

## Important Questions and Answers from other selected units

1. Deduce the expression for the magnetic dipole moment of an electron orbiting around the central nucleus.

**Ans:**

A revolving electron in an orbit of radius  $r$  moving with velocity  $v$  behaves as a current loop of effective current

$I = ve$  ( $v$  is frequency of revolution)

$$I = \frac{ve}{2\pi r}$$

Hence it acts like a magnetic dipole moment

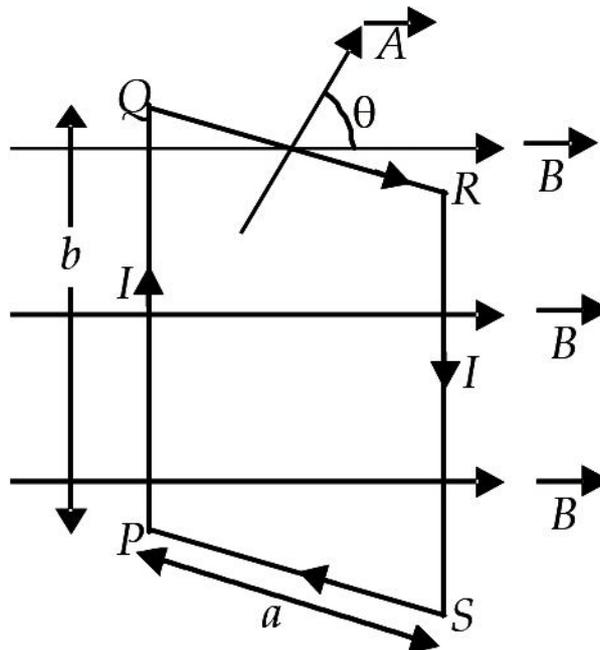
$$M = IA = \frac{ve}{2\pi r} \times \pi r^2 = \frac{evr}{2}$$

2. (a) With the help of a diagram, explain the principle and working of a moving coil galvanometer.  
(b) What is the importance of a radial magnetic field and how is it produced?  
(c) Why is it that while using a moving coil galvanometer as a voltmeter a high resistance in series is required whereas in an ammeter a shunt is used?

**Ans:**

**Principle :** Galvanometer works on the principle that when an electric current is passed through a coil placed in a magnetic field, it experiences a torque, whose magnitude is proportional to the strength of electric current passed through it.

**Working :**



When a rectangular loop  $PQRS$  (suspended through a torsion head) of sides ' $a$ ' and ' $b$ ' carrying current  $I$  is placed in uniform magnetic field  $\vec{B}$  such that area vector  $\vec{A}$  makes an angle  $\theta$  with direction of magnetic field, then forces on the arms  $QR$  and  $SP$  of loop are equal, opposite and collinear, thereby perfectly cancel each other, whereas forces on the arms  $PQ$  and  $RS$  of loop are equal and opposite but not collinear, so they give rise to torque on the loop.

or  $\tau = IAB\sin\theta$  [where  $A = ab$ ]

and if loop has  $N$  turns, then  $\tau = NIAB\sin\theta$

Due to this torque, the coil is deflected by an angle  $\alpha$ , where it is balanced by restoring torque  $C\alpha$ , developed in suspension strip.  $C$  is restoring torque per unit deflection or torsional constant of the strip.

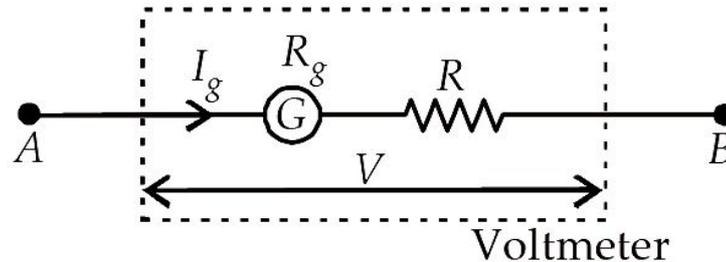
So, by measuring  $\alpha$ , we can measure current  $I$  in the coil.

$$NIAB = C\alpha$$

$$I = \frac{C}{NAB}\alpha$$

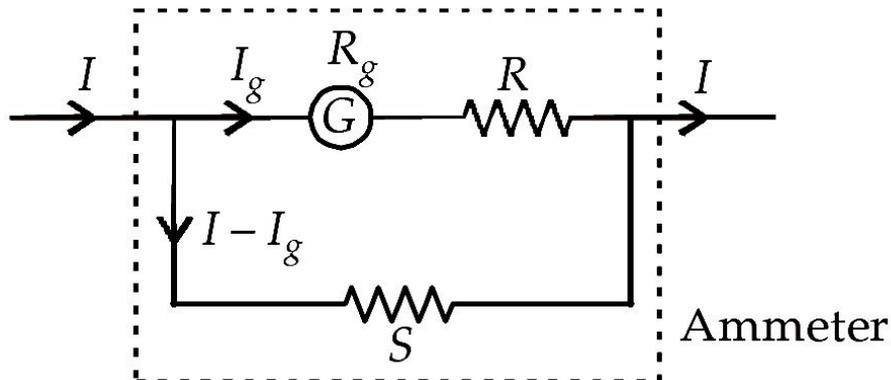
(b) In order to make torque on the coil independent of angle  $\theta$ , the plane of coil should always remain parallel to the field. For this purpose a radial magnetic field is applied.

(c) A galvanometer can be converted into a voltmeter by connecting high resistance in series with it, so that most of the voltage applied drops across it, enabling the galvanometer to measure much larger voltages.



$$\text{or } R = \frac{V}{I_g} - R_g$$

A galvanometer can be converted into an ammeter by connecting a low shunt resistance in parallel to it, so that most of the current by passes through the shunt resistance, enabling the galvanometer to measure much larger currents.



$$V_s = V_g \quad \text{or} \quad (I - I_g)S = I_g R_g$$

$$\text{or } S = \frac{I_g R_g}{I - I_g}$$

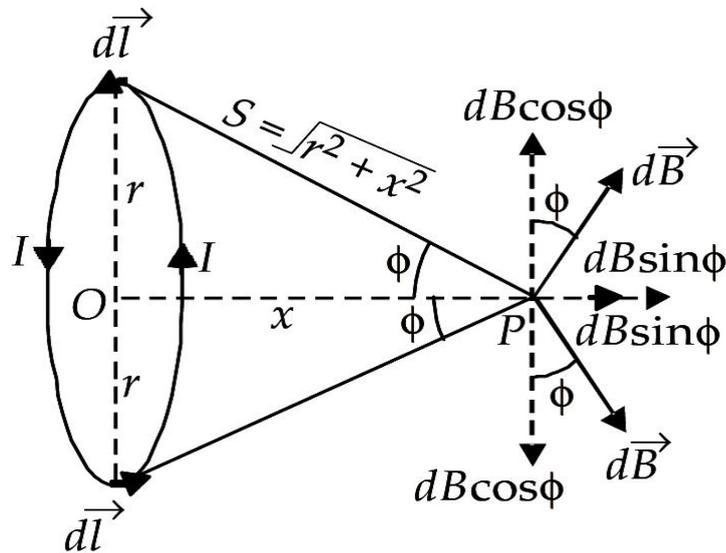
3. State BiotSavart law, giving the mathematical expression for it. Use this law to derive the expression for the magnetic field due to a circular coil carrying current at a point along its axis. How does a circular loop carrying current behaves as a magnet

**Ans:**

A current carrying wire produces a magnetic field around it. BiotSavart law states that magnitude of intensity of small magnetic field due to current  $I$  carrying element  $\vec{dl}$  at any point  $P$  at distance  $r$  from it is given by:

$$|d\vec{B}| = \frac{\mu_0}{4\pi} \frac{I dl \sin \theta}{r^2}$$

Magnetic field on the axis of circular coil



Small magnetic field due to current element  $Id\vec{l}$  of circular coil of radius  $r$  at point  $P$  at distance  $x$  from its centre is

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin 90^\circ}{r^2} = \frac{\mu_0}{4\pi} \frac{Idl}{(r^2 + x^2)}$$

Component  $dB \cos \phi$  due to current element at point  $P$  is cancelled by equal and opposite component  $dB \cos \phi$  of another diagonally opposite current element, whereas the sine components  $dB \sin \phi$  add up to give net magnetic field along the axis. So net magnetic field at point  $P$  due to entire loop is

$$B = \oint dB \sin \phi = \int_0^{2\pi} \frac{\mu_0}{4\pi} \frac{Idl}{(r^2 + x^2)} \cdot \frac{r}{(r^2 + x^2)^{1/2}}$$

$$\Rightarrow B = \frac{\mu_0 Ir}{4\pi(r^2 + x^2)^{3/2}} \int_0^{2\pi} dl$$

$$\Rightarrow B = \frac{\mu_0 Ir}{4\pi(r^2 + x^2)^{3/2}} \cdot 2\pi r$$

$$\Rightarrow B = \frac{\mu_0 Ir^2}{2(r^2 + x^2)^{3/2}} \text{ directed along the axis,}$$

(a) towards the coil if current in it is in clockwise direction.

(b) away from the coil if current in it is in anticlockwise direction.

**4. Draw a schematic sketch of a cyclotron. Explain briefly how it works and how it is used to accelerate the charged particles.**

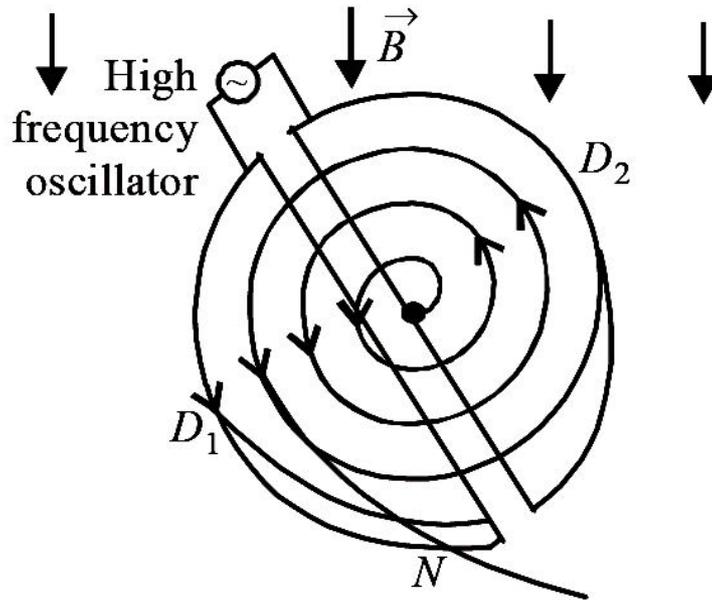
**(i) Show that time period of ions in a cyclotron is independent of both the speed and radius of circular path.**

**(ii) What is resonance condition? How is it used to accelerate the charged particles?**

**(iii) Show that cyclotron frequency is independent of energy of the particle. Is there an upper limit on the energy acquired by the particle? Give reason.**

**Ans:** Cyclotron is used to accelerate the charged particles to large velocities or large kinetic energies. In cyclotron, both electric field  $\vec{E}$  and magnetic field  $\vec{B}$  are applied normally to velocity  $\vec{v}$  of the charged particle, such that electric field accelerates the charged particle and magnetic field makes the charged particle move in circular paths repeatedly, so that charged particle is accelerated to large velocities and hence large kinetic energies, under the combined effect of electric and magnetic fields.

A charged particle produced at point  $P$  by a source is accelerated towards Dee  $D_1$  due to applied electric field, but moves along semicircular path of radius  $r = \frac{mv}{qB}$  in  $D_1$  due to force of magnetic field on it.



When it reaches the gap between the two dees, polarities of the dees is changed by oscillator and now the charged particle is accelerated towards  $D_2$ , where it follows semicircular path of increased radius with increased velocity. This process repeats itself again and again and charged particle spends the same time inside a dee irrespective of its velocity or the radius of circular path, as

$$t = \frac{\pi r}{v} = \frac{\pi}{v} \cdot \frac{mv}{qB} = \frac{\pi m}{qB}$$

So, time period of its motion is  $T = 2t = \frac{2\pi m}{qB}$

Thus time period is independent of both the speed and radius of circular path.

(ii) Frequency of motion of charged particle is  $\nu = \frac{1}{T} = \frac{qB}{2\pi m}$

When this frequency  $\nu$  becomes equal to the frequency  $\nu_a$  of the applied alternating voltage source or oscillator, then it is called resonance condition.

This ensure that the ions always get accelerated across the gap. Inside the dees the particles travel in a region free of the electric field. The increase in their kinetic energy is  $qV$  each time they cross from one dee to another. This is known as '**cyclotron frequency**', which is independent of radius  $r$  of semicircular path followed by charged particle or its velocity  $v$ . So if we set the oscillator at this frequency, it automatically changes the polarities of the two dees.

When charged particle reaches near the periphery of dee, it is moving in a circular path of maximum radius equal to radius  $R$  of dee and posses maximum kinetic energy

$$K.E_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}m \frac{q^2 B^2 R^2}{m^2} = \frac{q^2 B^2 R^2}{2m} \text{ when it is extracted from dees at point } N.$$

5. (i) State Faraday's law of electromagnetic induction. (ii) A jet plane is travelling towards west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of  $5 \times 10^{-4}$  T and the dip angle is  $30^\circ$ ?

