

MATHEMATICS

MINIMUM LEVEL MATERIAL

for

CLASS – X

2017 – 18



Project Planned By
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PREFACE

It gives me great pleasure in presenting the Minimum Level Study Material in Mathematics for Class X. It is in accordance with the latest CBSE syllabus of the session 2017-18.

I am extremely thankful to Honourable Shri D. Manivannan, Deputy Commissioner, KVS RO Hyderabad and respected sir Shri. E. Krishna Murthy, Principal, KV Gachibowli, who blessed and motivates me to complete this project work. This study material has been designed in such a way that all the minimum level learning scoring chapters with sufficient number of previous years Board Exam important questions for practice are covered. This is very useful for all level of students to get quality result.

I avail this opportunity to convey my sincere thanks to respected sir, Shri U. N. Khaware, Additional Commissioner(Acad), KVS Headquarter, New Delhi, respected sir, Shri S. Vijay Kumar, Joint Commissioner(Acad), KVS Headquarter, New Delhi, respected sir Shri P. V. Sairanga Rao, Deputy Commissioner(Acad), KVS Headquarter, New Delhi, respected sir Shri. D. Manivannan, Deputy Commissioner, KVS RO Hyderabad, respected sir Shri Isampal, Deputy Commissioner, KVS RO Bhopal, respected sir Shri P. Deva Kumar, Deputy Commissioner, KVS RO Bangalore, respected sir Shri Nagendra Goyal, Deputy Commissioner, KVS RO Ranchi, respected sir Shri Y. Arun Kumar, Deputy Commissioner, KVS RO Agra, respected sir Shri Sirimala Sambanna, Deputy Commissioner, KVS RO Jammu, respected sir Shri. K. L. Nagaraju, Retd. Assistant Commissioner, KVS RO Bangalore, respected sir Shri.Gangadharaiah, Retd. Assistant Commissioner, KVS RO Bangalore and respected Shri M.K. Kulshreshtha, Retd. Assistant Commissioner, KVS RO Chandigarh for their blessings, motivation and encouragement in bringing out this project in such an excellent form.

I also extend my special thanks to respected sir Shri. P. S. Raju, Principal, KV No. 1 Uppal, respected madam Smt. Nirmala Kumari M., Principal, KV Mysore & respected sir Shri. M. Vishwanatham, Principal, KV Raichur for their kind suggestions and motivation while preparing this Question Bank. I would like to place on record my thanks to respected sir Shri. P. K. Chandran, Principal, presently working in KV Bambolim. I have started my career in KVS under his guidance, suggestions and motivation.

Inspite of my best efforts to make this notes error free, some errors might have gone unnoticed. I shall be grateful to the students and teacher if the same are brought to my notice. You may send your valuable suggestions, feedback or queries through email to kumarsir34@gmail.com that would be verified by me and the corrections would be incorporated in the next year Question Bank.

M. S. KUMARSWAMY

**DEDICATED
TO
MY FATHER**

LATE SHRI. M. S. MALLAYYA

**MINIMUM LEVEL DAILY REVISION SYLLABUS
FOR REMEDIAL STUDENTS
MATHEMATICS: CLASS X**

S. NO.	CHAPTER/TOPIC	MARKS COVERED AS PER LATEST CBSE SAMPLE PAPERS
1	Real Numbers – Full Chapter	6
2	Polynomials – Full Chapter	3
3	Arithmetic Progression – Full Chapter	7
4	Triangles Theorem	5
5	Circles – Full Chapter	3
6	Construction – Full Chapter	4
7	Areas related to Circles	3
8	Statistics – Full Chapter	7
9	Probability – Full Chapter	4
Total Marks		41

All Remedial Students have to complete the above chapters/topics thoroughly with 100% perfection and then they can also concentrate the below topics for Board Exam:

*Linear Equation in two variables – **Graph Questions, Comparing the ratios of coefficients based questions.**

*Quadratic Equations – **imp questions**

*Triangles – **1 mark imp questions**

*Coordinate Geometry – **imp questions**

*Trigonometry – **imp questions**

*Surface Areas and Volumes – **imp questions**



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CHAPTER – 1

REAL NUMBERS

EUCLID'S DIVISION LEMMA

Given positive integers a and b , there exist unique integers q and r satisfying $a = bq + r$, where $0 \leq r < b$.

Here we call 'a' as dividend, 'b' as divisor, 'q' as quotient and 'r' as remainder.

\therefore Dividend = (Divisor \times Quotient) + Remainder

If in Euclid's lemma $r = 0$ then b would be HCF of 'a' and 'b'.

IMPORTANT QUESTIONS

Show that any positive even integer is of the form $6q$, or $6q + 2$, or $6q + 4$, where q is some integer.

Solution: Let x be any positive integer such that $x > 6$. Then, by Euclid's algorithm,

$x = 6q + r$ for some integer $q \geq 0$ and $0 \leq r < 6$.

Therefore, $x = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Now, $6q$ is an even integer being a multiple of 2.

We know that the sum of two even integers are always even integers.

Therefore, $6q + 2$ and $6q + 4$ are even integers

Hence any positive even integer is of the form $6q$, or $6q + 2$, or $6q + 4$, where q is some integer.

Questions for practice

1. Show that any positive even integer is of the form $4q$, or $4q + 2$, where q is some integer.
2. Show that any positive odd integer is of the form $4q + 1$, or $4q + 3$, where q is some integer.
3. Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.
4. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m .
5. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.
6. Use Euclid's division lemma to show that the square of an odd positive integer can be of the form $6q + 1$ or $6q + 3$ for some integer q .
7. Use Euclid's division lemma to prove that one and only one out of n , $n + 2$ and $n + 4$ is divisible by 3, where n is any positive integer.
8. Use Euclid's division lemma to prove that one of any three consecutive positive integers must be divisible by 3.
9. For any positive integer n , use Euclid's division lemma to prove that $n^3 - n$ is divisible by 6.
10. Use Euclid's division lemma to show that one and only one out of n , $n + 4$, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5, where n is any positive integer.

EUCLID'S DIVISION ALGORITHM

Euclid's division algorithm is a technique to compute the Highest Common Factor (HCF) of two given positive integers. Recall that the HCF of two positive integers a and b is the largest positive integer d that divides both a and b .

To obtain the HCF of two positive integers, say c and d , with $c > d$, follow the steps below:

Step 1 : Apply Euclid's division lemma, to c and d . So, we find whole numbers, q and r such that $c = dq + r$, $0 \leq r < d$.

Step 2 : If $r = 0$, d is the HCF of c and d . If $r \neq 0$ apply the division lemma to d and r .

Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

This algorithm works because $HCF(c, d) = HCF(d, r)$ where the symbol $HCF(c, d)$ denotes the HCF of c and d , etc.

IMPORTANT QUESTIONS

Use Euclid's division algorithm to find the HCF of 867 and 255

Solution: Since $867 > 255$, we apply the division lemma to 867 and 255 to obtain

$$867 = 255 \times 3 + 102$$

Since remainder $102 \neq 0$, we apply the division lemma to 255 and 102 to obtain

$$255 = 102 \times 2 + 51$$

We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain

$$102 = 51 \times 2 + 0$$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51,

Therefore, HCF of 867 and 255 is 51.

Questions for practice

1. Use Euclid's algorithm to find the HCF of 4052 and 12576.
2. Use Euclid's division algorithm to find the HCF of 135 and 225.
3. Use Euclid's division algorithm to find the HCF of 196 and 38220.
4. Use Euclid's division algorithm to find the HCF of 455 and 42.
5. Using Euclid's division algorithm, find which of the following pairs of numbers are co-prime: (i) 231, 396 (ii) 847, 2160
6. If the HCF of 65 and 117 is expressible in the form $65m - 117$, then find the value of m .

The Fundamental Theorem of Arithmetic

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

The prime factorisation of a natural number is unique, except for the order of its factors.

❖ Property of HCF and LCM of two positive integers 'a' and 'b':

➤ $HCF(a, b) \times LCM(a, b) = a \times b$

➤ $LCM(a, b) = \frac{a \times b}{HCF(a, b)}$

➤ $HCF(a, b) = \frac{a \times b}{LCM(a, b)}$

PRIME FACTORISATION METHOD TO FIND HCF AND LCM

$HCF(a, b)$ = Product of the smallest power of each common prime factor in the numbers.

$LCM(a, b)$ = Product of the greatest power of each prime factor, involved in the numbers.

IMPORTANT QUESTIONS

Find the LCM and HCF of 510 and 92 and verify that $LCM \times HCF =$ product of the two numbers

Solution: $510 = 2 \times 3 \times 5 \times 17$

$$92 = 2 \times 2 \times 23 = 2^2 \times 23$$

$$HCF = 2$$

$$LCM = 2^2 \times 3 \times 5 \times 17 \times 23 = 23460$$

$$\text{Product of two numbers} = 510 \times 92 = 46920$$

$$\text{HCF} \times \text{LCM} = 2 \times 23460 = 46920$$

Hence, product of two numbers = HCF \times LCM

Questions for practice

1. Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.
2. Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.
3. Find the LCM and HCF of the following pairs of integers and verify that LCM \times HCF = product of the two numbers: (i) 26 and 91 (ii) 336 and 54
4. Find the LCM and HCF of the following integers by applying the prime factorisation method: (i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25
5. Explain why $3 \times 5 \times 7 + 7$ is a composite number.
6. Can the number 6^n , n being a natural number, end with the digit 5? Give reasons.
7. Can the number 4^n , n being a natural number, end with the digit 0? Give reasons.
8. Given that HCF (306, 657) = 9, find LCM (306, 657).
9. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers, then find the HCF (a, b).
10. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then find the LCM (p, q).

IRRATIONALITY OF NUMBERS

IMPORTANT QUESTIONS

Prove that $\sqrt{5}$ is an irrational number.

Solution: Let $\sqrt{5}$ is a rational number then we have

$$\sqrt{5} = \frac{p}{q}, \text{ where } p \text{ and } q \text{ are co-primes.}$$

$$\Rightarrow p = \sqrt{5}q$$

Squaring both sides, we get

$$p^2 = 5q^2$$

$$\Rightarrow p^2 \text{ is divisible by } 5$$

$$\Rightarrow p \text{ is also divisible by } 5$$

So, assume $p = 5m$ where m is any integer.

