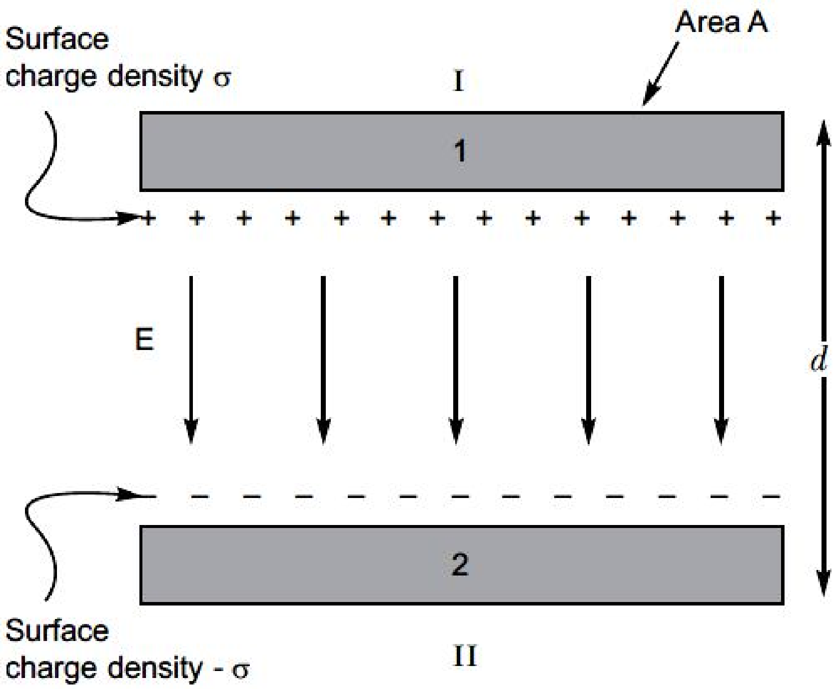
$$\Rightarrow \frac{q_1}{R_1} = \frac{q_2}{R_2} \Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2}$$

Hence, the ratio of surface charge densities

$$\frac{\sigma_1}{\sigma_2} = \frac{\frac{q_1}{4\pi R_1^2}}{\frac{q_2}{4\pi R_2^2}} = \frac{q_1 R_2^2}{q_2 R_1^2}$$

49. Derive the expression for the capacitance of a parallel plate capacitor having plate area A and plate separation *d*.

Ans.



In the region between the plates the net electric field is equal to the sum of the electric fields due to the two charged plates. Thus, the net electric field is given by

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

The electric field is constant in the region between the plates. Therefore, the potential difference between the plates will be

$$V = Ed = \frac{\sigma d}{\varepsilon_0}$$

Now, capacitance $C = \frac{Q}{V} = \frac{Q\varepsilon_0}{\sigma d}$

Surface charge density $\sigma = \frac{Q}{A}$, where A is the area of cross-section of the plates.

$$C = \frac{Q\varepsilon_0 A}{Qd} = \frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{R_1}{R_2} \times \frac{R_2^{\ 2}}{R_1^{\ 2}} = \frac{R_2}{R_1}$$

50. Derive an expression for the energy stored in a parallel plate capacitor. On charging a parallel plate capacitor to a potential *V*, the spacing between the plates is halved, and a dielectric medium of E_r = 10 is introduced between the plates, without disconnecting the d.c. source. Explain, using suitable expressions, how the
(i) capacitance,
(ii) electric field and
(iii) energy density of the capacitor change.
Ans.

(i) $C = \frac{k\varepsilon_0 A}{\frac{d}{2}} = \frac{2k\varepsilon_0 A}{d} = 2kC_0 = 2\times 10C_0$

$\Rightarrow C = 20C_0$

(ii) As battery remains connected so, potential difference
V remains same across the capacitor.

$\therefore E = \frac{V}{\frac{d}{2}} = \frac{2V}{d} \Rightarrow E = 2E_0$

(iii) Initial energy density = $\frac{1}{2}\varepsilon_0 E_0^{\ 2}$

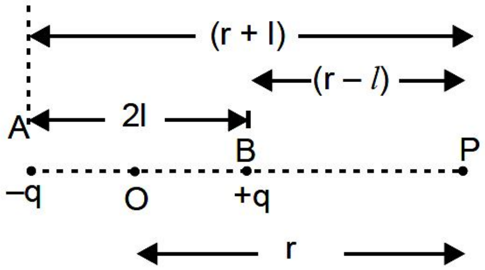
Final energy density $= \frac{1}{2}\varepsilon_0 E^2$

$= \frac{1}{2}\varepsilon_0 (2E_0)^2 = 4 \times \frac{1}{2}\varepsilon_0 E_0^{\ 2}$

or Final energy density = 4 Initial energy density.

51. Derive the expression for the electric potential at any point along the axial line of an electric dipole ?

Ans. Electric Potential due to an electric dipole at axial point. Consider an electric dipole *AB*, having charges – *q* and + *q* at points *A* and *B* respectively. The separation between the charges is 2*l*.



Electric dipole moment, $\vec{p} = q.2l$, directed from – *q* to + *q*.

Consider a point *P* on the axis of dipole at a distance *r* from mid-point *O* of dipole.

The distance of point *P* from charge + *q* is *BP* =*r* – *l*

The distance of point *P* from charge - *q* is *AP* =*r* + *l*

Let V_1 and V_2 be the potentials at *P* due to charges + *q* and - *q* respectively. Then

$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l)}$ and $V_2 = \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r+l)}$

$\therefore$ Resultant potential at *P* due to dipole

$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l)} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r+l)}$

$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-l)} - \frac{1}{(r+l)}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+l)-(r-l)}{(r-l)(r+l)}\right]$

$\Rightarrow V = \frac{1}{4\pi\varepsilon_0}\frac{q.2l}{(r^2 - l^2)}$

As q . $2l = p$ (dipole moment)

$\therefore V = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2 - l^2)}$

If point *P* is far away from the dipole, then $r >> l$

$\therefore V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$