**Ans:** Faraday's law of electromagnetic induction states that whenever there is change in magnetic flux linked with the circuit, an emf is induced in it, whose magnitude is directly proportional to the rate of change of magnetic flux linked with the circuit. i.e.  $\varepsilon = \frac{d\phi}{dt}$

(ii) EMF induced across the ends of the wings of plane is

$$\varepsilon = vB_v l = vB \sin \delta \cdot l$$

$$\Rightarrow \varepsilon = (1800 \times \frac{5}{18} \text{ m/s}) \times (5 \times 10^{-4} \text{ T}) \times \sin 30^\circ \times 25$$

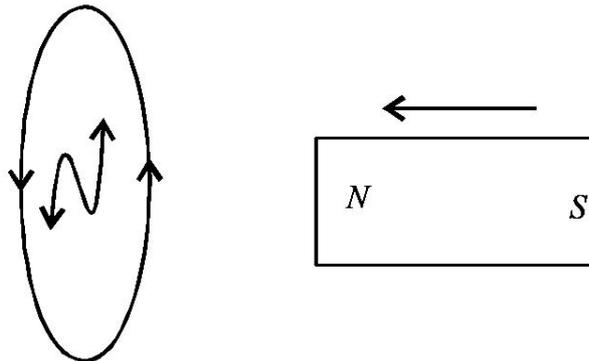
$$\Rightarrow \varepsilon = 500 \times 5 \times 10^{-4} \times \frac{1}{2} \times 25 = 3.125 \text{ V}$$

6. (a) State Lenz's law. Give one example to illustrate this law. "The Lenz's law is a consequence of the principle of conservation of energy". Justify this statement.

(b) Deduce an expression for the mutual inductance of two long coaxial solenoids but having different radii and different number of turns.

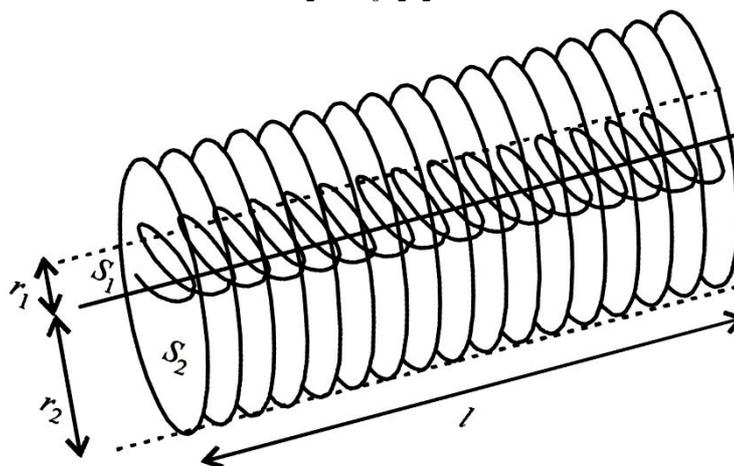
Ans:

(a) Lenz's law states that the "induced current in a circuit always flows in such a direction that it opposes the change in magnetic flux linked with the circuit or the very cause that has produced it".



When the *N* pole of a magnet is moved towards a coil, the induced current in the coil flows in anticlockwise direction on the side of magnet, so as to acquire north polarity and oppose the motion of the magnet towards the coil, by applying repulsive force on it. Lenz's law is in accordance with law of conservation of energy. Whenever magnetic flux linked with a circuit changes, it induces an EMF in it. The induced current set up in the circuit flows in such a direction that it opposes the change in magnetic flux linked with the circuit. In order to continue the change in magnetic flux linked with the circuit, some work is to be done or some energy is to be spent against the opposition offered by induced EMF. This energy spent by the external source ultimately appears in the circuit in the form of electrical energy.

(b) Magnetic field due to current  $I_2$  in  $S_2$  is  $B_2 = \mu_0 n_2 I_2$



The magnetic flux linked with solenoid  $S_1$  is  $\phi_{12} = N_1 B_2 A \cos 0^\circ$

$$\Rightarrow \phi_{12} = (n_1 l) \cdot (\mu_0 n_2 I_2) \cdot (\pi r_1^2) \cdot 1$$

$$\Rightarrow \frac{\phi_{12}}{I_2} = \mu_0 n_1 n_2 \pi r_1^2 l$$

This gives the mutual inductance of two long coaxial solenoids.

7. (i) Draw a labelled diagram of a step-up transformer. Explain its working principle. Deduce the expression for the secondary to primary voltage in terms of the number of turns in the two coils. In an ideal transformer, how is this ratio related to the currents in the two coils? How is the transformer used in large scale transmission and distribution of electrical energy over long distances?

(ii) Write any two sources of energy loss in a transformer.

Ans:

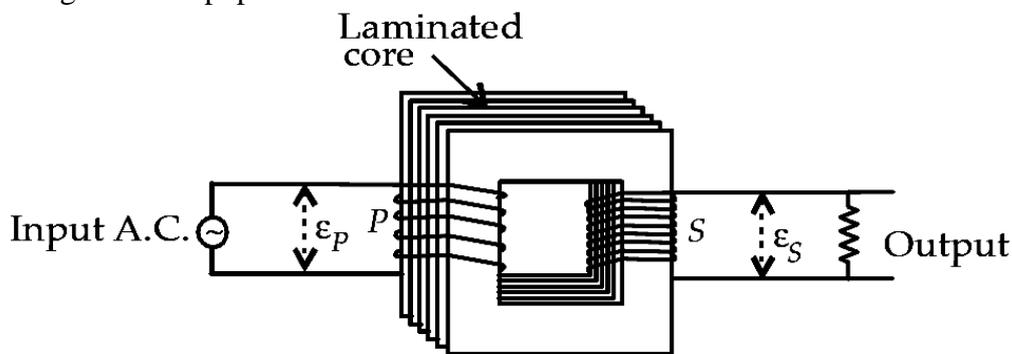
**Transformer:** Transformer is a device by which an alternating voltage may be decreased or increased. This is based on the principle of mutual-induction.

**Construction:** It consists of laminated core of soft iron, on which two coils of insulated copper wire are separately wound. These coils are kept insulated from each other and from the iron-core, but are coupled through mutual induction. The number of turns in these coils are different. Out of these coils one coil is

called *primary coil* and other is called the *secondary coil*. The terminals of primary coils are connected to AC mains and the terminals of the secondary coil are connected to external circuit in which alternating current of desired voltage is required.

**Step up Transformer:** It transforms the alternating low voltage to alternating high voltage and in this the number of turns in secondary coil is more than that in primary coil. (i. e. ,  $N_S > N_P$ ).

A schematic diagram of stepup transformer is shown below.



### Working Principle

It works on the principle of mutual induction. It consists of two coils primary  $P$  and secondary  $S$  wound on a laminated soft iron core. The input voltage is applied across the primary coil and output voltage is obtained across the secondary coil.

Magnetic flux  $\phi_S$  and  $\phi_P$  linked with secondary and primary coils at any instant are proportional to the number of turns  $N_S$  and  $N_P$  in secondary and primary coils i.e.,

$$\frac{\phi_S}{\phi_P} = \frac{N_S}{N_P} \text{ or } -\frac{d\phi_S}{dt} = \frac{N_S}{N_P} \left( -\frac{d\phi_P}{dt} \right)$$

$$\text{or } \varepsilon_S = \frac{N_S}{N_P} \varepsilon_P \text{ or } \frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P}$$

where  $\frac{N_S}{N_P}$  is called transformation ratio of transformer.

In step up transformer,  $\varepsilon_S > \varepsilon_P$  and  $N_S > N_P$  and transformation ratio  $> 1$

In an ideal transformer, there is no loss of energy or it is 100% efficient. Then

Power input = Power output

$$\text{or } \varepsilon_P I_P = \varepsilon_S I_S \text{ or } \frac{\varepsilon_S}{\varepsilon_P} = \frac{I_P}{I_S}$$

where  $I_S$  and  $I_P$  are currents in secondary and primary coils of transformer.

Transformer is mainly used in long distance transmission of electrical energy. At the electric power producing station, a stepup transformer is used which increases the alternating voltage upto several kilo volts, thereby decreasing the electric current flowing through transmission wires, As Joule's heating is proportional to square of current, so this decreases the loss of electrical energy across

transmission wires. Further a stepdown transformer is used to decrease the alternating voltage at substation before distributing electrical energy for domestic use.

(ii) The two sources of energy loss in a transformer :

(1) Copper loss is the energy loss in the form of heat in copper coils of a transformer. This is due to joule heating of conducting wires. These are minimized using thick wires.

(2) Iron loss is the energy loss in the form of heat in the iron core of the transformer. This is due to formation of eddy currents in iron core. It is minimized by taking laminated cores.

8. (a) State the working principle of an AC generator with the help of a labelled diagram. Derive an expression for the instantaneous value of the emf induced in coil. Why is the emf maximum when the plane of the armature is parallel to the magnetic field?

(b) A 100turn coil of area  $0.1 \text{ m}^2$  rotates at half a revolution per second. It is placed in a magnetic field  $0.01 \text{ T}$  perpendicular to the axis of rotation of the coil. Calculate the maximum voltage generated in the coil.

**Ans:(a) AC generator:** A dynamo or generator is a device which converts mechanical energy into electrical energy. It is based on the principle of electromagnetic induction.

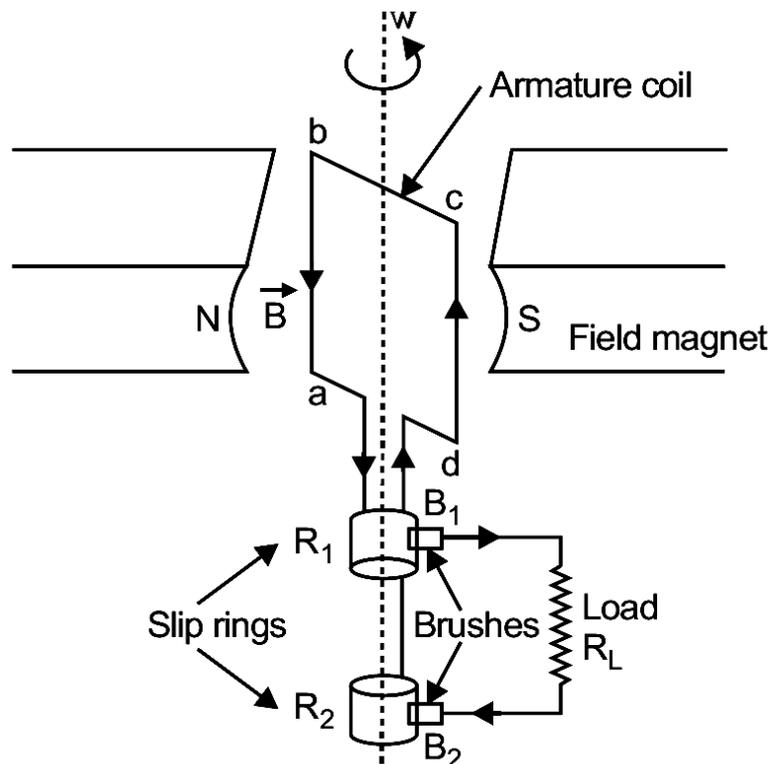
**Construction:** It consists of the four main parts:

(i) **Field Magnet:** It produces the magnetic field. In the case of a low power dynamo, the magnetic field is generated by a permanent magnet, while in the case of large power dynamo, the magnetic field is produced by an electromagnet.

(ii) **Armature:** It consists of a large number of turns of insulated wire in the soft iron drum or ring. It can revolve round an axle between the two poles of the field magnet. The drum or ring serves the two purposes : (i) It serves as a support to coils and (ii) It increases the magnetic field due to air core being replaced by an iron core.

(iii) **Slip Rings:** The slip rings  $R_1$  and  $R_2$  are the two metal rings to which the ends of armature coil are connected. These rings are fixed to the shaft which rotates the armature coil so that the rings also rotate along with the armature.

(iv) **Brushes:** These are two flexible metal plates or carbon rods ( $B_1$  and  $B_2$ ) which are fixed and constantly touch the revolving rings. The output current in external load  $R_L$  is taken through these brushes.



**Working:** When the armature coil is rotated in the strong magnetic field, the magnetic flux linked with the coil changes and the current is induced in the coil, its direction being given by Fleming's

right hand rule. Considering the armature to be in vertical position and as it rotates in anticlockwise direction, the wire ab moves upward and cd downward, so that the direction of induced current is shown in fig. In the external circuit, the current flows along  $B_1R_L B_2$ . The direction of current remains unchanged during the first half turn of armature. During the second half revolution, the wire ab moves downward and cd upward, so the direction of current is reversed and in external circuit it flows along  $B_2R_L B_1$ . Thus the direction of induced emf and current changes in the external circuit after each half revolution. If  $N$  is the number of turns in coil,  $f$  the frequency of rotation,  $A$  area of coil and  $B$  the magnetic induction, then induced emf

$$e = -\frac{d\phi}{dt} = \frac{d}{dt}\{NBA(\cos 2\pi ft)\} = 2\pi NBAf \sin 2\pi ft$$

Obviously, the emf produced is alternating and hence the current is also alternating. Current produced by an ac generator cannot be measured by moving coil ammeter; because the average value of ac over full cycle is zero.