Squaring both sides, we get $p^2 = 25m^2$

$$\text{But } p^2 = 5q^2$$

$$\text{Therefore, } 5q^2 = 25m^2$$

$$\Rightarrow q^2 = 5m^2$$

$$\Rightarrow q^2 \text{ is divisible by } 5$$

$$\Rightarrow q \text{ is also divisible by } 5$$

From above we conclude that p and q has one common factor i.e. 5 which contradicts that p and q are co-primes.

Therefore our assumption is wrong.

Hence, $\sqrt{5}$ is an irrational number.

Questions for practice

1. Prove that $\sqrt{2}$ is an irrational number.
2. Prove that $\sqrt{3}$ is an irrational number.
3. Prove that $2 + 5\sqrt{3}$ is an irrational number.
4. Prove that $3 - 2\sqrt{5}$ is an irrational number.
5. Prove that $\sqrt{2} + \sqrt{3}$ is an irrational number.

RATIONAL NUMBERS AND THEIR DECIMAL EXPANSIONS

Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form $2^m \cdot 5^n$, where m, n are non-negative integers. Then x has a decimal expansion which terminates.

Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^m \cdot 5^n$, where m, n are non-negative integers. Then x has a decimal expansion which is non-terminating repeating (recurring).

IMPORTANT QUESTIONS

Without actually performing the long division, state whether the rational numbers $\frac{987}{10500}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

Solution: Given rational number $\frac{987}{10500}$ is not in the simplest form. Dividing numerator and

denominator by 21 we get $\frac{987}{10500} = \frac{987 \div 21}{10500 \div 21} = \frac{47}{500}$ which is in the form of $\frac{p}{q}$

Now $q = 500 = 2^2 \times 5^3$ which is in the form of $2^m \cdot 5^n$, where m, n are non-negative integers. Therefore the given rational number has terminating decimal expansion.

Questions for practice

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$ (ii) $\frac{129}{2^2 5^7 7^5}$ (iii) $\frac{77}{210}$ (iv) $\frac{14587}{1250}$ (v) $\frac{833}{2^2 5^5 7^2}$

CHAPTER – 2 POLYNOMIALS

QUADRATIC POLYNOMIAL

Relationship between zeroes and coefficients

General form of Quadratic polynomial: $ax^2 + bx + c$, $a \neq 0$

$$\text{Sum of zeroes } (\alpha + \beta) = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{b}{a}$$

$$\text{Product of zeroes } (\alpha\beta) = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$$

IMPORTANT QUESTIONS

Find a quadratic polynomial, the sum and product of whose zeroes are – 3 and 2, respectively.

Solution: Here, $\alpha + \beta = -3$ and $\alpha\beta = 2$

We know that quadratic polynomial is given by $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$
 $= x^2 - (-3)x + 2 = x^2 + 3x + 2$

Hence, required quadratic polynomial is $x^2 + 3x + 2$

Find a quadratic polynomial, whose zeroes are – 3 and 2.

Solution: Here, $\alpha = -3$ and $\beta = 2$.

Now, $\alpha + \beta = -3 + 2 = -1$ and $\alpha\beta = (-3)(2) = -6$

We know that quadratic polynomial is given by $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$
 $= x^2 - (-1)x + (-6) = x^2 + x - 6$

Hence, required quadratic polynomial is $x^2 + x - 6$

Find the zeroes of the quadratic polynomial $x^2 - 2x - 8$ and verify the relationship between the zeroes and the coefficients.

Solution: Here, $p(x) = x^2 - 2x - 8 = 0$

$$x^2 - 4x + 2x - 8 = 0 \Rightarrow x(x - 4) + 2(x - 4) = 0 \Rightarrow (x - 4)(x + 2) = 0$$

$$\Rightarrow x = 4, -2$$

Now, $a = 1$, $b = -2$, $c = -8$, $\alpha = 4$, $\beta = -2$

$$\text{Sum of zeroes, } \alpha + \beta = 4 + (-2) = 2 \text{ and } \frac{-b}{a} = \frac{-(-2)}{1} = 2 \quad \therefore \alpha + \beta = \frac{-b}{a}$$

$$\text{Product of zeroes, } \alpha\beta = 4(-2) = -8 \text{ and } \frac{c}{a} = \frac{-8}{1} = -8 \quad \therefore \alpha\beta = \frac{c}{a}$$

Hence verified.

Questions for practice

1. Find a quadratic polynomial, the sum and product of whose zeroes are – 5 and 3, respectively.
2. Find a quadratic polynomial, whose zeroes are – 4 and 1, respectively.
3. Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.
4. Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.
5. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.
6. Find the zeroes of the quadratic polynomial $3x^2 - x - 4$ and verify the relationship between the zeroes and the coefficients.
7. Find the zeroes of the quadratic polynomial $4x^2 - 4x + 1$ and verify the relationship between the zeroes and the coefficients.

DIVISION ALGORITHM FOR POLYNOMIALS

If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x) \times q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

- ❖ If $r(x) = 0$, then $g(x)$ is a factor of $p(x)$.
- ❖ Dividend = Divisor \times Quotient + Remainder

IMPORTANT QUESTIONS

Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$, and verify the division algorithm.

Solution:

$$\begin{array}{r} x - 2 \\ -x^2 + x - 1 \overline{) -x^3 + 3x^2 - 3x + 5} \\ \underline{-x^3 + x^2 - x} \\ 2x^2 - 2x + 5 \\ \underline{2x^2 - 2x + 2} \\ 3 \end{array}$$

So, quotient = $x - 2$, remainder = 3.

Now, Divisor \times Quotient + Remainder

$$\begin{aligned} &= (-x^2 + x - 1)(x - 2) + 3 \\ &= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3 \\ &= -x^3 + 3x^2 - 3x + 5 \\ &= \text{Dividend} \end{aligned}$$

Hence, the division algorithm is verified.

Questions for Practice

1. Divide $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$.
2. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following :
 - (i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$
 - (ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$
 - (iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$
3. Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.
4. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.
5. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.
6. If the remainder on division of $x^3 + 2x^2 + kx + 3$ by $x - 3$ is 21, find the quotient and the value of k . Hence, find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$.
7. Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeroes of the two polynomials.

CHAPTER – 5

ARITHMETIC PROGRESSION

***n*th Term of an ARITHMETIC PROGRESSION (AP)**

*n*th term a_n of the AP with first term a and common difference d is given by

$$a_n = a + (n - 1) d.$$

IMPORTANT QUESTIONS

Find the 15th term of the 21, 24, 27, . . .

Solution: Here, $a = 21$, $d = 24 - 21 = 3$

We know that $a_n = a + (n - 1)d$

So, $a_{15} = a + 14d = 21 + 14(3) = 21 + 42 = 63$

Which term of the AP : 3, 9, 15, 21, . . . , is 99?

Solution: Here, $a = 3$, $d = 9 - 3 = 6$

We know that $a_n = a + (n - 1)d$

Let $a_n = 99 \Rightarrow a + (n - 1)d = 99$

$\Rightarrow 3 + (n - 1)6 = 99 \Rightarrow (n - 1)6 = 99 - 3 = 96$

$\Rightarrow n - 1 = \frac{96}{6} = 16 \Rightarrow n = 16 + 1 = 17$

Hence, 17th term of the given AP is 99

Determine the AP whose 3rd term is 5 and the 7th term is 9.

Solution: We have $a_3 = a + (3 - 1) d = a + 2d = 5$ (1)

and $a_7 = a + (7 - 1) d = a + 6d = 9$ (2)

Solving the pair of linear equations (1) and (2), we get $a = 3$, $d = 1$

Hence, the required AP is 3, 4, 5, 6, 7, . . .

Questions for practice

1. Find the 10th term of the AP : 2, 7, 12, . . .
2. Which term of the AP : 21, 18, 15, . . . is -81 ?
3. Which term of the AP : 3, 8, 13, 18, . . . , is 78?
4. How many two-digit numbers are divisible by 3?
5. How many three-digit numbers are divisible by 7?
6. How many multiples of 4 lie between 10 and 250?
7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.
8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.
9. If the 3rd and the 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?
10. Which term of the AP : 3, 15, 27, 39, . . . will be 132 more than its 54th term?
11. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.
12. The sum of 4th term and 8th term of an AP is 24 and the sum of 6th and 10th terms is 44. Find the AP.
13. The sum of 5th term and 9th term of an AP is 72 and the sum of 7th and 12th terms is 97. Find the AP.
14. If the numbers $n - 2$, $4n - 1$ and $5n + 2$ are in AP, find the value of n .
15. Find the value of the middle most term (s) of the AP : $-11, -7, -3, \dots, 49$.
16. The sum of the first three terms of an AP is 33. If the product of the first and the third term exceeds the second term by 29, find the AP.
17. The sum of the 5th and the 7th terms of an AP is 52 and the 10th term is 46. Find the AP.

18. Find the 20th term of the AP whose 7th term is 24 less than the 11th term, first term being 12.
19. If the 9th term of an AP is zero, prove that its 29th term is twice its 19th term.
20. Which term of the AP: 53, 48, 43,... is the first negative term?
21. A sum of Rs 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.
22. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

***n*th Term from the end of an ARITHMETIC PROGRESSION (AP)**

Let the last term of an AP be ' l ' and the common difference of an AP is ' d ' then the n th term from the end of an AP is given by

$$l_n = l - (n - 1) d.$$

IMPORTANT QUESTIONS

Find the 11th term from the last term (towards the first term) of the AP : 10, 7, 4, . . . , - 62.

Solution : Here, $a = 10$, $d = 7 - 10 = -3$, $l = -62$,

We know that n th term from the last is given by $l_n = l - (n - 1) d$.

$$\therefore l_{11} = l - 10d = -62 - 10(-3) = -62 + 30 = -32$$

Questions for practice

- Find the 20th term from the last term of the AP : 3, 8, 13, . . . , 253.
- Find the 10th term from the last term of the AP : 4, 9, 14, . . . , 254.
- Find the 6th term from the end of the AP 17, 14, 11, (-40).
- Find the 8th term from the end of the AP 7, 10, 13, 184.
- Find the 10th term from the last term of the AP : 8, 10, 12, . . . , 126.

Sum of First n Terms of an ARITHMETIC PROGRESSION (AP)

The sum of the first n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where a = first term, d = common difference and n = number of terms.

or

$$S_n = \frac{n}{2}[a + l]$$

where l = last term

IMPORTANT QUESTIONS

Find the sum of the first 22 terms of the AP : 8, 3, -2, . . .

Solution : Here, $a = 8$, $d = 3 - 8 = -5$, $n = 22$.