The source of energy generation is the mechanical energy of rotation of armature coil.

When plane of armature coil is parallel to magnetic field, then  $\sin\omega t = 1$ , so emf is maximum, the maximum value is  $e_0 = NBA\omega$ .

(b)  $N = 100$ ,  $A = 0.1 \text{ m}^2$ ,  $n = \frac{1}{2} \text{ s}^{-1}$   $B = 0.01 \text{ T}$

Maximum voltage generated in the coil is

$$e_0 = NBA\omega = NBA \times 2\pi v$$

$$\text{or } e_0 = 100 \times 0.01 \times 0.1 \times 2 \times 3.14 \times \frac{1}{2}$$

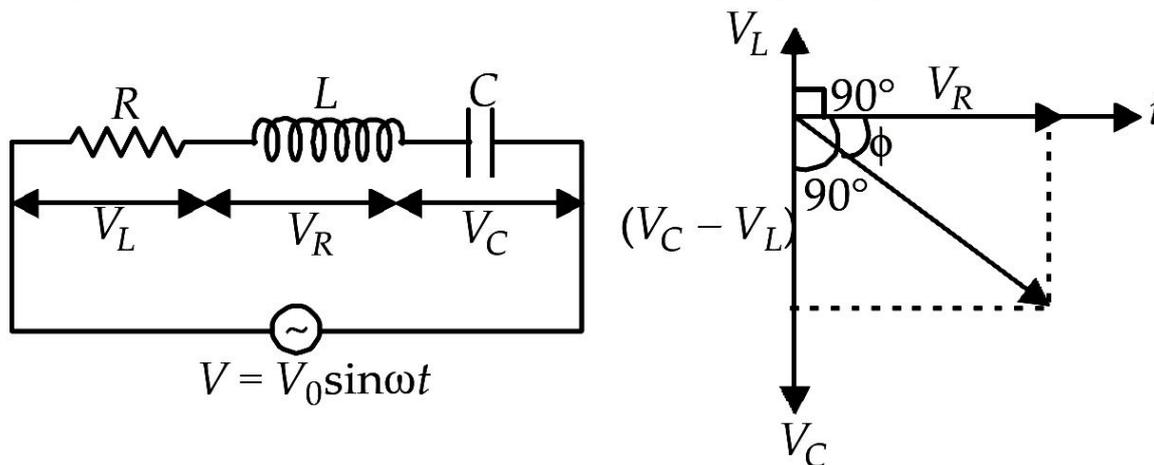
$$\text{or } e_0 = 0.314 \text{ V.}$$

**9. Derive an expression for the impedance of a series LCR circuit connected to an AC supply of variable frequency. Plot a graph showing variation of current with the frequency of the applied voltage.**

**Explain briefly how the phenomenon of resonance in the circuit can be used in the tuning mechanism of a radio or a TV set.**

**Ans:**

Expression for impedance in LCR series circuit : Suppose resistance  $R$ , inductance  $L$  and capacitance  $C$  are connected in series and an alternating source of voltage  $V = V_0\sin\omega t$  is applied across it as shown in figure. On account of being in series, the current  $i$  flowing through all of them is the same.



Consider the voltage across resistance  $R$  is  $V_R$ , voltage across inductance  $L$  is  $V_L$  and voltage across capacitance  $C$  is  $V_C$ . The voltage  $V_R$  and current  $i$  are in the same phase, the voltage  $V_L$  will lead the current by angle  $90^\circ$  while the voltage  $V_C$  will lag behind the current by angle  $90^\circ$  (figure). Clearly  $V_C$  and  $V_L$  are in opposite directions, therefore their resultant potential difference =  $V_C - V_L$  (if  $V_C > V_L$ ).

Thus  $V_R$  and  $(V_C - V_L)$  are mutually perpendicular and the phase difference between them is  $90^\circ$ . As applied voltage across the circuit is  $V$ , the resultant of  $V_R$  and  $(V_C - V_L)$  will also be  $V$ . From figure,

$$V^2 = V_R^2 + (V_C - V_L)^2$$

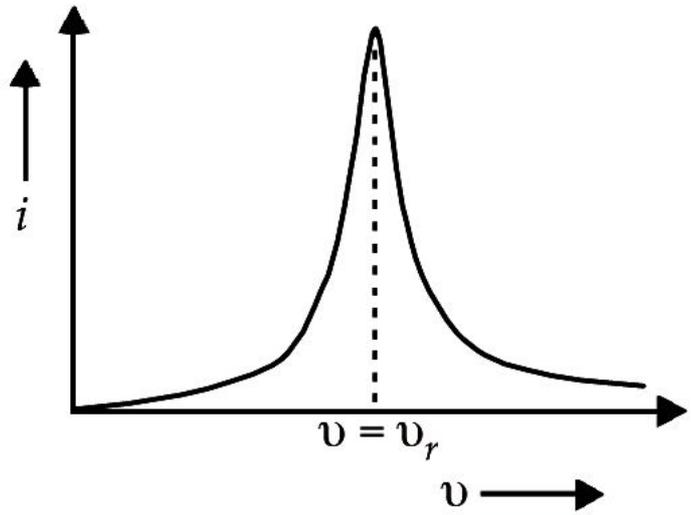
$$\Rightarrow V = \sqrt{V_R^2 + (V_C - V_L)^2} \quad \dots\dots\dots(i)$$

But  $V_R = Ri$ ,  $V_C = X_C i$   
 and  $V_L = X_L i \quad \dots\dots\dots(ii)$

where  $X_C = \frac{1}{\omega C}$  = capacitance reactance and

$X_L = \omega L$  = inductive reactance

$$\therefore \text{Impedance of circuit, } Z = \frac{V}{i} = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$$



The practical application of series resonance circuit is in radio and T.V. receiver sets. The antenna of a radio/T.V. intercepts signals from many broadcasting stations. To receive one particular radio station./T.V. channel, we tune our receiver set by changing the capacitance of a capacitor in the tuning circuit of the set such that resonance frequency of the circuit becomes equal to the frequency of the desired station. Therefore, resonance occurs. The amplitude of current with the frequency of the signal from the desired station becomes maximum and it is received in our set.

**10. Describe briefly how a diffraction pattern is obtained on a screen due to a single narrow slit illuminated by a monochromatic source of light. Hence obtain the conditions for the angular width of secondary maxima and secondary minima.**

**Ans: Diffraction of light at a single slit :** When monochromatic light is made incident on a single slit, we get diffraction pattern on a screen placed behind the slit. The diffraction pattern contains bright and dark bands, the intensity of central band is maximum and goes on decreasing on both sides.

**Explanation :** Let  $AB$  be a slit of width ‘ $a$ ’ and a parallel beam of monochromatic light is incident on it. According to Fresnel the diffraction pattern is the result of superposition of a large number of waves, starting from different points of illuminated slit.

Let  $\theta$  be the angle of diffraction for waves reaching at point  $P$  of screen and  $AN$  the perpendicular dropped from  $A$  on wave diffracted from  $B$ .

The path difference between rays diffracted at points  $A$  and  $B$ ,

$$\Delta = BP - AP = BN$$

In  $\Delta ANB$ ,  $\angle ANB = 90^\circ \therefore$  and  $\angle BAN = \theta$

As  $AB =$  width of slit =  $a$

$$\therefore \text{Path difference, } \Delta = a \sin \theta \quad \dots\dots\dots (i)$$

To find the effect of all coherent waves at  $P$ , we have to sum up their contribution, each with a different phase. This was done by Fresnel by rigorous calculations, but the main features may be explained by simple arguments given below :

At the central point  $C$  of the screen, the angle  $\theta$  is zero. Hence the waves starting from all points of slit arrive in the same phase. This gives maximum intensity at the central point  $C$ .

If point  $P$  on screen is such that the path difference between rays starting from edges  $A$  and  $B$  is  $\lambda$ , then path difference

$$a \sin \theta = \lambda \Rightarrow \sin \theta = \frac{\lambda}{a}$$

If angle  $\theta$  is small,  $\sin \theta = \theta = \frac{\lambda}{a}$  .....(ii)

**Minima :** Now we divide the slit into two equal halves  $AO$  and  $OB$ , each of width  $\frac{a}{2}$ . Now for every

point,  $M_1$  in  $AO$ , there is a corresponding point  $M_2$  in  $OB$ , such that  $M_1M_2 = \frac{a}{2}$ ; Then path difference

between waves arriving at  $P$  and starting from  $M_1$  and  $M_2$  will be  $\frac{a}{2} \sin \theta = \frac{\lambda}{2}$ . This means that the contributions from the two halves of slit  $AO$  and  $OB$  are opposite in phase and so cancel each other. Thus equation (2) gives the angle of diffraction at which intensity falls to zero. Similarly it may be shown that the intensity is zero for  $\sin \theta = \frac{n\lambda}{a}$ , with  $n$  as integer.

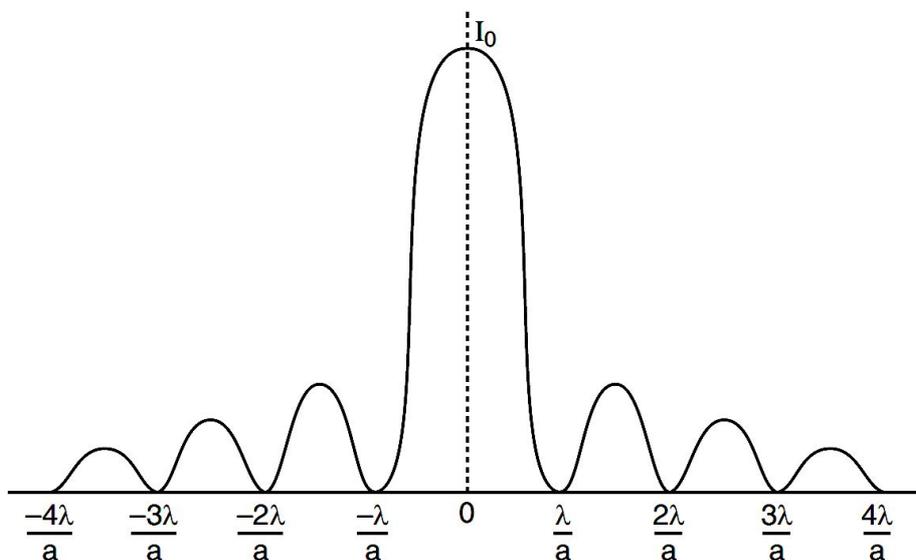
Thus the general condition of **minima** is

$$a \sin \theta = n\lambda \quad \text{.....(iii)}$$

**Secondary Maxima :** Let us now consider angle  $\theta$  such that  $\sin \theta = \theta = \frac{3\lambda}{2a}$

which is midway between two dark bands given by

$$\sin \theta = \theta = \frac{\lambda}{a} \quad \text{and} \quad \sin \theta = \theta = \frac{2\lambda}{a}$$



Let us now divide the slit into three parts. If we take the first two of parts of slit, the path difference between rays diffracted from the extreme ends of the first two parts

$$\frac{2}{3} a \sin \theta = \frac{2}{3} a \times \frac{3\lambda}{2a} = \lambda$$

Then the first two parts will have a path difference of  $\frac{\lambda}{2}$  and cancel the effect of each other. The

remaining third part will contribute to the intensity at a point between two minima. Clearly there will be a maxima between first two minima, but this maxima will be of much weaker intensity than central maximum. This is called *first secondary maxima*. In a similar manner we can show that there

are secondary maxima between any two consecutive minima; and the intensity of maxima will go on decreasing with increase of order of maxima. In general the position of  $n$ th maxima will be given by

$$a \sin \theta = \left( n + \frac{1}{2} \right) \lambda, [n = 1, 2, 3, 4, \dots] \dots\dots\dots(iv)$$

The intensity of secondary maxima decrease with increase of order  $n$  because with increasing  $n$ , the contribution of slit decreases.

For  $n = 2$ , it is one-fifth, for  $n = 3$ , it is one-seventh and so on.

(b) Angular width of secondary maxima

$$a \cdot \theta = \left( n + \frac{1}{2} \right) \lambda$$

$$\Rightarrow \theta = \left( n + \frac{1}{2} \right) \frac{\lambda}{a}$$

and Linear width  $\theta = \frac{y}{D}$

$$\Rightarrow y = D \cdot \theta = \left( n + \frac{1}{2} \right) \frac{\lambda D}{a}$$

If  $n = 1$ , and  $\lambda_1 = 590 \text{ nm}$ ,

$$y_1 = \left( 1 + \frac{1}{2} \right) \frac{\lambda_1 D}{a} = \frac{3\lambda_1 D}{2a}$$

If  $n = 1$   $\lambda_2 = 596 \text{ nm}$

$$y_2 = \left( 1 + \frac{1}{2} \right) \frac{\lambda_2 D}{a} = \frac{3\lambda_2 D}{2a}$$

$$\text{Linear separation} = y_2 - y_1 = \frac{3(\lambda_2 - \lambda_1)D}{2a}$$

$$= \frac{3(596 - 590) \times 10^{-9} \times 1.5}{2 \times 2 \times 10^{-6}} = \frac{3 \times 6 \times 10^{-3} \times 1.5}{4}$$

$$= 4.5 \times 1.5 \times 10^{-3}$$

$$= 6.75 \times 10^{-3} = 6.75 \text{ mm}$$

11. (a) Write three characteristic features to distinguish between the interference fringes in Young's double slit experiment and the diffraction pattern obtained due to a narrow single slit.  
 (b) A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is a distance of 2.5 mm away from the centre. Find the width of the slit.

Ans:

Interference	Diffraction
(i) It is due to the superposition of two waves coming from two coherent sources.	(i) It is due to the superposition of secondary wavelets originating from different parts of the same wavefront.
(ii) The width of the interference bands is equal.	(ii) The width of the diffraction bands is not the same.
(iii) The intensity of all maxima (fringes) is same.	(iii) The intensity of central maximum is maximum and goes on decreasing rapidly with increase of order of maxima.

(b) The distance of  $n$ th bright fringe from central fringe is,  $y_n = \frac{n\lambda D}{d}$

$$\text{Width, } d = \frac{n\lambda D}{y_n} = \frac{1 \times 500 \times 10^{-9} \times 1}{2.5 \times 10^{-3}} = 2 \times 10^{-4} \text{ m} = 0.2 \text{ mm}$$

12. (a) For a ray of light travelling from a denser medium of refractive index  $n_1$  to a rarer medium of refractive index  $n_2$ , prove that  $\frac{n_2}{n_1} = \sin i_c$ , where  $i_c$  is the critical angle of incidence for the media.

(b) Explain with the help of a diagram, how the above principle is used for transmission of video signals using optical fibres.

Ans:

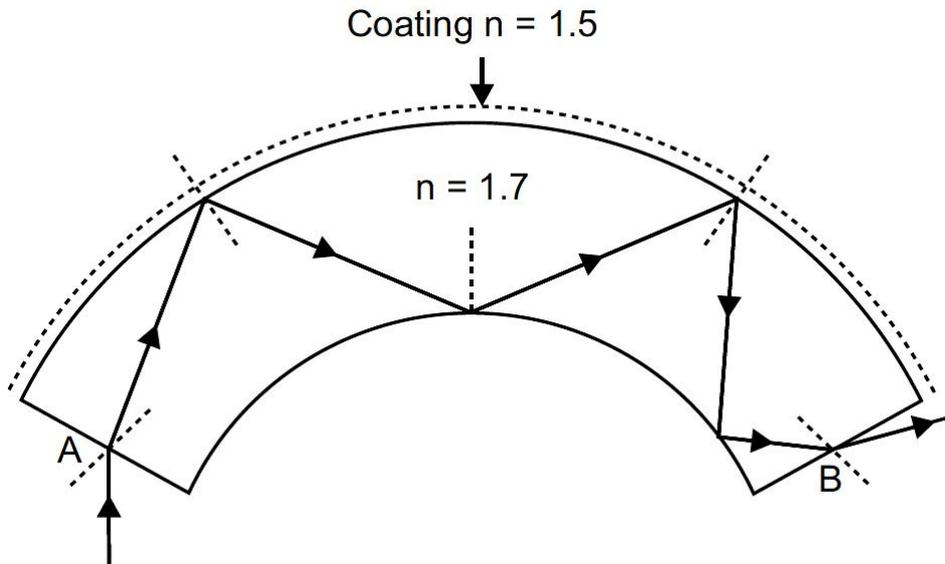
(a) Snell's laws is  $\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$  ..... (i)

Critical angle is the angle of incidence in denser medium for which angle of refraction in rarer medium is  $90^\circ$  i.e.,  $i = i_c, r = 90^\circ$

$\therefore$  From (i)  $\frac{\sin i}{\sin 90^\circ} = \frac{n_2}{n_1} \Rightarrow \frac{\sin i_c}{1} = \frac{n_2}{n_1} \Rightarrow \frac{n_2}{n_1} = \sin i_c$

(b) Transmission of video signals using optical fibre.

*An optical fibre is a device based on total internal reflection by which a light signal may be transmitted from one place to another with a negligible loss of energy.* It is a very long and thin pipe of quartz ( $n = 1.7$ ) of thickness nearly  $\approx 10^{-4}$  m coated all around with a material of refractive index 1.5. A large number of such fibres held together form a *light pipe* and are used for communication of light signals. When a light ray is incident on one end at a small angle of incidence, it suffers refraction from air to quartz and strikes the quartz-coating interface at an angle more than the critical angle and so suffers total internal reflection and strikes the opposite face again at an angle greater than critical angle and so again suffers total internal reflection. Thus the ray within the fibre suffers multiple total internal reflections and finally strikes the other end at an angle less than critical angle for quartz-air interface and emerges in air.

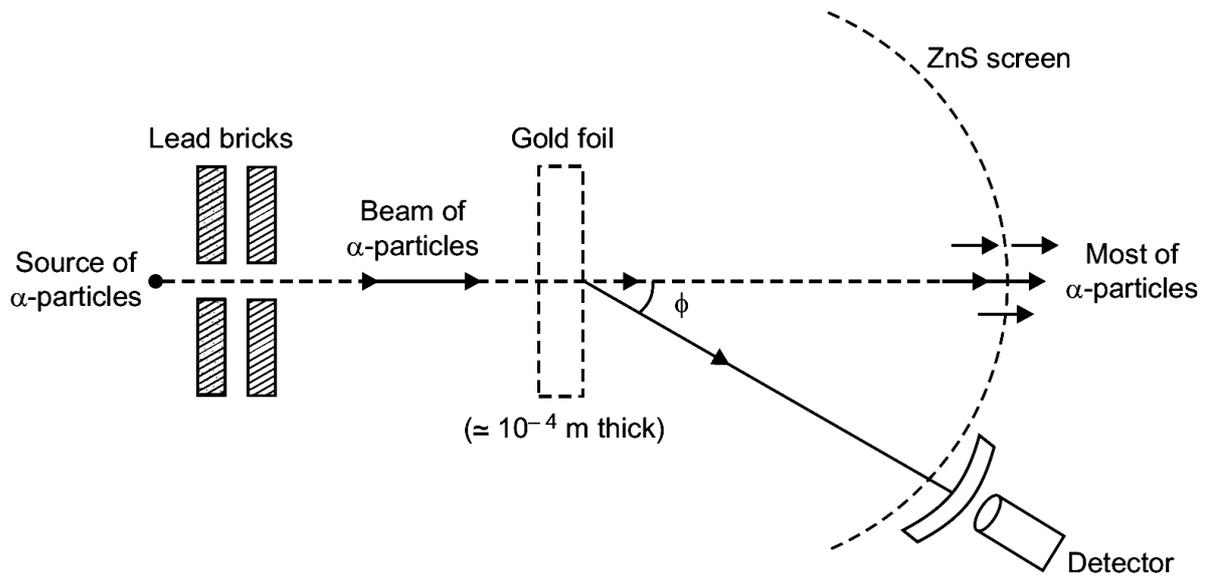


As there is no loss of energy in total internal reflection, the light signal is transmitted by this device without any appreciable loss of energy.

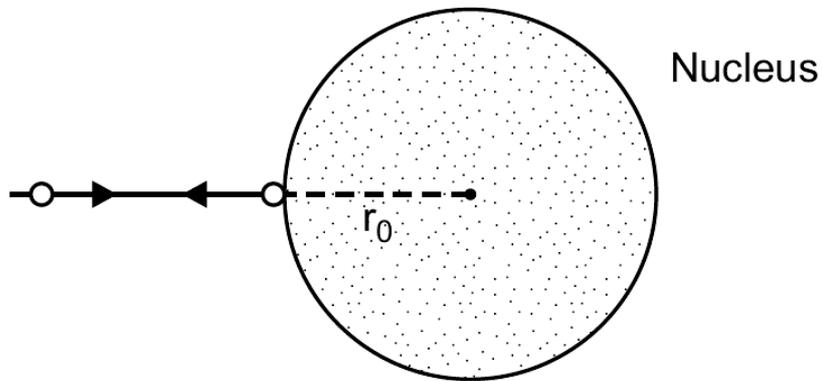
13. Draw a schematic arrangement of the Geiger-Marsden experiment. How did the scattering of  $\alpha$ -particles of a thin foil of gold provide an important way to determine an upper limit on the size of the nucleus? Explain briefly.

Ans:

The Schematic arrangement of Geiger-Marsdon Experiment (also known as Rutherford Scattering Experiment) is shown in fig.



**Observations:** (i) Only a small fraction of number of  $\alpha$ -particles rebound back. This shows that the number of  $\alpha$ -particles undergoing head on collision is very small. The conclusion is that the entire positive charge of atom is concentrated in a small volume called the **nucleus**.



At the distance of head on approach, the entire kinetic energy of  $\alpha$ -particle is converted into electrostatic potential energy. This distance of head on approach gives an upper limit of the size of nucleus (denoted by  $r_0$ ) and is given by

$$E_k = \frac{1}{4\pi\epsilon_0} \frac{(Ze)(2e)}{r_0}$$

$$\Rightarrow r_0 = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{E_k}$$

This is about  $10^{-14}$  m.

**14. Derive an expression for the de-Broglie wavelength associated with an electron accelerated through a potential V. Draw a schematic diagram of a localised-wave describing the wave nature of the moving electron.**

**Ans:**

**Expression for de Broglie Wavelength associated with Accelerated Electrons**

The de Broglie wavelength associated with electrons of momentum  $p$  is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \dots\dots\dots (i)$$

where  $m$  is mass and  $v$  is velocity of electron. If  $E_k$  is the kinetic energy of electron, then

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m} \quad \left(\because p = mv \Rightarrow v = \frac{p}{m}\right)$$

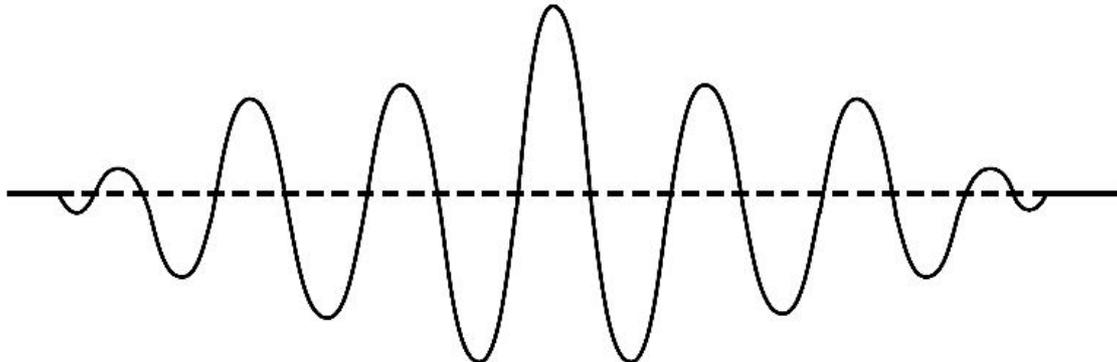
$$\Rightarrow p = \sqrt{2mE_k}$$

∴ Equation (i) gives  $\lambda = \frac{h}{\sqrt{2mE_k}}$  ..... (ii)

If  $V$  volt is accelerating potential of electron, then Kinetic energy,  $E_k = eV$

∴ Equation (ii) gives  $\lambda = \frac{h}{\sqrt{2meV}}$  ..... (iii)

This is the required expression for de Broglie wavelength associated with electron accelerated to potential of  $V$  volt. The diagram of wave packet describing the motion of a moving electron is shown.



**15. Draw a labelled ray diagram of a reflecting telescope. Mention its two advantages over the refracting telescope.**

**Ans:**

**Reflecting type telescope:**

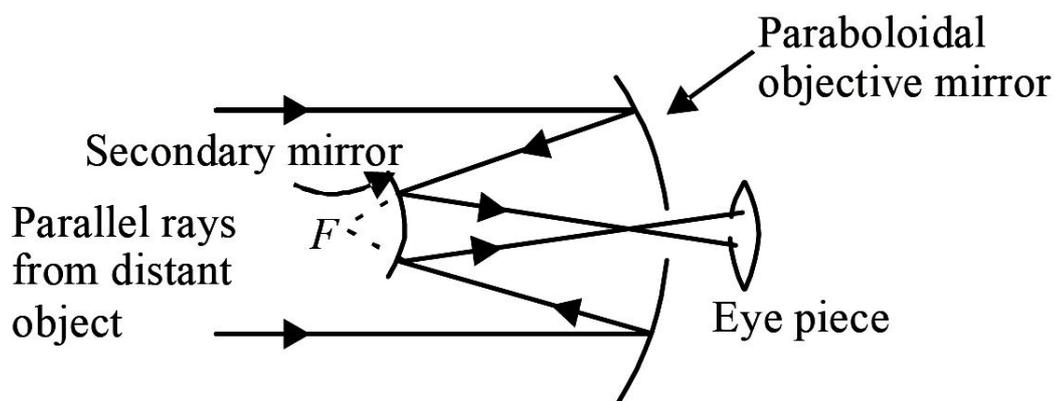
It is a telescope with concave parabolic mirror as objective. It has several advantages over refracting type telescope like having no chromatic aberration, no spherical aberration, has huge light gathering power and low cost.