We know that $S = \frac{n}{2}[2a + (n-1)d]$

$$\therefore S = \frac{22}{2}[16 + (22-1)(-5)] = 11(16 - 105) = 11(-89) = -979$$

So, the sum of the first 22 terms of the AP is - 979.

Questions for practice

- If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.
- How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

3. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?
4. Find the sum of first 24 terms of the list of numbers whose n th term is given by $a_n = 3 + 2n$
5. Find the sum of the first 40 positive integers divisible by 6.
6. Find the sum of the first 15 multiples of 8.
7. Find the sum of the odd numbers between 0 and 50.
8. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.
9. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.
10. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.
11. If $a_n = 3 - 4n$, show that a_1, a_2, a_3, \dots form an AP. Also find S_{20} .
12. In an AP, if $S_n = n(4n + 1)$, find the AP.
13. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k .
14. If S_n denotes the sum of first n terms of an AP, prove that $S_{12} = 3(S_8 - S_4)$
15. Find the sum of first 17 terms of an AP whose 4th and 9th terms are -15 and -30 respectively.
16. If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.
17. Find the sum of all the 11 terms of an AP whose middle most term is 30.
18. Find the sum of last ten terms of the AP: 8, 10, 12, ---, 126.
19. How many terms of the AP: $-15, -13, -11, \dots$ are needed to make the sum -55 ? Explain the reason for double answer.
20. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x .
21. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find : (i) the production in the 1st year (ii) the production in the 10th year (iii) the total production in first 7 years
22. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?
23. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?
24. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.
25. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?
26. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, What is the total length of such a spiral made up of thirteen consecutive semicircles?
27. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

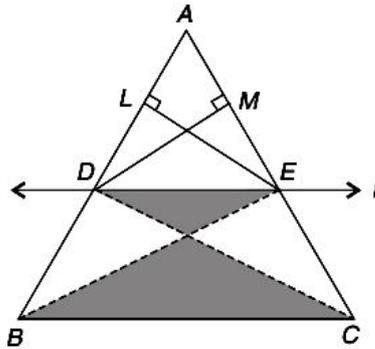
CHAPTER – 6 TRIANGLES

IMPORTANT THEOREMS

BASIC PROPORTIONALITY THEOREM OR THALES THEOREM

If a straight line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

GIVEN: A ΔABC and line ' l ' parallel to BC intersect AB at D and AC at E .



TO PROVE :

$$\frac{AD}{DB} = \frac{AE}{EC}$$

CONSTRUCTION : Join BE and CD . Draw $EL \perp$ to AB and $DM \perp$ to AC .

PROOF: We know that areas of the triangles on the same base and between same parallel lines are equal, hence we have :

$$\text{area} (\Delta BDE) = \text{area} (\Delta CDE) \quad \dots(i)$$

Now, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2} \times AD \times EL}{\frac{1}{2} \times DB \times EL} = \frac{AD}{DB} \quad \dots(ii)$$

Again, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \dots(iii)$$

Put value from (i) in (ii), we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{AD}{DB} \quad \dots(iv)$$

On comparing equation (ii) and (iii), we get

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{Hence Proved.}$$

COROLLARY :

$$(i) \frac{AB}{DB} = \frac{AC}{EC}$$

$$(ii) \frac{DB}{AD} = \frac{EC}{AE}$$

$$(iii) \frac{AB}{AD} = \frac{AC}{AE}$$

$$(iv) \frac{DB}{AB} = \frac{EC}{AC}$$

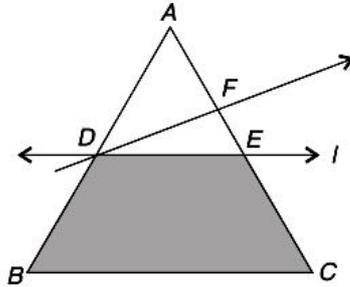
$$(v) \frac{AD}{AB} = \frac{AE}{AC}$$

**CONVERSE OF BASIC PROPORTIONALITY THEOREM
(CONVERSE OF THALES THEOREM)**

If a straight line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

GIVEN : A ΔABC and line 'l' intersecting the sides AB at D and AC at E such that :

$$\frac{AD}{DB} = \frac{AE}{EC}$$



TO PROVE : $l \parallel BC$.

PROOF : Let us suppose that the line l is not parallel to BC.

Then through D, there must be any other line which must be parallel to BC.

Let $DF \parallel BC$, such that $E \neq F$.

Since,

$$DF \parallel BC \quad \text{(by supposition)}$$

$$\frac{AD}{DB} = \frac{AF}{FC} \quad \dots(i) \text{ (Basic Proportionality Theorem)}$$

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \dots(ii) \quad \text{(Given)}$$

Comparing (i) and (ii), we get

$$\frac{AF}{FC} = \frac{AE}{EC}$$

Adding 1 to both sides, we get

$$\frac{AF}{FC} + 1 = \frac{AE}{EC} + 1$$

$$\Rightarrow \frac{AF + FC}{FC} = \frac{AE + EC}{EC}$$

$$\Rightarrow \frac{AC}{FC} = \frac{AC}{EC}$$

$$\Rightarrow \frac{1}{FC} = \frac{1}{EC}$$

$$\Rightarrow FC = EC$$

This shows that E and F must coincide, but it contradicts our supposition that $E \neq F$ and $DF \parallel BC$.

Hence, there is one and only line, $DE \parallel BC$, i.e.

$$\boxed{l \parallel BC}$$

Hence Proved.

AREAS OF SIMILAR TRIANGLES THEOREM

The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

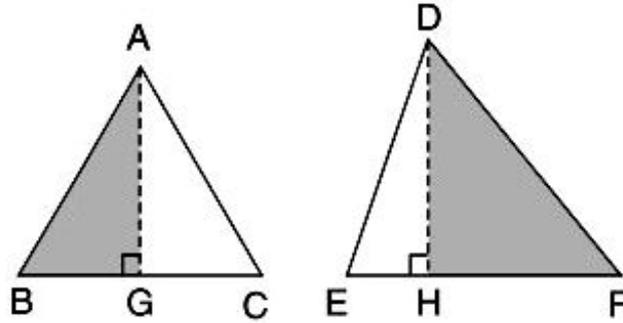
GIVEN :

$$\Delta ABC \sim \Delta DEF$$

TO PROVE :

$$\frac{\text{Area of } (\Delta ABC)}{\text{Area of } (\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$$

CONSTRUCTION : Draw $AG \perp BC$ and $DH \perp EF$.



PROOF :

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{\frac{1}{2} \times BC \times AG}{\frac{1}{2} \times EF \times DH} = \frac{BC}{EF} \times \frac{AG}{DH} \quad \dots(i)$$

(area of $\Delta = \frac{1}{2} \times \text{base} \times \text{height}$)

Now in triangle ABG and DEH , we have

$$\angle B = \angle E \quad \text{(since, } \Delta ABC \sim \Delta DEF \text{)}$$

$$\angle AGB = \angle DHE \quad \text{(each } 90^\circ \text{)}$$

Therefore, $\Delta ABG \sim \Delta DEH$ (by AA criterion)

Hence, $\frac{AB}{DE} = \frac{AG}{DH}$... (ii) (Using property of similar triangles)

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$... (iii) (since, $\Delta ABC \sim \Delta DEF$)

Comparing (ii) and (iii), we get

$$\frac{AG}{DH} = \frac{BC}{EF} \quad \dots(iv)$$

Using (i) and (iv), we get

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{BC}{EF} \times \frac{BC}{EF} = \frac{BC^2}{EF^2} \quad \dots(v)$$

$$\frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF} \quad \dots(vi) \text{ (since, } \Delta ABC \sim \Delta DEF \text{)}$$

Using (v) and (vi), we get

$$\boxed{\frac{\text{Area of } (\Delta ABC)}{\text{Area of } (\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}}$$

Hence Proved.

PYTHAGORAS THEOREM

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

GIVEN : ΔABC is right angled at B .

TO PROVE : $AC^2 = AB^2 + BC^2$.

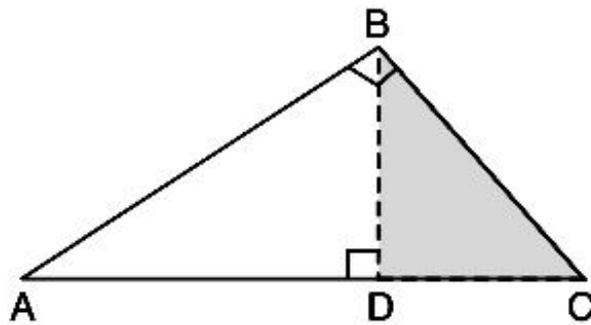
CONSTRUCTION : Draw $BD \perp AC$.

PROOF : Taking ΔADB and ΔABC

$$\angle B = \angle ADB \quad \text{(each } 90^\circ \text{)}$$

$$\angle A = \angle A \quad \text{(common)}$$

Therefore, $\Delta ADB \sim \Delta ABC$ (by AA criterion)



Hence,

$$\frac{AD}{AB} = \frac{AB}{AC}$$

⇒

$$AB^2 = AD \times AC$$

...(i)

Now taking ΔCDB and ΔCBA

$$\angle B = \angle BDC$$

(each 90°)

$$\angle C = \angle C$$

(common)

Therefore,

$$\Delta CDB \sim \Delta CBA$$

(by AA criterion)

Hence,

$$\frac{CD}{BC} = \frac{BC}{AC}$$

⇒

$$BC^2 = CD \times AC$$

...(ii)

Adding (i) and (ii), we get

⇒

$$AB^2 + BC^2 = AD \times AC + CD \times AC$$

⇒

$$AB^2 + BC^2 = AC \times (AD + CD)$$

⇒

$$AB^2 + BC^2 = AC \times AC$$

$$\boxed{AC^2 = AB^2 + BC^2}$$

Hence Proved.

Converse of Pythagoras theorem

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

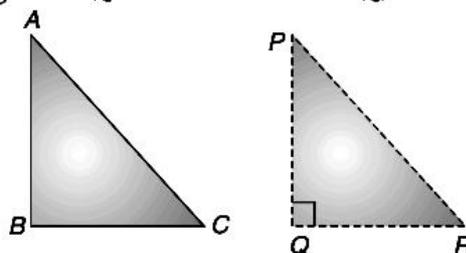
GIVEN :

$$AC^2 = AB^2 + BC^2$$

...(i)

TO PROVE : ΔABC is right angled at B.