The magnifying power of reflecting telescope is given by

$$M = + \frac{f_o}{f_e}$$

**(i) Cassegrain Reflecting Telescope**

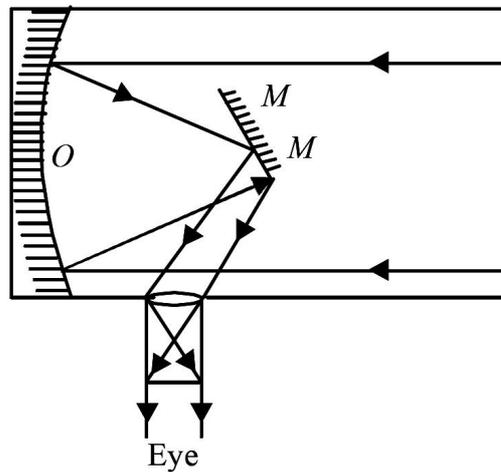
It consist of a large primary concave parabolic shape mirror having a hole at its centre. Another secondary convex mirror before the focus of primary mirror forms the image



The parallel rays from astronomical object are reflected by primary concave mirror and then are further reflected by convex mirror before getting focussed at eye piece. Eyepiece removes the defects from the image and also act as magnifier.

**(ii) Newtonian Telescope**

Primary concave mirror of large aperture as objective reflects the parallel rays from astronomical object.



Plane mirror  $M$  is placed at  $45^\circ$  with the axis of the tube. Light reflected from concave mirror falls on plane mirror  $M$  and further deviated to form a real image at eyepiece located at convenient place for observer. The eyepiece removes the defects from image and also act as magnifier.

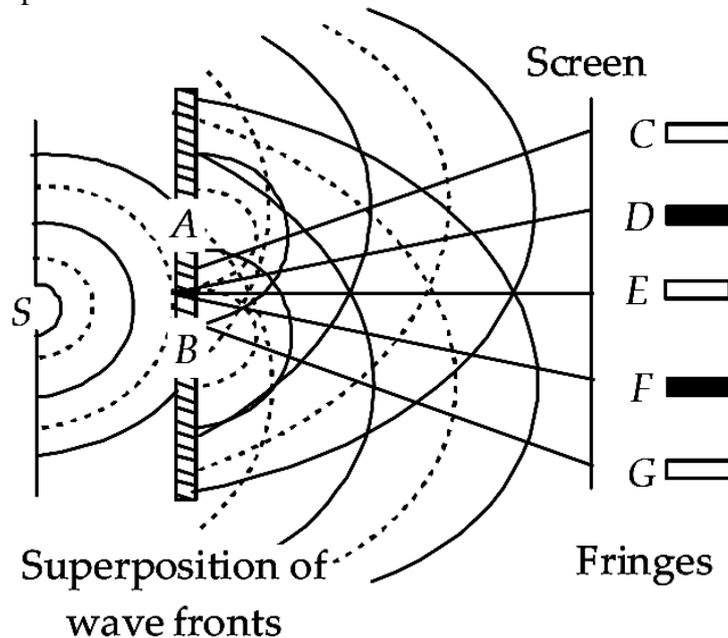
**Advantage of reflecting type telescope over refracting type :**

- (i) In refracting type the final image is formed after two times of partial refraction through the lens major losses in the intensity take places due to partial reflection and refractions. In reflecting type all the light intensity incident forms the final image as no loss of intensity can be ensured in reflection.
- (ii) Glass of lens offers different refractive index to different colours hence chromatic aberration due to which coloured image is formed take place in refracting type telescope. Reflecting telescope is free from chromatic aberration as no refraction.

**16. Describe Young’s double slit experiment to produce interference pattern due to a monochromatic source of light. Deduce the expression for the fringe width.**

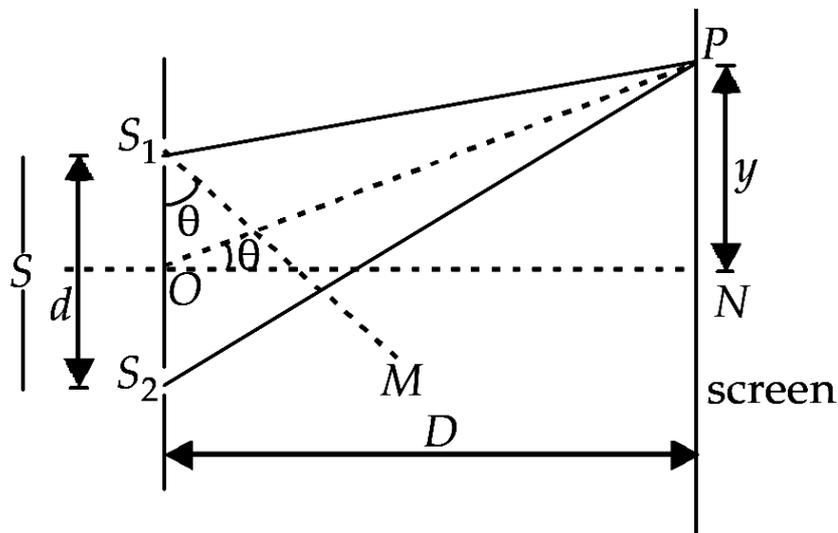
**Ans:**

Young’s double slit experiment :



$S$  is a narrow slit (of width about 1 mm) illuminated by a monochromatic source of light,  $S$ . At a suitable distance (about 10 cm) from  $S$ , there are two fine slits  $A$  and  $B$  about 0.5 mm apart placed symmetrically parallel to  $S$ . When a screen is placed at a large distance (about 2 m) from the slits  $A$  and  $B$ , alternate bright and dark fringes running parallel to the lengths of slits appear on the screen. These are the interference fringes. The fringes disappear when one of the slits  $A$  or  $B$  is covered.

Expression for fringe width : In Young’s double slit experiment we obtain two sources from a single source.



Here  $S_1P$  and  $S_2P$  are nearly parallel since the distance  $S_1S_2 = d$  is much less than  $D$ . The angle that these two lines make with the normal to the screen is taken as  $\theta$ .

Path difference between the waves reaching the point  $P$  on screen is

$$\Delta P = S_2P - S_1P = S_2P - MP = S_2M = d \sin \theta$$

As angle is very small

$$d \sin \theta \approx d \tan \theta$$

$$\text{i.e. } \Delta P = \frac{yd}{D} \left( \because \text{in } \triangle NOP, \tan \theta = \frac{y}{D} \right) \dots\dots\dots(i)$$

We know, that for maxima

$$\Delta P = n\lambda \dots\dots\dots(ii)$$

where,  $n = 1, 2, 3, \dots$

From equation (i) and (ii), we get

$$y_n = \frac{n\lambda D}{d}$$

Similarly for minima

$$y_n = \frac{(2n-1)\lambda D}{2d}$$

The fringe width is the separation between two consecutive maxima or minima,

$$\Delta y = \frac{\lambda D}{d}(n+1-n) = \frac{\lambda D}{d}$$

It is denoted by  $\beta$

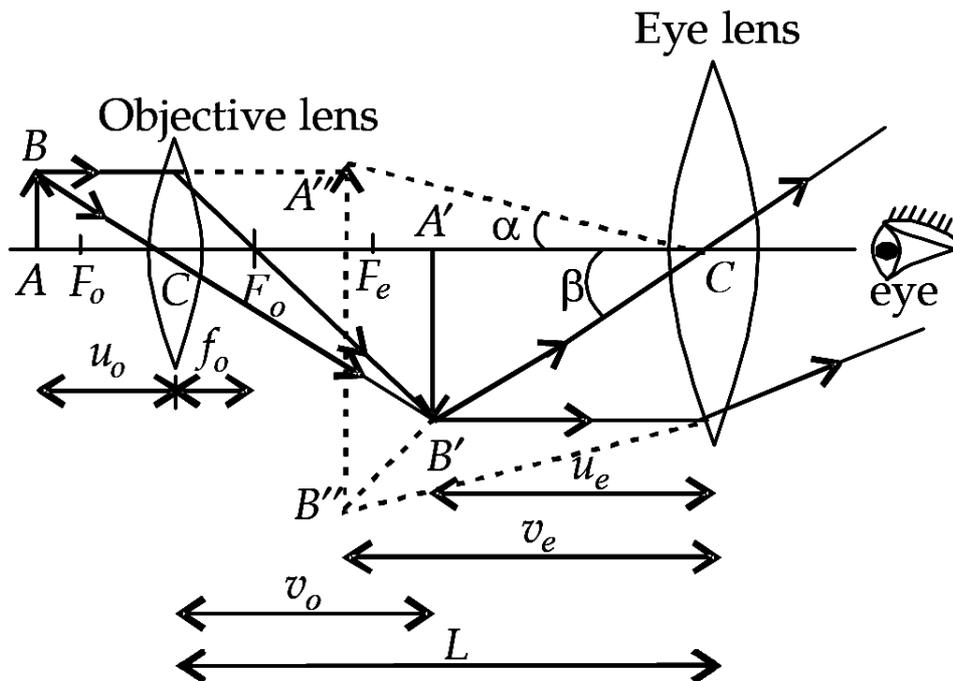
$$\beta = \frac{\lambda D}{d}$$

17. (i) Draw a neat labelled diagram of a compound microscope. Explain briefly its working.  
 (ii) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?

Ans:

- (i) Compound microscope is used to see extremely small objects. It consists of two lenses.  
 ➤ Objective lens of short aperture and short focal length  $f_o$   
 ➤ Eye lens of large aperture and short focal length  $f_e$

Ray diagram of a compound microscope is shown below.



**Working :** A real, inverted and enlarged image  $A'B'$  of a tiny object  $AB$ , is formed by objective. Eye lens is so adjusted that  $A'B'$  lies between its optical centre and principle focus  $F_e$ . A virtual and magnified image  $A''B''$  (erect w.r.t.  $A'B'$ ) is formed by the eye lens.

(ii) Both, the objective and the eye piece of a compound microscope should have short focal lengths to have greater magnifying power as magnifying power of a compound microscope is given by

$$M = -\frac{L}{f_o} \left( 1 + \frac{D}{f_e} \right)$$

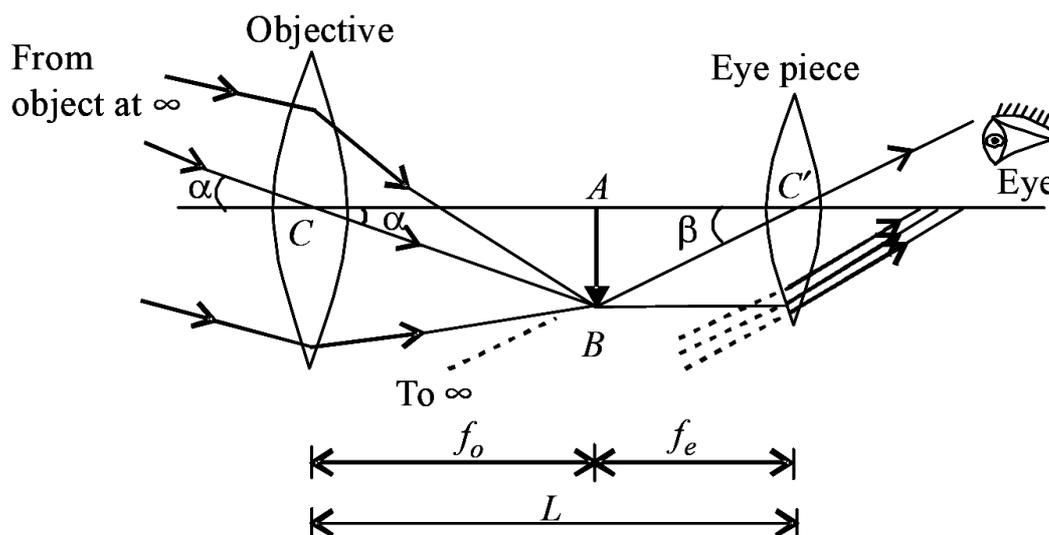
where

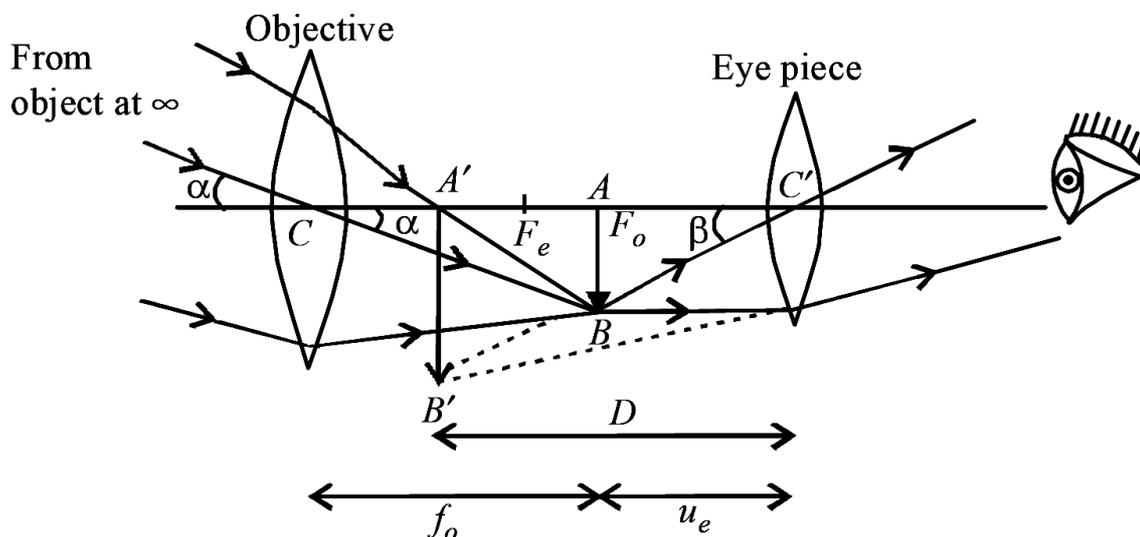
$L$  = length of microscope tube

$D$  = least distance of distinct vision.