CONSTRUCTION : Draw right ΔPQR such that $AB = PQ$, $BC = QR$ and $\angle Q = 90^\circ$.



PROOF : Using Pythagoras theorem in ΔPQR , we get

$$PR^2 = PQ^2 + QR^2$$

...(ii)

By construction,

$$AB = PQ$$

$$BC = QR, \text{ substituting these values in (ii), we get}$$

$$PR^2 = AB^2 + BC^2$$

...(iii)

Comparing (i) and (iii), we get

$$AC^2 = PR^2$$

⇒

$$AC = PR$$

...(iv)

In $\triangle ABC$ and $\triangle PQR$

$$AB = PQ \quad (\text{by construction})$$

$$BC = QR \quad (\text{by construction})$$

$$AC = PR \quad (\text{proved above in (iv)})$$

$$\Rightarrow \triangle ABC = \triangle PQR \quad (\text{by SSS congruence rule})$$

$$\Rightarrow \angle B = \angle Q \quad (\text{by cpct})$$

$$\text{But } \angle Q = 90^\circ \quad (\text{by construction})$$

$$\text{Hence, } \angle B = 90^\circ$$

$\triangle ABC$ is right angled at B .

Hence Proved.



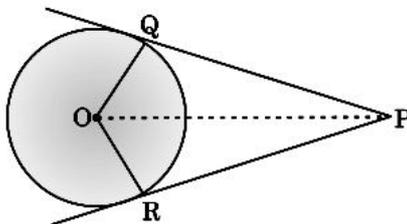
CHAPTER – 10 CIRCLES

THEOREMS

- 1) The tangent to a circle is perpendicular to the radius through the point of contact.
- 2) The lengths of tangents drawn from an external point to a circle are equal.

Given : A circle $C(O, r)$ and two tangents say PQ and PR from an external point P .

To prove : $PQ = PR$.



Construction : Join OQ , OR and OP .

Proof : In $\triangle OQP$ and $\triangle ORP$

$$OQ = OR$$

(radii of the same circle)

$$OP = OP$$

(Common)

$$\angle Q = \angle R = \text{each } 90^\circ \text{ (The tangent at any point of a circle is perpendicular to the radius through the point of contact)}$$

$$\text{Hence } \triangle OQP \cong \triangle ORP$$

(By RHS Criterion)

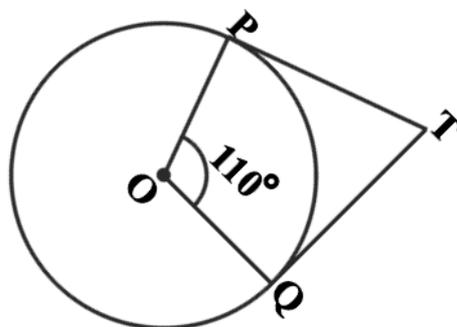
$$\therefore PQ = PR$$

(By CPCT)

Hence Proved.

IMPORTANT QUESTIONS

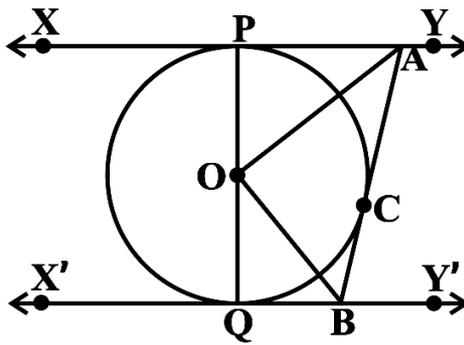
1. From a point Q , the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle
2. In the below figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then find $\angle PTQ$.



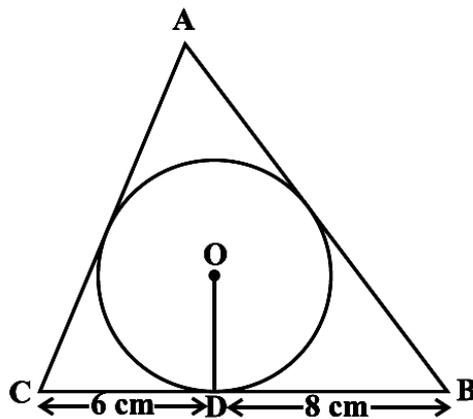
3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80° , then find $\angle POA$
4. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.
5. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
6. A quadrilateral $ABCD$ is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$
7. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.
8. Prove that the parallelogram circumscribing a circle is a rhombus.
9. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

10. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

11. XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and $X'Y'$ at B . Prove that $\angle AOB = 90^\circ$.



12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC .



13. Two tangents TP and TQ are drawn to a circle with centre O from an external point T . Prove that $\angle PTQ = 2 \angle OPQ$.

14. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T . Find the length TP .

15. Two tangents PQ and PR are drawn from an external point to a circle with centre O . Prove that $QORP$ is a cyclic quadrilateral.

16. If from an external point B of a circle with centre O , two tangents BC and BD are drawn such that $\angle DBC = 120^\circ$, prove that $BC + BD = BO$, i.e., $BO = 2BC$.

17. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

18. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A .

19. From an external point P , two tangents, PA and PB are drawn to a circle with centre O . At one point E on the circle tangent is drawn which intersects PA and PB at C and D , respectively. If $PA = 10$ cm, find the perimeter of the triangle PCD .

20. In a right triangle ABC in which $\angle B = 90^\circ$, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P . Prove that the tangent to the circle at P bisects BC .

CHAPTER – 11 CONSTRUCTIONS

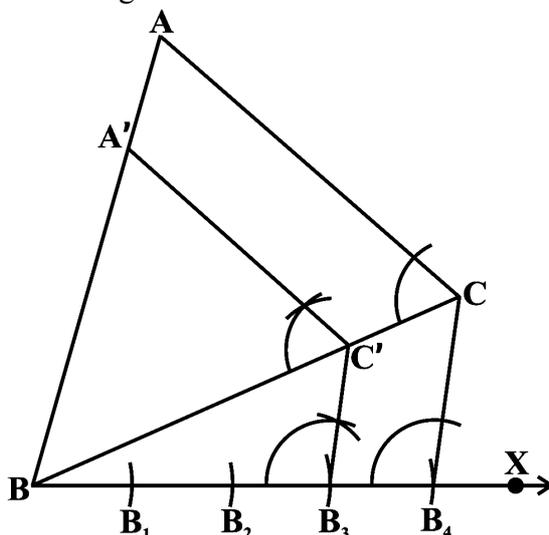
CONSTRUCTION OF SIMILAR TRIANGLE

Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{3}{4}$ of the corresponding sides of the triangle ABC (i.e., of scale factor $\frac{3}{4}$).

Steps of Construction :

- ☞ Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
- ☞ Locate 4 (the greater of 3 and 4 in $\frac{3}{4}$) points B_1, B_2, B_3 and B_4 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
- ☞ Join B_4C and draw a line through B_3 (the 3rd point, 3 being smaller of 3 and 4 in $\frac{3}{4}$) parallel to B_4C to intersect BC at C' .
- ☞ Draw a line through C' parallel to the line CA to intersect BA at A' (see below figure).

Then, $\Delta A'BC'$ is the required triangle.



Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{5}{3}$ of the corresponding sides of the triangle ABC (i.e., of scale factor $\frac{5}{3}$).

Steps of Construction :

- Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
- Locate 5 points (the greater of 5 and 3 in $\frac{5}{3}$) B_1, B_2, B_3, B_4 and B_5 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
- Join B_3 (the 3rd point, 3 being smaller of 3 and 5 in $\frac{5}{3}$) to C and draw a line through B_5 parallel to B_3C , intersecting the extended line segment BC at C' .
- Draw a line through C' parallel to CA intersecting the extended line segment BA at A' (see the below figure).
Then $A'BC'$ is the required triangle.

To construct the tangents to a circle from a point outside it.

Given : We are given a circle with centre 'O' and a point P outside it. We have to construct two tangents from P to the circle.

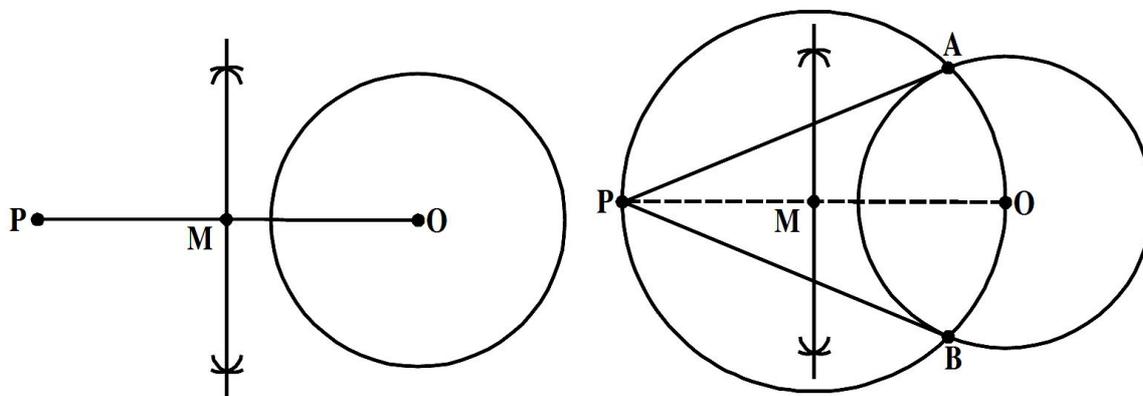
Steps of construction :

☞ Join PO and draw a perpendicular bisector of it. Let M be the midpoint of PO.

☞ Taking M as centre and PM or MO as radius, draw a circle. Let it intersect the given circle at the points A and B.

☞ Join PA and PB.

Then PA and PB are the required two tangents.



IMPORTANT QUESTIONS FOR PRACTICE

1. Construct an isosceles triangle whose base is 7 cm and altitude 4 cm and then construct another similar triangle whose sides are $\frac{3}{2}$ time the corresponding sides of the isosceles triangle.
2. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.
3. Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^\circ$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.
4. Draw a triangle ABC with side BC = 7 cm, $\angle B = 45^\circ$, $\angle A = 105^\circ$. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of $\triangle ABC$.
5. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.
6. Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.
7. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.
8. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.
9. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60° .
10. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

CHAPTER – 12

AREAS RELATED TO CIRCLES

AREA AND PERIMETER OF CIRCLE, QUADRANT, SEMICIRCLE

Area of Circle = πr^2 , Perimeter of Circle = Circumference = $2\pi r$

Area of Semicircle = $\frac{1}{2}\pi r^2$, Perimeter of Semicircle = $\pi r + 2r$

Area of Quadrant = $\frac{1}{4}\pi r^2$, Perimeter of Quadrant = $\frac{1}{2}\pi r + 2r$

IMPORTANT QUESTIONS

Find the diameter of the circle whose area is equal to the sum of the areas of the two circles of diameters 20 cm and 48 cm.

Solution: Here, radius r_1 of first circle = $20/2$ cm = 10 cm

and radius r_2 of the second circle = $48/2$ cm = 24 cm

Therefore, sum of their areas = $\pi r_1^2 + \pi r_2^2 = \pi (10)^2 + \pi (24)^2 = \pi \times 676$

Let the radius of the new circle be r cm. Its area = πr^2

Therefore, $\pi r^2 = \pi \times 676 \Rightarrow r^2 = 676 \Rightarrow r = 26$

Thus, radius of the new circle = 26 cm

Hence, diameter of the new circle = 2×26 cm = 52 cm

Questions for Practice

- The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.
- The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.
- The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?
- Find the area of a quadrant of a circle whose circumference is 22 cm.

AREAS OF SECTOR AND SEGMENT OF A CIRCLE

Area of the sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$, where r is the radius of the circle and θ the angle of the sector in degrees

length of an arc of a sector of angle $\theta = \frac{\theta}{360^\circ} \times 2\pi r$, where r is the radius of the circle and θ the angle of the sector in degrees

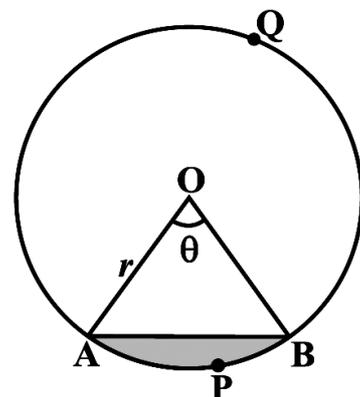
Area of the segment APB = Area of the sector OAPB – Area of Δ OAB

$$= \frac{\theta}{360^\circ} \times \pi r^2 - \text{area of } \Delta \text{ OAB}$$

☞ Area of the major sector OAQB = $\pi r^2 - \text{Area of the minor sector OAPB}$

☞ Area of major segment AQB = $\pi r^2 - \text{Area of the minor segment APB}$

☞ Area of segment of a circle = Area of the corresponding sector – Area of the corresponding triangle



IMPORTANT QUESTIONS

Find the area of the sector of a circle with radius 4 cm and of angle 30° . Also, find the area of the corresponding major sector (Use $\pi = 3.14$).

Solution : Here, radius, $r = 4$ cm, $\theta = 30^\circ$,

$$\begin{aligned}\text{We know that Area of sector} &= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{30^\circ}{360^\circ} \times 3.14 \times 4 \times 4 = \frac{1}{12} \times 3.14 \times 4 \times 4 \\ &= \frac{12.56}{3} = 4.19 \text{ cm}^2 \text{ (approx.)}\end{aligned}$$

Area of the corresponding major sector

$$= \pi r^2 - \text{area of sector OAPB}$$

$$= (3.14 \times 16 - 4.19) \text{ cm}^2$$

$$= 46.05 \text{ cm}^2 = 46.1 \text{ cm}^2 \text{ (approx.)}$$

A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding : (i) minor segment (ii) major sector. (Use $\pi = 3.14$)

Solutions: Here, radius, $r = 10$ cm, $\theta = 90^\circ$,

$$\text{We know that Area of minor sector} = \frac{\theta}{360^\circ} \times \pi r^2 = \frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10 = \frac{1}{4} \times 314 = 78.5 \text{ cm}^2$$

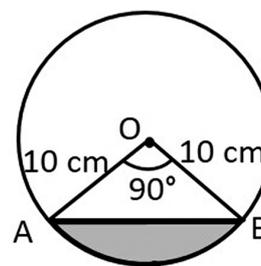
$$\text{and Area of triangle AOB} = \frac{1}{2} \times b \times h = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$$

Area of minor segment = Area of minor sector -

$$\text{Area of triangle AOB} = 78.5 - 50 = 28.5 \text{ cm}^2.$$

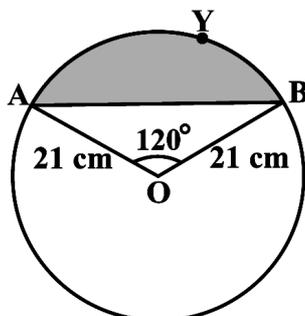
$$\text{Area of circle} = \pi r^2 = 3.14 \times 10 \times 10 = 314 \text{ cm}^2$$

$$\begin{aligned}\text{Area of major sector} &= \text{Area of circle} - \text{Area of minor sector} \\ &= 314 - 78.5 = 235.5 \text{ cm}^2\end{aligned}$$



Questions for Practice

1. Find the area of the segment AYB shown in below figure, if radius of the circle is 21 cm and $\angle AOB = 120^\circ$.



2. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60° .
3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.
4. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find (i) the area of that part of the field in which the horse can graze. (ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use $\pi = 3.14$)
5. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors. Find : (i) the total length of the silver wire required. (ii) the area of each sector of the brooch.
6. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find: (i) the length of the arc (ii) area of the sector formed by the arc (iii) area of the segment formed by the corresponding chord

7. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)
8. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)
9. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades.
10. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)

AREA OF SHADED REGION BASED QUESTIONS

IMPORTANT QUESTIONS

In the adjoining figure, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and the flower beds.

Solution: Here, side of square ABCD, $a = 56$ m

diagonal of square = $a\sqrt{2} = 56\sqrt{2}$

radius, $r = OA = OB = OC = OD = \frac{56\sqrt{2}}{2} = 28\sqrt{2}$ cm

Now, Area of sector OAB = Area of sector ODC

$$= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times r^2 = \frac{1}{4} \times \frac{22}{7} \times r^2$$

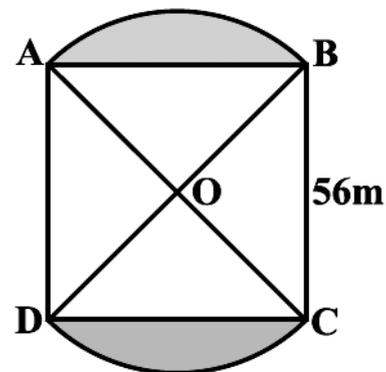
and Area of $\triangle OAD =$ Area of $\triangle OBC = \frac{1}{2} \times r \times r = \frac{1}{2} \times r^2$

Total area = Area of sector OAB + Area of sector ODC
+ Area of $\triangle OAD$ + Area of $\triangle OBC$

$$= \frac{1}{4} \times \frac{22}{7} \times r^2 + \frac{1}{4} \times \frac{22}{7} \times r^2 + \frac{1}{2} \times r^2 + \frac{1}{2} \times r^2$$

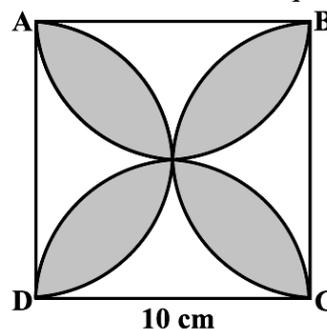
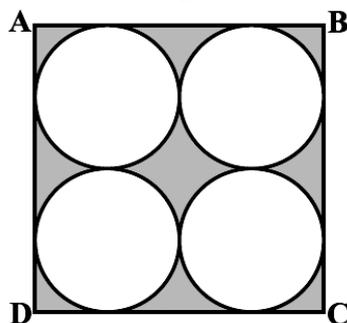
$$= 2 \times \frac{1}{4} \times \frac{22}{7} \times r^2 + 2 \times \frac{1}{2} \times r^2 = \frac{11}{7} \times r^2 + r^2 = \left(\frac{11}{7} + 1\right) r^2$$

$$= \frac{18}{7} \times 28 \times 28 \times 2 = 4032 \text{ cm}^2$$



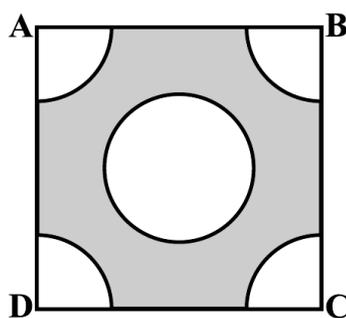
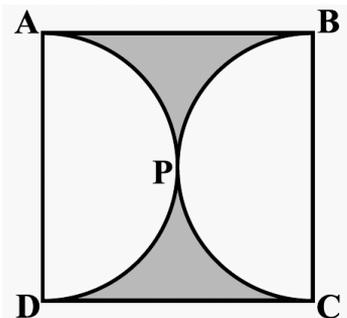
Questions for Practice

1. Find the area of the shaded region in below left figure, where ABCD is a square of side 14 cm.

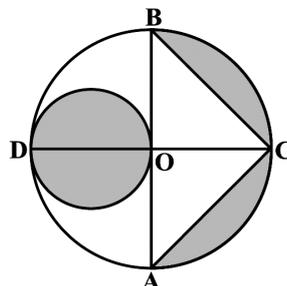
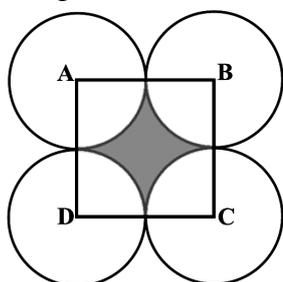


2. Find the area of the shaded design in above right figure, where ABCD is a square of side 10 cm and semicircles are drawn with each side of the square as diameter. (Use $\pi = 3.14$)

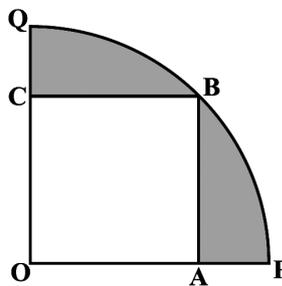
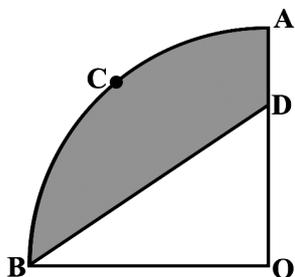
3. Find the area of the shaded region in below left figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.