18. Draw a labelled ray diagram of a refracting telescope. Define its magnifying power and write the expression for it. Write two important limitations of a refracting telescope over a reflecting type telescope.

Ans:





The magnifying power of a telescope is measured by the ratio of angle ( $\beta$ ) subtended by the final image on the eye to the angle ( $\alpha$ ) subtended by object on eye.

$$M = -\frac{f_o}{f_e} \text{ or } M = -\frac{f_o}{f_e} \left(1 + \frac{f_o}{D}\right)$$

where  $f_o$  is focal length of the objective and  $f_e$  is the focal length of eye piece.

Any two limitations of refracting type telescope

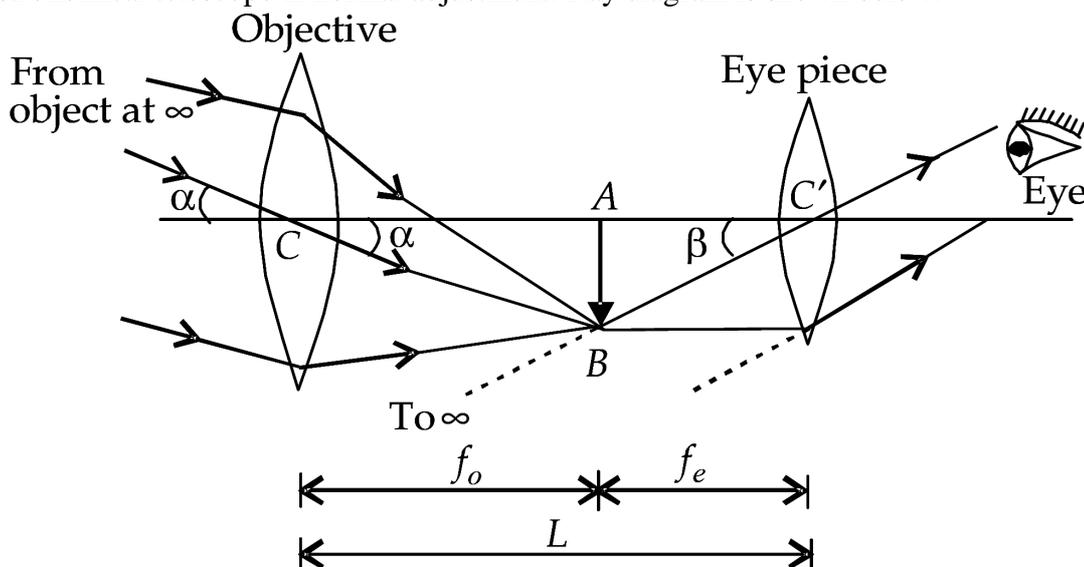
- (i) Image formed is of lesser intensity.
- (ii) Image is not free from chromatic aberration due to refraction.
- (iii) Image is not free from spherical aberration.
- (iv) Objective of telescope should have a large aperture for resolving power.

19. (i) Draw a neat labelled ray diagram of an astronomical telescope in normal adjustment. Explain briefly its working.

(ii) An astronomical telescope uses two lenses of powers 10 D and 1 D. What is its magnifying power in normal adjustment?

Ans:

(i) An astronomical telescope in normal adjustment. Ray diagram is shown below.



It is used to see distant objects.

It consists of two lenses:

- Objective of large aperture and large focal length  $f_o$
- Eyepiece of small aperture and short focal length  $f_e$

**Working :** A parallel beam of light from an astronomical object at infinity is made to fall on objective lens. It forms a real, inverted and diminished image  $AB$  of the object. In normal

adjustment,  $AB$  lies at focus of the eye piece. So a highly magnified, erect image (w.r.t.  $AB$ ) is formed at infinity.

(ii) Here, power of objective lens = 1 D

Power of eye piece = 10 D

In normal adjustment

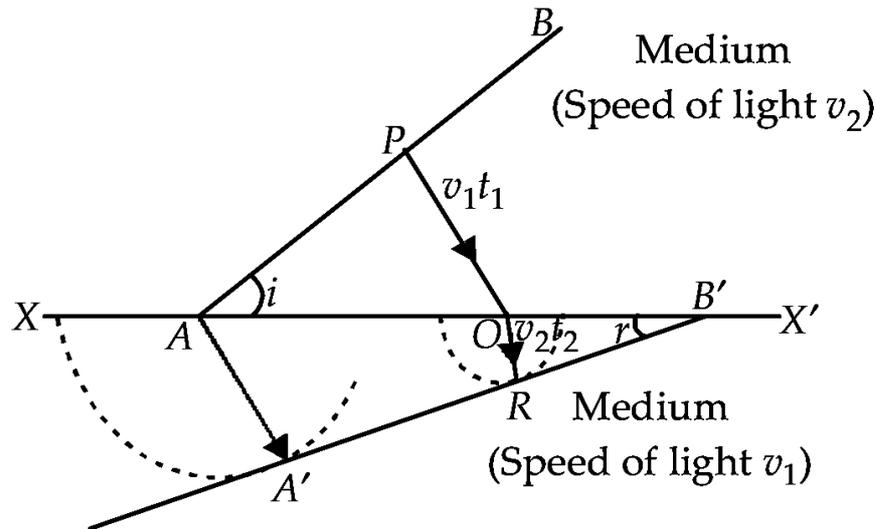
$$\text{Magnifying power, } M = -\frac{f_0}{f_e} = -\frac{P_e}{P_0}$$

$$\Rightarrow M = -10$$

**20. Use Huygen's principle to verify the laws of refraction.**

**Ans:**

Verification of Snell's law of refraction by using Huygen's principle.



We take a plane wavefront  $AB$  incident at a plane surface  $XX'$ . We use secondary wavelets starting at different times. We get refracted wavefront only when time taken by light to travel along different rays from one wavefront to another is same. We take any arbitrary ray starting from point  $P$  on incident wavefront to refracted wavefront at point  $R$ .

Let total time be  $t$

$$t = \frac{PO}{v_1} + \frac{OR}{v_2} = \frac{AO \sin i}{v_1} + \frac{(AB' - AO) \sin r}{v_2}$$

$$\Rightarrow t = \frac{AB' \sin r}{v_2} + AO \left( \frac{\sin i}{v_1} - \frac{\sin r}{v_2} \right)$$

As time should be independent of the ray to be considered, the coefficient of  $AO$  in the above equation should be zero.

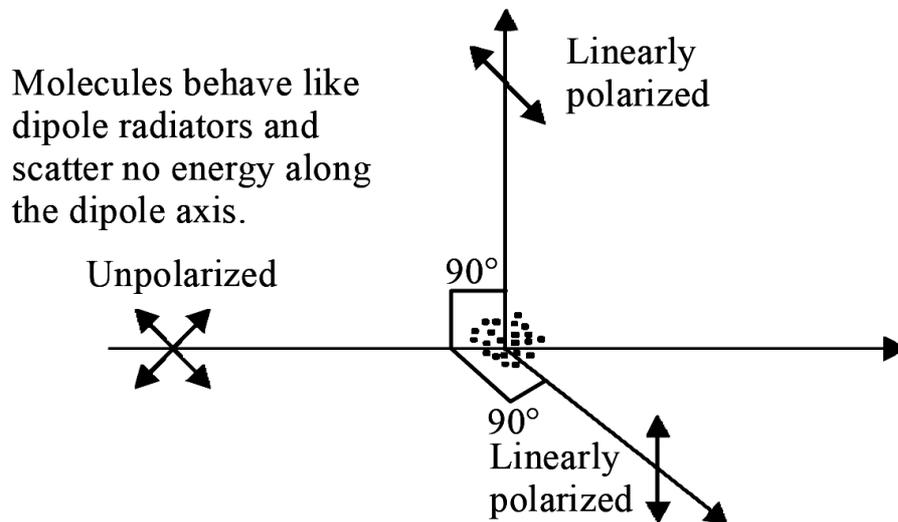
That is,  $\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = {}^1\mu_2$ , where  ${}^1\mu_2$  is called refractive index of medium 2 w.r.t. medium 1. This is

Snell's law of refraction.

- 21. (a) What is linearly polarized light? Describe briefly using a diagram how sunlight is polarised.**  
**(b) Unpolarised light is incident on a polaroid. How would the intensity of transmitted light change when the polaroid is rotated?**

**Ans:**

(a) If the electric field vector of a light wave vibrates just in one direction perpendicular to the direction of the propagation then it is said to be linearly polarised.



Unpolarised light incident on air molecules is scattered and gets polarized.

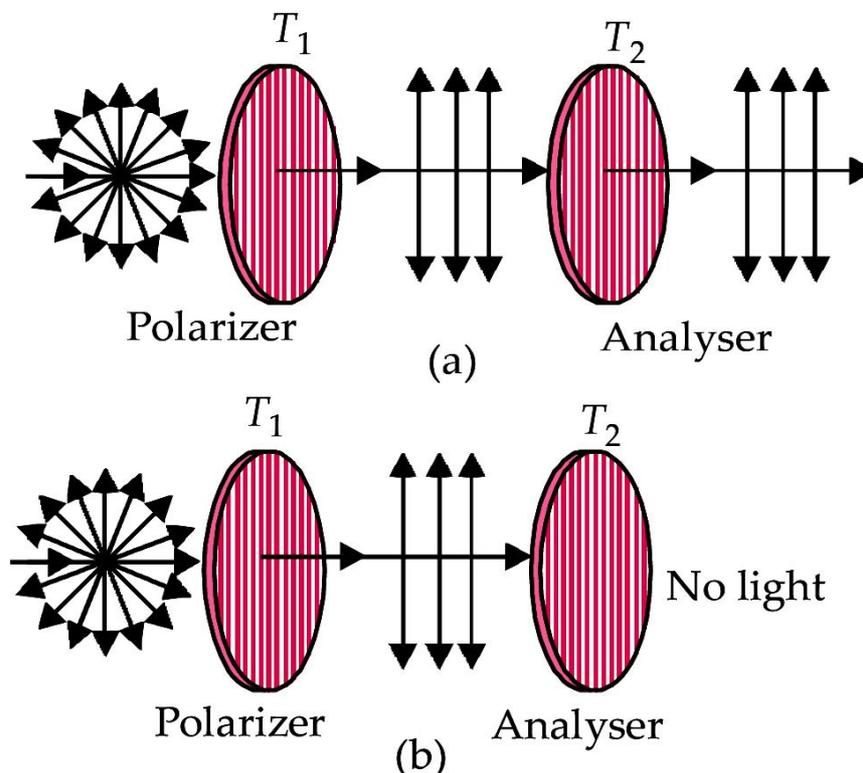
(b) Same/Unchanged/constant

22. (a) Describe briefly, with the help of suitable diagram, how the transverse nature of light can be demonstrated by the phenomenon of polarization.

(b) When unpolarized light passes from air to transparent medium, under what condition does the reflected light get polarized?

Ans:

(a) If two thin plates of tourmaline crystals  $T_1$  and  $T_2$  are rotated with the same angular velocity in the same direction as shown in the figure below, no change in intensity of transmitted light is observed.



The phenomenon can be explained only when we assume that light waves are transverse. Now the unpolarized light falling on  $T_1$  has transverse vibrations of electric vector lying in all possible directions. The crystal  $T_1$  allows only those vibrations to pass through it, which are parallel to its axis. When the crystal  $T_2$  is introduced with its axis kept parallel to the axis of  $T_1$ , the vibrations of electric vector transmitted by  $T_1$  are also transmitted through  $T_2$ . However, when axis of  $T_2$  is perpendicular to axis of  $T_1$ , vibrations of electric vector transmitted from  $T_1$  are normal to the axis of  $T_2$ . Therefore,  $T_2$  does not allow them to pass and hence eye receives no light. Light coming out of

the crystal  $T_1$  is said to be polarized *i.e.* it has vibrations of electric vector which are restricted only in one direction (*i.e.* parallel to the optic axis of crystal  $T_1$ ).

Since the intensity of polarized light on passing through a tourmaline crystal changes, with the relative orientation of its crystallographic axes with that of polariser, therefore, light must consist of transverse waves.