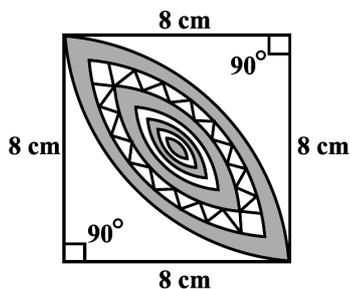
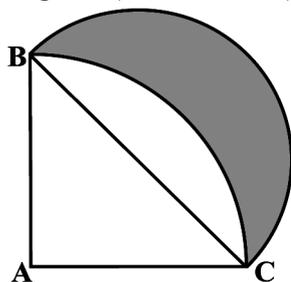
4. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in above right sided figure. Find the area of the remaining portion of the square.
5. In the below left figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.



6. In the above right sided figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.
7. In the below left figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.

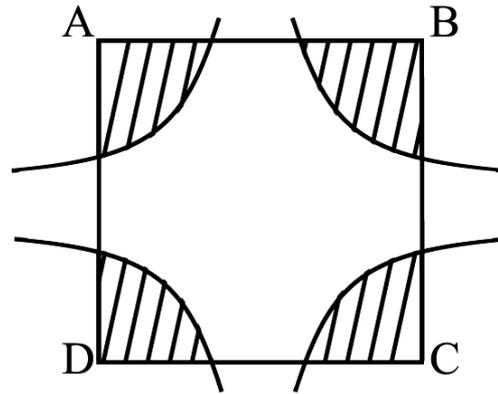
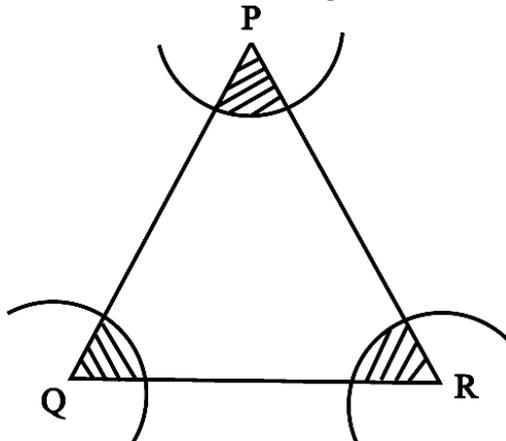


8. In the above right sided figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the (i) quadrant OACB, (ii) shaded region.
9. In the below figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use $\pi = 3.14$)

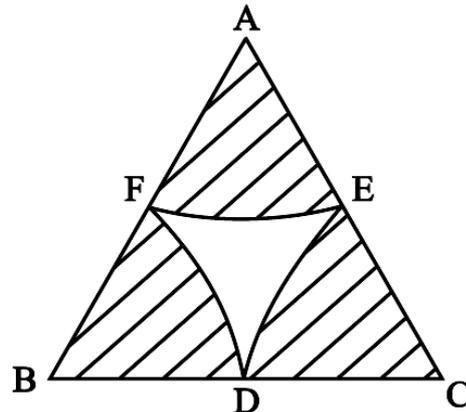
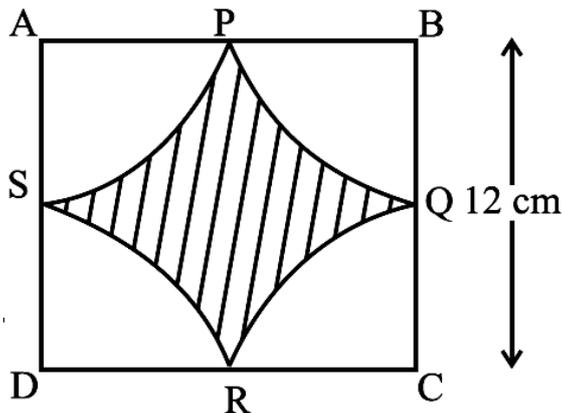


10. Calculate the area of the designed region in above right sided figure, common between the two quadrants of circles of radius 8 cm each.

11. In the below figure, arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.



12. In the above right sided figure, arcs have been drawn of radius 21 cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of the shaded region.
13. A circular park is surrounded by a road 21 m wide. If the radius of the park is 105 m, find the area of the road.
14. Find the area of the shaded region in the below figure, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA, respectively of a square ABCD (Use $\pi = 3.14$).



15. In the above right sided figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm. to intersect the sides BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region (Use $\pi = 3.14$).

CHAPTER – 14 STATISTICS

MEAN OF GROUPED DATA

Direct method

$$\text{Mean, } \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Assume mean method or Short-cut method

$$\text{Mean, } \bar{x} = A + \frac{\sum f_i d_i}{\sum f_i} \text{ where } d_i = x_i - A$$

Step Deviation method

$$\text{Mean, } \bar{x} = A + \frac{\sum f_i u_i}{\sum f_i} \times h \text{ where } u = \frac{x_i - A}{h}$$

IMPORTANT QUESTIONS

The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95
Number of cities	3	10	11	8	3

Solution:

Literacy rate (in %)	Number of Cities 'f'	Class mark 'x'	$u = \frac{x - A}{h}$	fu
45 – 55	3	50	-2	-6
55 – 65	10	60	-1	-10
65 – 75	11	70	0	0
75 – 85	8	80	1	8
85 – 95	3	90	2	6
Total	35			-2

Here, $\sum fu = -2$, $\sum f = 35$, $A = 70$, $h = 10$

$$\text{Mean, } \bar{x} = A + \frac{\sum fu}{\sum f} \times h = \Rightarrow \bar{x} = 70 + \frac{-2}{35} \times 10 = 70 - \frac{20}{35} = 70 - \frac{4}{7} = 70 - 0.57 \Rightarrow \bar{x} = 69.43$$

Questions for Practice

1. Find the mean of the following data:

Class Interval	10 – 25	25 – 40	40 – 55	55 – 70	70 – 85	85 – 100
Frequency	2	3	7	6	6	6

2. Find the mean percentage of female teachers of the following data:

Percentage of female teachers	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75	75 – 85
Number of States/U.T	6	11	7	4	4	2	1

3. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
Number of houses	1	2	1	5	6	2	3

4. Find the mean daily wages of the workers of the factory by using an appropriate method for the following data:

Daily wages (in Rs)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

5. Find the mean number of mangoes kept in a packing box for the following data:

Number of mangoes	50 – 52	53 – 55	56 – 58	59 – 61	62 – 64
Number of boxes	15	110	135	115	25

6. Find the mean daily expenditure on food for the following data:

Daily expenditure (in Rs.)	100 – 150	150 – 200	200 – 250	250 – 300	300 – 350
Number of households	4	5	12	2	2

MODE OF GROUPED DATA

$$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where l = lower limit of the modal class,

h = size of the class interval (assuming all class sizes to be equal),

f_1 = frequency of the modal class,

f_0 = frequency of the class preceding the modal class,

f_2 = frequency of the class succeeding the modal class.

IMPORTANT QUESTIONS

Find the mean, mode and median for the following frequency distribution.

Class	0-10	10-20	20-30	30-40	40-50	Total
Frequency	8	16	36	34	6	100

Solution:

Here, highest frequency is 36 which belongs to class 20 – 30. So, modal class is 20 – 30,
 $l = 20$, $f_0 = 16$, $f_1 = 36$, $f_2 = 34$, $h = 10$

We know that $Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

$$\Rightarrow Mode = 20 + \frac{36 - 16}{2(36) - 16 - 34} \times 10$$

$$\Rightarrow Mode = 20 + \frac{20}{72 - 50} \times 10 = 20 + \frac{200}{22} = 20 + 9.09 = 29.09$$

Questions for Practice

1. The frequency distribution table of agriculture holdings in a village is given below:

Area of land(in ha)	1-3	3-5	5-7	7-9	9-11	11-13
No. of families	20	45	80	55	40	12

Find the modal agriculture holdings of the village.

2. Find the mode age of the patients from the following distribution :

Age(in years)	6-15	16-25	26-35	36-45	46-55	56-65
No. of patients	6	11	21	23	14	5

3. Find the mode of the following frequency distribution:

Class	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

4. Find the modal height of maximum number of students from the following distribution:

Height(in cm)	160-162	163-165	166-168	169-171	172-174
No. of students	15	118	142	127	18

5. A survey regarding the heights (in cms) of 50 girls of a class was conducted and the following data was obtained.

Height(in cm)	120-130	130-140	140-150	150-160	160-170	Total
No. of girls	2	8	12	20	8	50

Find the mode of the above data.

- **Cumulative Frequency:** The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class.

MEDIAN OF GROUPED DATA

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

where l = lower limit of median class,

n = number of observations,

cf = cumulative frequency of class preceding the median class,

f = frequency of median class,

h = class size (assuming class size to be equal).

EMPIRICAL FORMULA

$$3\text{Median} = \text{Mode} + 2\text{Mean}$$

IMPORTANT QUESTIONS

Find the median of the following frequency distribution:

Class	75-84	85-94	95-104	105-114	115-124	125-134	135-144
Frequency	8	11	26	31	18	4	2

Solution:

Class	True Class limits	Frequency	cf
75-84	74.5 – 84.5	8	8
85-94	84.5 – 94.5	11	19
95-104	94.5 – 104.5	26	45
105-114	104.5 – 114.5	31	76
115-124	114.5 – 124.5	18	94
125-134	124.5 – 134.5	4	98
135-144	134.5 – 144.5	2	100
Total		100	

Here, $n = 100 \Rightarrow \frac{n}{2} = 50$ which belongs to 104.5 – 114.5

So, $l = 104.5$, $cf = 45$, $f = 31$, $h = 10$

We know that $Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$

$$\Rightarrow Median = 104.5 + \frac{50 - 45}{31} \times 10 \Rightarrow Median = 104.5 + \frac{50}{31} = 104.5 + 1.61 = 106.11$$

Questions for Practice

1. The percentage of marks obtained by 100 students in an examination are given below:

Marks	30-35	35-40	40-45	45-50	50-55	55-60	60-65
No. of Students	14	16	18	23	18	8	3

Determine the median percentage of marks.

2. Weekly income of 600 families is as under:

Income(in Rs.)	0-1000	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000
No. of Families	250	190	100	40	15	5

Compute the median income.

3. Find the median of the following frequency distribution:

Marks	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40
Number of students	8	12	20	12	18	13	10	7

4. The following table gives the distribution of the life time of 500 neon lamps:

Life time (in hrs)	1500 – 2000	2000 – 2500	2500 – 3000	3000 – 3500	3500 – 4000	4000 – 4500	4500 – 5000
Number of Lamps	24	86	90	115	95	72	18

Find the median life time of a lamp.

5. Find the median marks for the following distribution:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of Students	6	15	29	41	60	70

6. Find the median wages for the following frequency distribution:

Wages per day	61-70	71-80	81-90	91-100	101-110	111-120
No. of workers	5	15	20	30	10	8

7. Find the median marks for the following distribution:

Marks	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
No. of Students	2	3	6	7	14	12	4	2

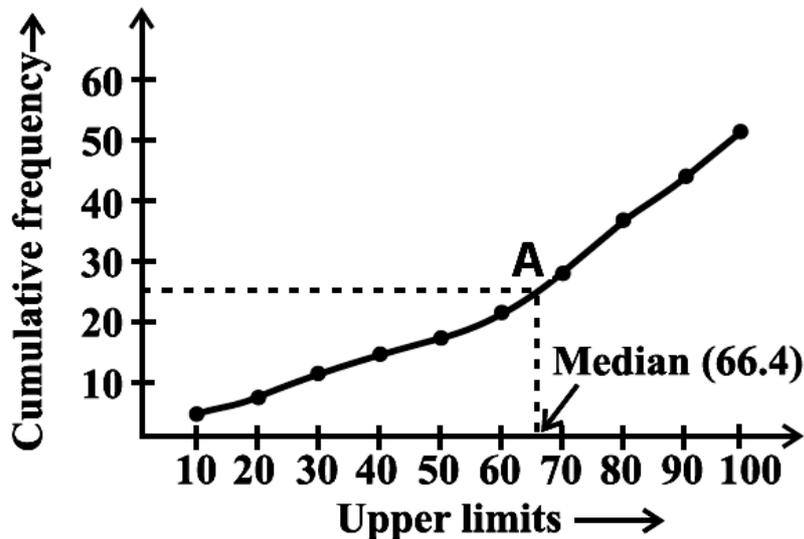
❖ CUMULATIVE FREQUENCY CURVE IS ALSO KNOWN AS 'OGIVE'.

There are three methods of drawing ogive:

1. LESS THAN METHOD

Steps involved in calculating median using less than Ogive approach-

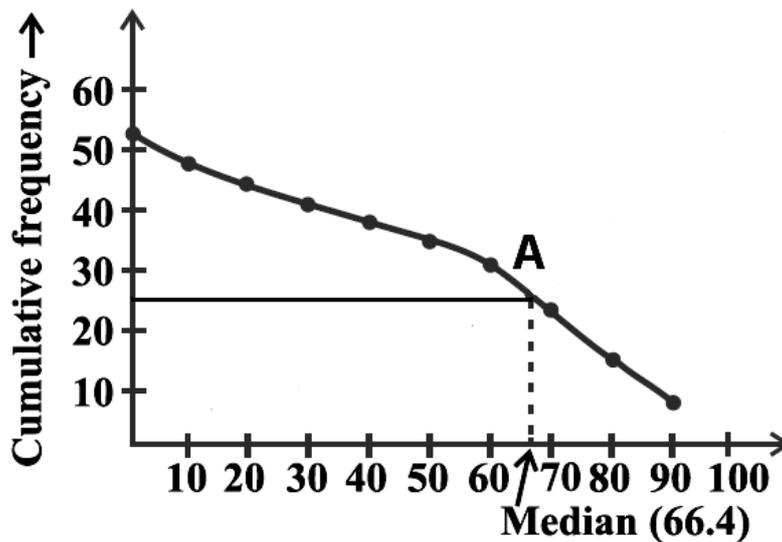
- Convert the series into a 'less than' cumulative frequency distribution.
- Let N be the total number of students whose data is given. N will also be the cumulative frequency of the last interval. Find the $(N/2)^{\text{th}}$ item and mark it on the y-axis.
- Draw a perpendicular from that point to the right to cut the Ogive curve at point A.
- From point A where the Ogive curve is cut, draw a perpendicular on the x-axis. The point at which it touches the x-axis will be the median value of the series as shown in the graph.



2. MORE THAN METHOD

Steps involved in calculating median using more than Ogive approach-

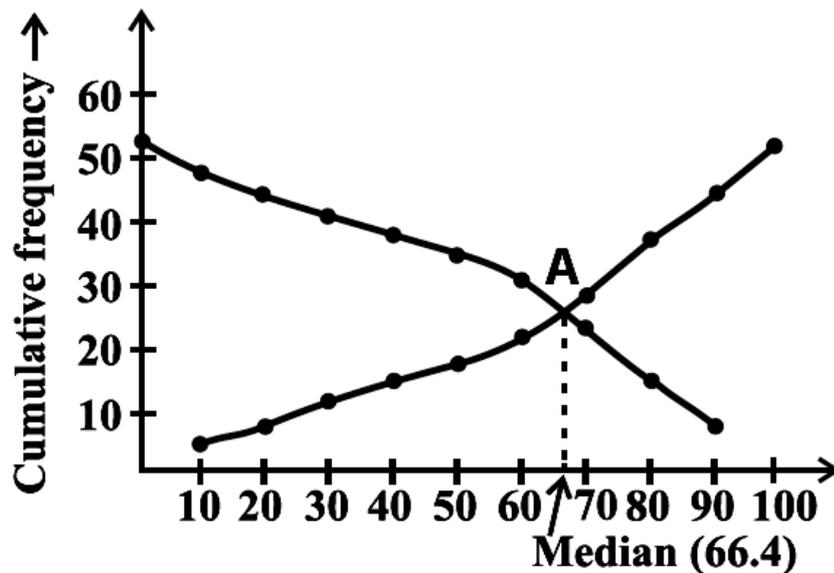
- Convert the series into a 'more than' cumulative frequency distribution.
- Let N be the total number of students whose data is given. N will also be the cumulative frequency of the last interval. Find the $(N/2)^{\text{th}}$ item and mark it on the y-axis.
- Draw a perpendicular from that point to the right to cut the Ogive curve at point A.
- From point A where the Ogive curve is cut, draw a perpendicular on the x-axis. The point at which it touches the x-axis will be the median value of the series as shown in the graph.



3. LESS THAN AND MORE THAN OGIVE METHOD

Another way of graphical determination of median is through simultaneous graphic presentation of both the less than and more than Ogives.

- Mark the point A where the Ogive curves cut each other.
- Draw a perpendicular from A on the x-axis. The corresponding value on the x-axis would be the median value.



- ❖ The median of grouped data can be obtained graphically as the x -coordinate of the point of intersection of the two ogives for this data.

IMPORTANT QUESTIONS

The following distribution gives the daily income of 50 workers of a factory.

Daily income (in Rs)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

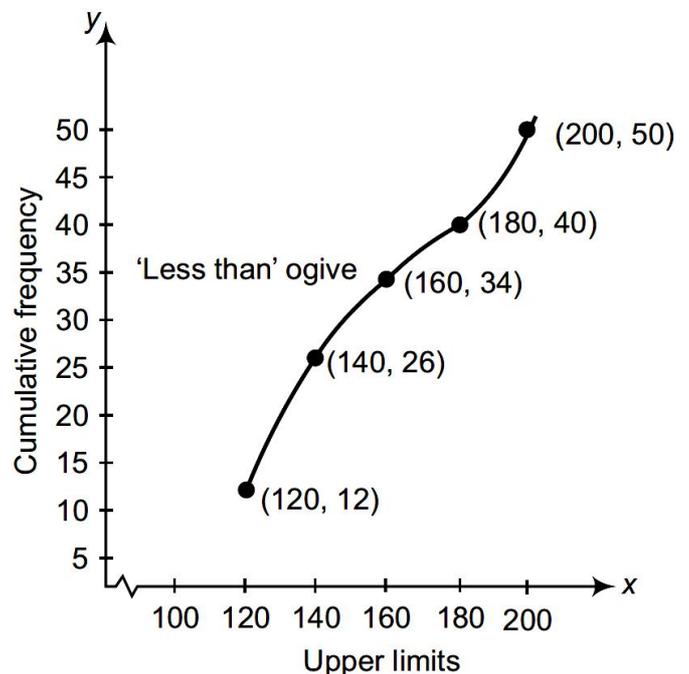
Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

Solution:

Cumulative frequency less than type

Daily income (in Rs)	Less than type cf
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

On the graph, we will plot the points (120, 12), (140, 26), (160, 34), (180, 40) and (200, 50).



Questions for Practice

- The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (in kg/ha)	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution, and draw its ogive.

2. For the following distribution, draw the cumulative frequency curve more than type and hence obtain the median from the graph.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

3. Draw less than ogive for the following frequency distribution:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	5	8	6	10	6	6

Also find the median from the graph and verify that by using the formula.

4. The table given below shows the frequency distribution of the cores obtained by 200 candidates in a BCA examination.

Score	200-250	250-300	300-350	350-400	400-450	450-500	500-550	550-600
No. of students	30	15	45	20	25	40	10	15

Draw cumulative frequency curves by using (i) less than type and (ii) more than type. Hence find median

5. Draw less than and more than ogive for the following frequency distribution:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	8	5	10	6	6	6

Also find the median from the graph and verify that by using the formula.

CHAPTER – 15 PROBABILITY

PROBABILITY

The theoretical probability (also called classical probability) of an event A, written as P(A), is defined as

$$P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Number of all possible outcomes of the experiment}}$$

COMPLIMENTARY EVENTS AND PROBABILITY

We denote the event 'not E' by \bar{E} . This is called the **complement** event of event E.

So, $P(E) + P(\bar{E}) = 1$

i.e., $P(E) + P(\bar{E}) = 1$, which gives us $P(\bar{E}) = 1 - P(E)$.

- ☞ The probability of an event which is impossible to occur is 0. Such an event is called an **impossible event**.
- ☞ The probability of an event which is sure (or certain) to occur is 1. Such an event is called a **sure event** or a **certain event**.
- ☞ The probability of an event E is a number P(E) such that $0 \leq P(E) \leq 1$
- ☞ An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.

DECK OF CARDS AND PROBABILITY

A deck of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each. They are black spades (♠) red hearts (♥), red diamonds (♦) and black clubs (♣).

The cards in each suit are Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2. Kings, Queens and Jacks are called face cards.