(b) The reflected ray is totally plane polarised, when reflected rays and refracted rays are perpendicular to each other.

**23. (a) Write Einstein's photoelectric equation and point out the characteristic properties of photons on which this equation is based. Briefly explain the observed features which can be explained by this equation.**

**(b) Define the terms (i) 'cutoff voltage and (ii) threshold frequency' in relation to the phenomenon of photoelectric effect.**

**Using Einstein's photoelectric equation show how the cutoff voltage and threshold frequency for a given photosensitive material can be determined with the help of a suitable plot/ graph.**

**Ans: (a)** Einstein's photoelectric equation

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = h\nu - \nu_0$$

Characteristics properties :

(i) In the interaction of photons with free electrons, the entire energy of photon is absorbed.

(ii) Energy of photon is directly proportional to frequency.

(iii) In photon electron collision, the total energy and momentum remain constant.

Three features :

(i) There is no time lag between the incidence of radiation and emission of electrons from the surface.

(ii) The number of electrons emitted per second, *i.e.*, photoelectric current, is directly proportional to the intensity of the incident radiations.

(iii) There is a minimum frequency of the incident radiations below which emission of electrons cannot occur.

(iv) The maximum KE of electrons increases proportionally, with increase in the frequency of incident radiations.

(b)

**Cutoff Voltage :** The minimum negative  $V_0$  potential applied to the plate or anode, (A) for which the photoelectric current just becomes zero.

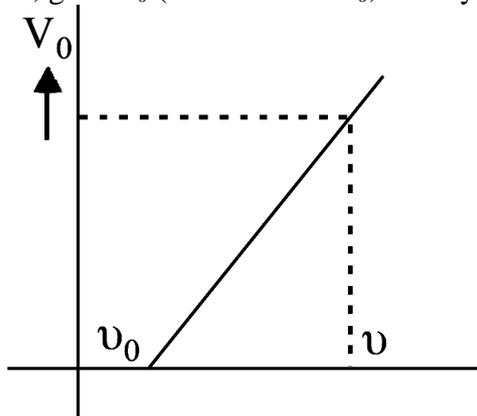
**Threshold frequency :** The minimum frequency of incident radiation which is required to have photo electrons emitted from a given metal surface.

As per Einstein's photoelectric equation

$$eV_0 = h\nu - h\nu_0 \text{ for } \nu > \nu_0$$

$$V_0 = \frac{h}{e}(\nu - \nu_0)$$

Hence the intercept, on the yaxis, gives  $\nu_0$  (one can read  $V_0$ , for any  $\nu$ , from the graph)



24. Using Bohr's postulates, derive the expression for the frequency of radiation emitted when electron in hydrogen atom undergoes transition from higher energy state (quantum number  $n_i$ ) to the lower state, ( $n_f$ ). When electron in hydrogen atom jumps from energy state  $n_i = 4$  to  $n_f = 3, 2, 1$ , identify the spectral series to which the emission lines belong.

Ans: Since,  $\frac{mv^2}{r_n} = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r_n^2}$  and  $mvr_n = \frac{nh}{2\pi}$

Therefore,  $r_n = \frac{\epsilon_0 h^2 n^2}{\pi m e^2}$  ..... (i)

Total energy,  $E_n = \frac{1}{2} mv_n^2 - \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r_n} = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2r_n} - \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r_n}$

$\Rightarrow E_n = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2r_n} = -\frac{1}{8\epsilon_0^2} \cdot \frac{me^4}{h^2 n^2}$  [using (i)]

$\Rightarrow E_n = \frac{-Rhc}{n^2}$  where Rydberg constant,  $R = \frac{me^4}{8\epsilon_0^2 h^3 c}$

Energy emitted  $\Delta E = E_i - E_f$

$\Delta E = Rhc \left[ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right]$

But  $\Delta E = hu$

$\therefore \nu = Rc \left[ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right]$

or  $\nu = \frac{me^4}{8\epsilon_0^2 h^3} \left[ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right]$

Paschen, Balmer, Lyman

25. (a) Using de Broglie's hypothesis, explain with the help of a suitable diagram, Bohr's second postulate of quantization of energy levels in a hydrogen atom.

(b) The ground state energy of hydrogen atom is  $-13.6$  eV. What are the kinetic and potential energies of the electron in this state?

Ans:

(a) According to de Broglie's hypothesis,

$\lambda = \frac{h}{mv}$  .....(i)

According to de Broglie's condition of stationary orbits, the stationary orbits are those which contain complete de-Broglie wavelength.

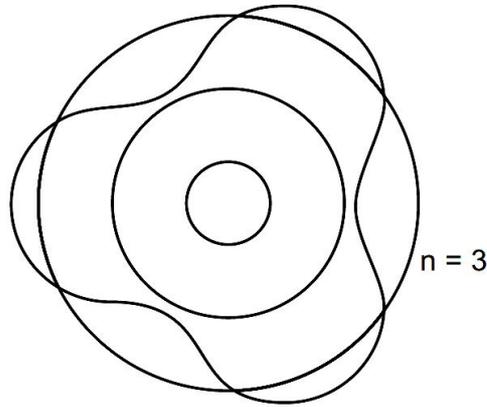
$2\pi r = n\lambda$  .....(ii)

Substituting value of  $\lambda$  from (ii) in (i), we get

$2\pi r = n \frac{h}{mv}$

$\Rightarrow mvr = n \frac{h}{2\pi}$  .....(iii)

This is Bohr's postulate of quantisation of energy levels.



(b) Kinetic energy,  $K = \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2r} \dots(i)$

Potential energy,  $U = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r} \dots(ii)$

Total energy  $E = K + U = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2r} \dots(iii)$

Comparing equations (i), (ii), (iii), we have

$$K = -E \text{ and } U = 2E$$

Given  $E = -13.6 \text{ eV}$  (in ground state)

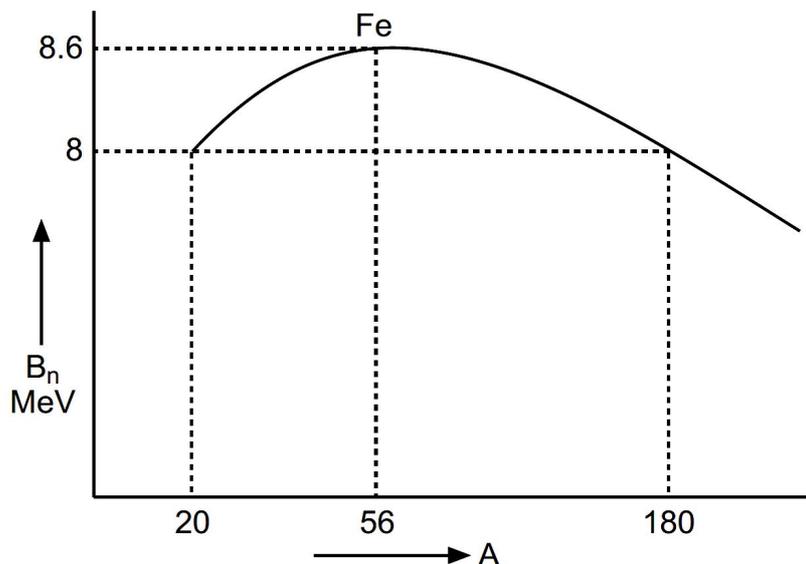
$\therefore$  Kinetic energy,  $K = 13.6 \text{ eV}$

Potential energy  $U = 2 \times (-13.6 \text{ eV}) = -27.2 \text{ eV}$

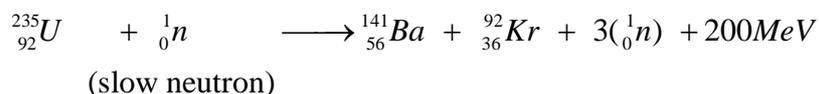
**26. Draw a plot showing the variation of binding energy per nucleon versus the mass number  $A$ . Explain with the help of this plot the release of energy in the processes of nuclear fission and fusion.**

**Ans:**

The variation of binding energy per nucleon versus mass number is shown in figure.

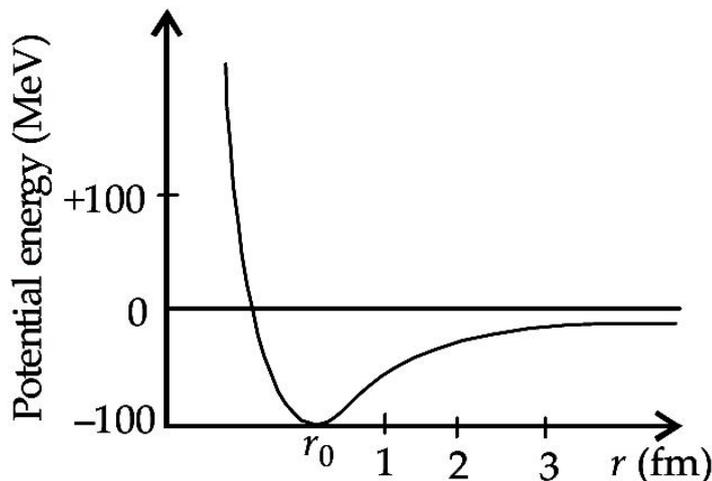


The binding energy curve indicates that binding energy for nucleon of heavy nuclei is less than that of middle nuclei. Clearly a heavy nucleus breaks into two lighter nuclei then binding energy per nucleon will increase and energy will be released in the process. This process is called nuclear fission. Nuclear fission reaction is



27. Draw a plot of potential energy of a pair of nucleons as a function of their separation. Write two important conclusions which you can draw regarding the nature of nuclear forces.

Ans:



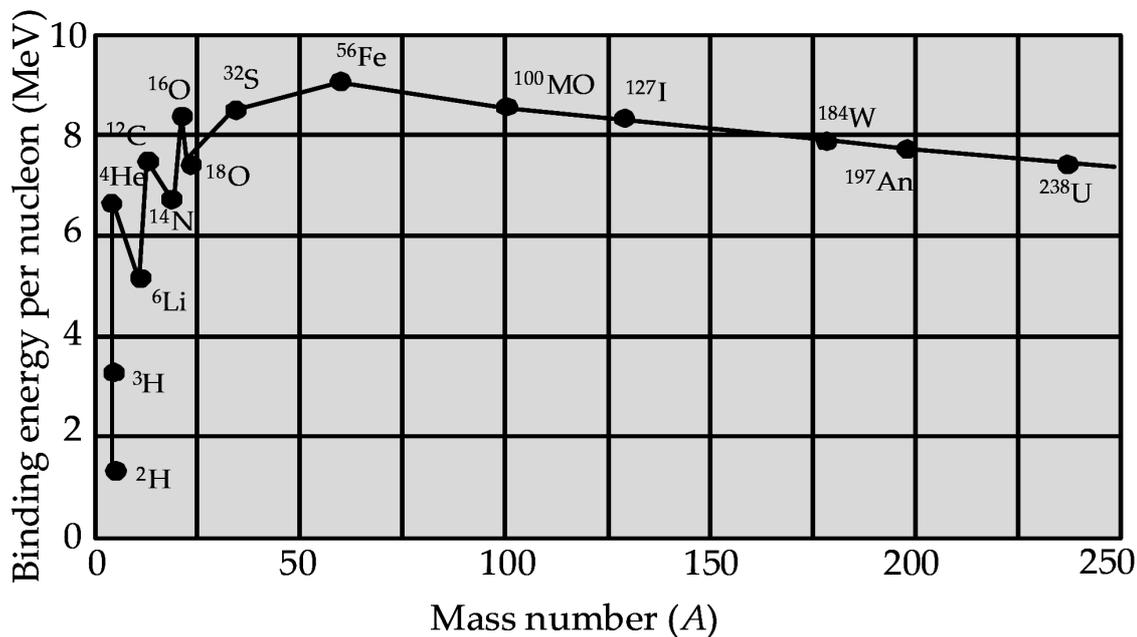
From the above plot, following conclusions can be drawn.

- (i) Nuclear forces are short range forces
- (ii) For a separation greater than  $r_0$ , the nuclear forces are attractive and for separation less than  $r_0$ , the nuclear forces are strongly repulsive.

28. Draw a plot of the binding energy per nucleon as a function of mass number for a large number of nuclei,  $2 \leq A \leq 240$ . How do you explain the constancy of binding energy per nucleon in the range  $30 < A < 170$  using the property that nuclear force is short ranged?

Ans:

The variation of binding energy per nucleon versus mass number is shown in figure.



The binding energy per nucleon as a function of mass number.

Inferences from graph

1. The nuclei having mass number below 20 and above 180 have relatively small binding energy and hence they are unstable.
2. The nuclei having mass number 56 and about 56 have maximum binding energy – 5.8 MeV and so they are most stable.
3. Some nuclei have peaks, e.g.,  ${}^4_2\text{He}$ ,  ${}^{12}_6\text{C}$ ,  ${}^{16}_8\text{O}$ ; this indicates that these nuclei are relatively more stable than their neighbours.

**Explanation:** When a heavy nucleus ( $A \geq 235$  say) breaks into two lighter nuclei (nuclear fission), the binding energy per nucleon increases *i.e.*, nucleons get more tightly bound. This implies that energy would be released in nuclear fission.

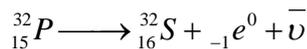
When two very light nuclei ( $A \leq 10$ ) join to form a heavy nucleus, the binding is energy per nucleon of fused heavier nucleus more than the binding energy per nucleon of lighter nuclei, so again energy would be released in nuclear **fusion**.

29. (a) Write symbolically the  $\beta^-$  decay process of  ${}^{32}_{15}P$ .

(b) Derive an expression for the average life of a radionuclide. Give its relationship with the half life.

**Ans:**

(a)  $\beta^-$  decay process of  ${}^{32}_{15}P$



(b) The average or mean life of a radioactive substance is defined as the time for which the active nuclei of the atoms of the radioactive substance exit. In mean life  $T_a$ , both number of nuclei,  $N$  and rate of disintegration,  $R$  reduce to  $1/e$  of their initial values.

*i.e.*, when  $t = T_a$  then  $N = \frac{N_0}{e}$

Using it in equation  $N = N_0 e^{-\lambda t}$ , we get  $\frac{N_0}{e} = N_0 e^{-\lambda t}$

$$\Rightarrow e^{-1} = e^{-\lambda T_a}$$

$$\Rightarrow 1 = \lambda T_a \Rightarrow T_a = \frac{1}{\lambda}$$

so, finally mean life is reciprocal of decay constant  $\lambda$ .

Also half life  $T_{1/2} = \frac{0.693}{\lambda} = 0.693 T_a$ .

30. State the law of radioactive decay. Plot a graph showing the number ( $N$ ) of undecayed nuclei as a function of time ( $t$ ) for a given radioactive sample having half life  $T_{1/2}$ . Depict in the plot the number of undecayed nuclei at (i)  $t = 3T_{1/2}$  and (ii)  $t = 5T_{1/2}$ .

**Ans:**

**Radioactive Decay Law :** Laws of radioactive decay

(i) Radioactivity is a nuclear phenomenon. It is independent of all physical and chemical conditions.

(ii) The disintegration is random and spontaneous. It is a matter of chance for any atom to disintegrate first.

(iii) The radioactive substance emit  $\alpha$  or  $\beta$  particles.

These rays originate from the nuclei of disintegrating atom and form fresh radioactive products.

(iv) The rate of decay of atoms is proportional to the number of undecayed radioactive atoms present at any instant.

If  $N$  is the number of undecayed atoms in a radioactive substance at any time  $t$ ,  $dN$  the number of atoms disintegrating in time  $dt$ , the rate of decay is  $\frac{dN}{dt}$  so that

$$-\frac{dN}{dt} \propto N \text{ or } \frac{dN}{dt} = -\lambda N \dots\dots\dots(i)$$

where  $\lambda$  is a constant of proportionality called the decay (or disintegration) constant, equation (i) results

$$N = N_0 e^{-\lambda t} \dots\dots\dots(ii)$$

where  $N_0$  = initial number of undecayed radioactive atoms.

If  $N_0$  is the initial number of radioactive atoms present then in a half life time  $T_{1/2}$ , the number of undecayed radioactive atoms will be  $N_0/2$  and in next half  $N_0/4$  and so on.

Using,  $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

According to problem

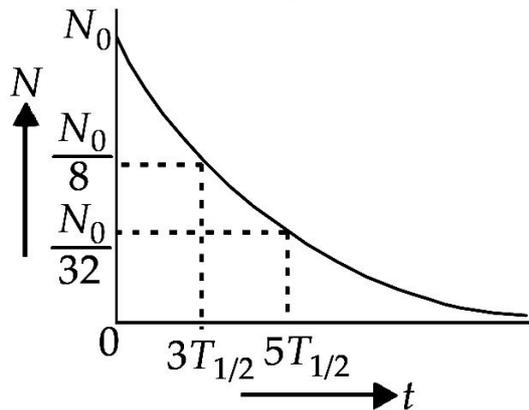
$$t = 3T_{1/2}$$

$$\therefore \frac{N}{N_0} = \left(\frac{1}{2}\right)^3 \Rightarrow N = N_0 \left(\frac{1}{2}\right)^3 = \frac{N_0}{8}$$

and at  $t = 5T_{1/2}$

$$N = \frac{N_0}{32}$$

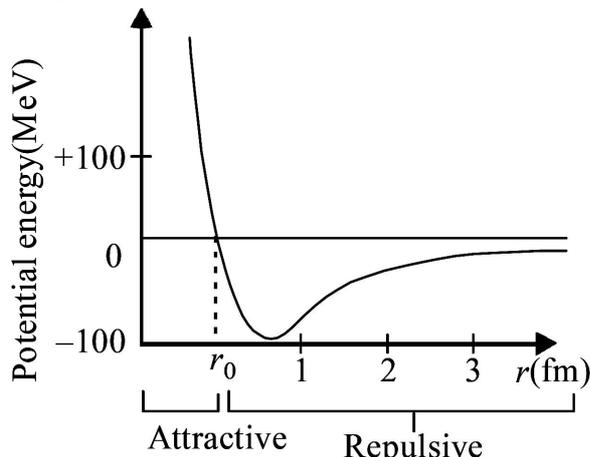
The graph shown below for the number of undecayed nuclei at  $t = 3T_{1/2}$  and  $t = 5T_{1/2}$ .



**31. Draw a plot of potential energy of a pair of nucleons as a function of their separations. Mark the regions where the nuclear force is (i) attractive and (ii) repulsive. Write the characteristic features of nuclear forces.**

**Ans:**

The nuclear force must be of short range because its influence does not exist far beyond its nuclear 'surface'. The graph of potential energy of a pair of nucleons as function of their separation is as shown. It depicts the short range character of nuclear force. It is attractive for a separation greater than  $r_0$  ( $< 1$  fm), but becomes strongly repulsive for separations less than  $r_0$ . This region is known as hard core. Nuclear attractive force is strongest when the separation is about 1 fm, or potential energy of two nucleons is minimum.



Properties of nuclear force are:

- Nuclear forces are short range forces and are strongly
- Nuclear forces above 4.2. Fermi are negligible, whereas below 1 Fermi, they become repulsive in nature. It is this repulsive nature below 1 Fermi, which prevents the nucleus from collapsing under strong attractive force.
- Nuclear forces are charge independent. The same magnitude of nuclear force act between a pair of protons, pair of proton and neutron and pair of neutrons. The attractive nuclear force is due to exchange of p mesons ( $\pi^0, \pi^+, \pi^-$ ) between them.

- Nuclear force of one nucleon at a time is only with nearest neighbouring nucleon and not with all
- neighbouring nucleons. Thus nuclear forces are saturated forces.
- Nuclear forces are strongest forces in nature and are  $10^2$  times stronger than electrostatic force and  $10^{38}$  times stronger than the gravitational force, in their own small range of few fermi.

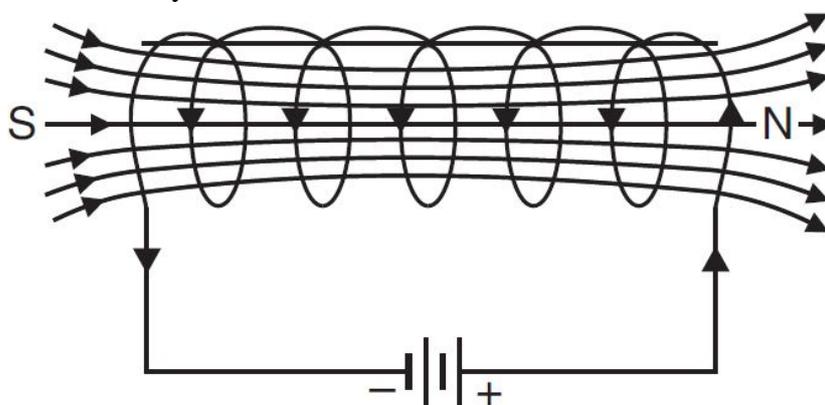
32. (a) State Ampere's circuital law. (b) Use it to derive an expression for magnetic field inside, along the axis of an air cored solenoid. (c) Sketch the magnetic field lines for a finite solenoid. How are these field lines different from the electric field lines from an electric dipole?

Ans:

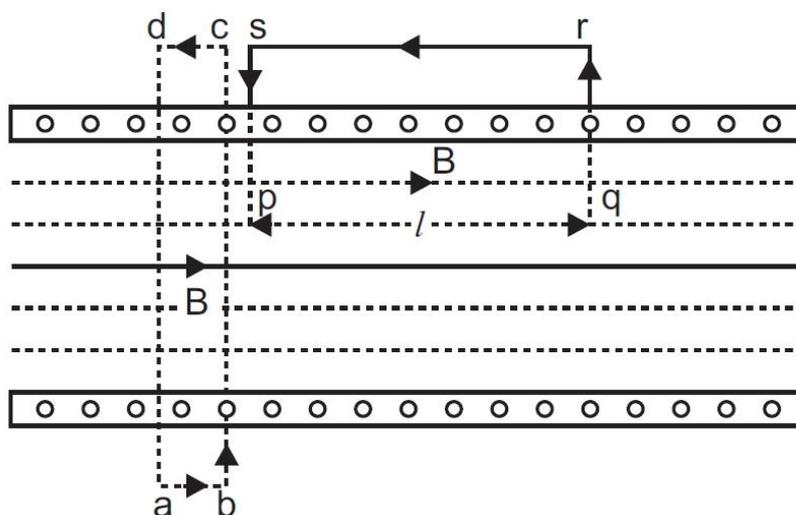
(a) It states that the line integral of magnetic field induction along a closed path is equal to  $\mu_0$  -times the current enclosed by the path i.e.,  $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

(b) **Magnetic Field Due to a Current Carrying Long Solenoid:**

A solenoid is a long wire wound in the form of a close-packed helix, carrying current. To construct a solenoid a large number of closely packed turns of insulated copper wire are wound on a cylindrical tube of card-board or china clay.



When an electric current is passed through the solenoid, a magnetic field is produced within the solenoid. If the solenoid is long and the successive insulated copper turns have no gaps, then the magnetic field within the solenoid is uniform; with practically no magnetic field outside it. The reason is that the solenoid may be supposed to be formed of a large number of circular current elements. The magnetic field due to a circular loop is along its axis and the current in upper and lower straight parts of solenoid is equal and opposite. Due to this the magnetic field in a direction perpendicular to the axis of solenoid is zero and so the resultant magnetic field is along the axis of the solenoid.



If there are ' $n$ ' number of turns per metre length of solenoid and  $I$  amperes is the current flowing, then magnetic field at axis of long solenoid is  $B = \mu_0 n I$

If there are  $N$  turns in length  $l$  of wire, then  $n = \frac{N}{l}$  or  $B = \frac{\mu_0 NI}{l}$

**Derivation:** Consider a symmetrical long solenoid having number of turns per unit length equal to  $n$ . Let  $I$  be the current flowing in the solenoid, then by right hand rule, the magnetic field is parallel to the axis of the solenoid.

**Field outside the solenoid:** Consider a closed path  $abcd$ . Applying Ampere's law to this path

$$\oint \vec{B} \cdot d\vec{l} = \mu \times 0 \text{ (since net current enclosed by path is zero)}$$

As  $dl \neq 0 \therefore B = 0$

This means that the magnetic field outside the solenoid is zero.

**Field Inside the solenoid:** Consider a closed path  $pqrs$ . The line integral of magnetic field  $\vec{B}$  along path  $pqrs$  is

$$\oint_{pqrs} \vec{B} \cdot d\vec{l} = \int_{pq} \vec{B} \cdot d\vec{l} + \int_{qr} \vec{B} \cdot d\vec{l} + \int_{rs} \vec{B} \cdot d\vec{l} + \int_{sp} \vec{B} \cdot d\vec{l} \quad \dots\dots(i)$$

For path  $pq$ ,  $\vec{B}$  and  $d\vec{l}$  are along the same direction,

$$\therefore \int_{pq} \vec{B} \cdot d\vec{l} = \int B dl = Bl \text{ (since } pq = l)$$

For paths  $qr$  and  $sp$ ,  $\vec{B}$  and  $d\vec{l}$  are mutually perpendicular.

$$\therefore \int_{qr} \vec{B} \cdot d\vec{l} = \int_{sp} \vec{B} \cdot d\vec{l} = \int B dl \cos 90^\circ = 0$$

For path  $rs$ ,  $B = 0$  (since field is zero outside a solenoid)

$$\therefore \int_{rs} \vec{B} \cdot d\vec{l} = 0$$

In view of these, equation (i) gives

$$\int_{pqrs} \vec{B} \cdot d\vec{l} = \int_{pq} \vec{B} \cdot d\vec{l} = Bl \quad \dots\dots(ii)$$

By Ampere's law  $\oint \vec{B} \cdot d\vec{l} = \mu_0 \times \text{net current enclosed by path}$

$$\therefore Bl = \mu_0 (nl I) \Rightarrow B = \mu_0 nI$$

This is the well known result.

(c) The magnetic field lines of magnet (or current carrying solenoid) form continuous closed loops and are directed from  $N$  to  $S$  pole outside the magnet and  $S$  to  $N$  pole inside the magnet and forms closed loops while in the case of an electric dipole the field lines begin from positive charge and end on negative charge or escape to infinity.