Example set of 52 poker playing cards

Suit	Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
Clubs													
Diamonds													
Hearts													
Spades													

IMPORTANT QUESTIONS

Two dice are thrown together. Find the probability that the sum of the numbers on the top of the dice is (i) 9 (ii) 10

Solution:

Here, total number of outcomes, $n(s) = 36$

(i) Let A be the event of getting the sum of the numbers on the top of the dice is 9 then we have $n(A) = 4$ i.e. (3, 6), (4, 5), (5, 4), (6, 3)

Therefore, Probability of getting the sum of the numbers on the top of the dice is 9, $P(A) = \frac{n(A)}{n(S)}$

$$\Rightarrow P(A) = \frac{4}{36} = \frac{1}{9}$$

(ii) Let B be the event of getting the sum of the numbers on the top of the dice is 10 then we have $n(B) = 3$ i.e. (4, 6), (5, 5), (6, 4)

Therefore, Probability of getting the sum of the numbers on the top of the dice is 10, $P(B) = \frac{n(B)}{n(S)}$

$$\Rightarrow P(B) = \frac{3}{36} = \frac{1}{12}$$

One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) red colour ace card (ii) a face card or a spade card (iii) a black face card

Solution:

Here, total number of outcomes, $n(s) = 52$

(i) Let A be the event of getting red colour ace card and we know that the number of red ace card is 2 then we have, $n(A) = 2$

Therefore, Probability of getting red colour ace card, $P(A) = \frac{n(A)}{n(S)}$

$$\Rightarrow P(A) = \frac{2}{52} = \frac{1}{26}$$

(ii) Let B be the event of getting a face card or a spade card and we know that there are 12 face cards, 13 spade cards and 3 face cards are spade then we have, $n(B) = 12 + 13 - 3 = 22$

Therefore, Probability of getting a face card or a spade card, $P(B) = \frac{n(B)}{n(S)}$

$$\Rightarrow P(B) = \frac{22}{52} = \frac{11}{26}$$

(ii) Let B be the event of getting a black face card and we know that there are 6 face cards are black then we have, $n(C) = 6$

Therefore, Probability of getting a black face card, $P(C) = \frac{n(C)}{n(S)}$

$$\Rightarrow P(C) = \frac{6}{52} = \frac{3}{26}$$

Questions for Practice

- Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is (i) 6 (ii) 12 (iii) 7
- A die is thrown twice. What is the probability that (i) 5 will not come up either time? (ii) 5 will come up at least once?
- A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that (i) She will buy it ? (ii) She will not buy it ?

4. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour (ii) a face card (iii) a red face card (iv) the jack of hearts (v) a spade (vi) the queen of diamonds
5. Five cards—the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random. (i) What is the probability that the card is the queen? (ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?
6. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.
7. A piggy bank contains hundred 50p coins, fifty Re 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 p coin ? (ii) will not be a Rs 5 coin?
8. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red ? (ii) white ? (iii) not green?
9. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?
(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective ?
10. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.
11. A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Jimmy, a trader, will only accept the shirts which are good, but Sujatha, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that (i) it is acceptable to Jimmy? (ii) it is acceptable to Sujatha?
12. Two customers are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on (i) the same day? (ii) consecutive days? (iii) different days?
13. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is *double* that of a red ball, determine the number of blue balls in the bag.
14. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball? If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x .
15. A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue marbles in the jar.

CHAPTER – 3

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

ALGEBRAIC INTERPRETATION OF PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

The pair of linear equations represented by these lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

1. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then the pair of linear equations has exactly one solution.
2. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the pair of linear equations has infinitely many solutions.
3. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then the pair of linear equations has no solution.

S. No.	Pair of lines	Compare the ratios	Graphical representation	Algebraic interpretation
1	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique solution (Exactly one solution)
2	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
3	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

IMPORTANT QUESTIONS

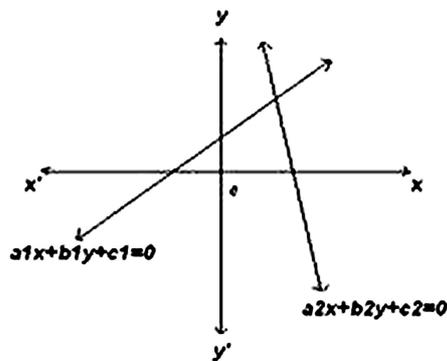
1. On comparing the ratios $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:
 (i) $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ (ii) $9x + 3y + 12 = 0$ and $18x + 6y + 24 = 0$
 (iii) $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$.
2. On comparing the ratios $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.
 (i) $3x + 2y = 5$; $2x - 3y = 7$ (ii) $2x - 3y = 8$; $4x - 6y = 9$
 (iii) $5x - 3y = 11$; $-10x + 6y = -22$
3. Find the number of solutions of the following pair of linear equations:
 $x + 2y - 8 = 0$
 $2x + 4y = 16$
4. Write whether the following pair of linear equations is consistent or not.
 $x + y = 14$, $x - y = 4$
5. Given the linear equation $3x + 4y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is parallel lines.
6. Find the value of k so that the following system of equations has no solution:
 $3x - y - 5 = 0$, $6x - 2y + k = 0$
7. Find the value of k so that the following system of equation has infinite solutions:
 $3x - y - 5 = 0$, $6x - 2y + k = 0$
8. For which values of p, does the pair of equations given below has unique solution?
 $4x + py + 8 = 0$ and $2x + 2y + 2 = 0$

9. Determine k for which the system of equations has infinite solutions:
 $4x + y = 3$ and $8x + 2y = 5k$
10. Find whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident:
 $2x - 3y + 6 = 0$; $4x - 5y + 2 = 0$
11. Find the value of k for which the system $3x + ky = 7$, $2x - 5y = 1$ will have infinitely many solutions.
12. For what value of k , the system of equations $2x - ky + 3 = 0$, $4x + 6y - 5 = 0$ is consistent?
13. For what value of k , the system of equations $kx - 3y + 6 = 0$, $4x - 6y + 15 = 0$ represents parallel lines?
14. For what value of p , the pair of linear equations $5x + 7y = 10$, $2x + 3y = p$ has a unique solution.
15. Find the value of m for which the pair of linear equations has infinitely many solutions.
 $2x + 3y - 7 = 0$ and $(m - 1)x + (m + 1)y = (3m - 1)$
16. For what value of p will the following pair of linear equations have infinitely many solutions?
 $(p - 3)x + 3y = p$; $px + py = 12$
17. For what value of k will the system of linear equations has infinite number of solutions?
 $kx + 4y = k - 4$, $16x + ky = k$
18. Find the values of a and b for which the following system of linear equations has infinite number of solutions:
 $2x - 3y = 7$, $(a + b)x - (a + b - 3)y = 4a + b$
19. For what value of k will the equations $x + 2y + 7 = 0$, $2x + ky + 14 = 0$ represent coincident lines?
20. For what value of k , the following system of equations $2x + ky = 1$, $3x - 5y = 7$ has (i) a unique solution (ii) no solution

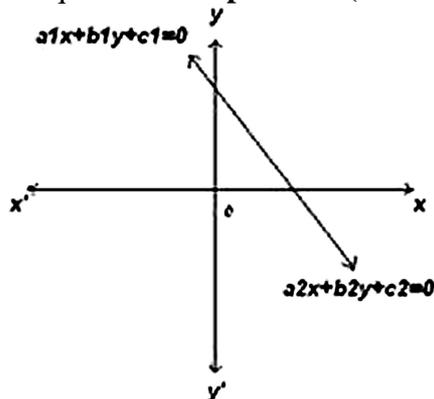
GRAPHICAL METHOD OF SOLUTION OF A PAIR OF LINEAR EQUATIONS

The graph of a pair of linear equations in two variables is represented by two lines.

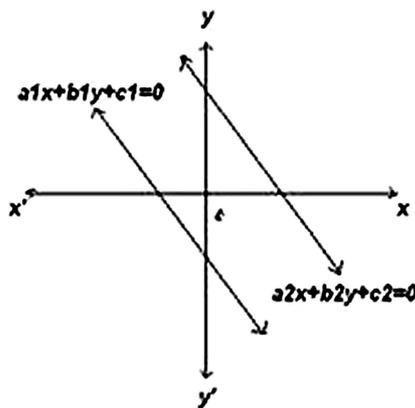
1. If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is **consistent**.



2. If the lines coincide, then there are infinitely many solutions — each point on the line being a solution. In this case, the pair of equations is **dependent (consistent)**.



3. If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is **inconsistent**.



IMPORTANT QUESTIONS

Solve the equation graphically: $x + 3y = 6$ and $2x - 3y = 12$.

Solution: Given that

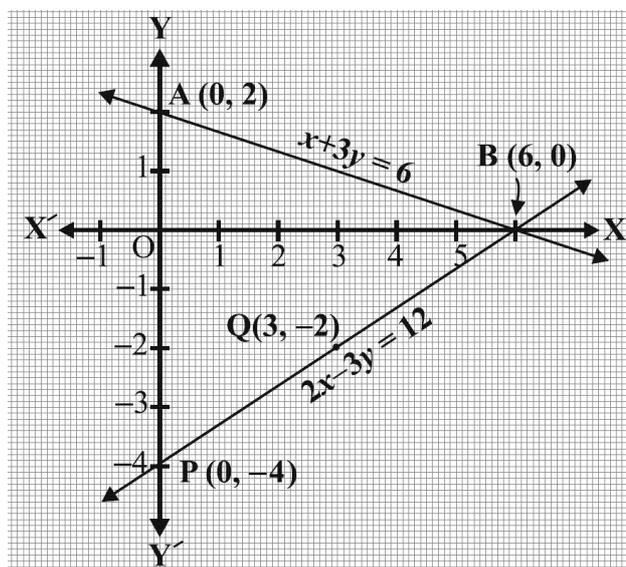
$$x + 3y = 6 \Rightarrow 3y = 6 - x \Rightarrow y = \frac{6 - x}{3}$$

x	0	3	6
y	2	1	0

and $2x - 3y = 12 \Rightarrow 3y = 2x - 12 \Rightarrow y = \frac{2x - 12}{3}$

x	0	3	6
y	-4	-2	0

Now plot the points and join the points to form the lines AB and PQ as shown in graph
Since point B(6, 0) common to both the lines AB and PQ. Therefore, the solution of the pair of linear equations is $x = 6$ and $y = 0$



Questions for Practice

- Determine by drawing graphs, whether the following pair of linear equations has a unique solution or not: $3x + 4y = 12$; $y = 2$
- Determine by drawing graphs, whether the following pair of linear equations has a unique solution or not: $2x - 5 = 0$, $y + 4 = 0$.
- Draw the graphs of the equations $4x - y - 8 = 0$ and $2x - 3y + 6 = 0$.
Also, determine the vertices of the triangle formed by the lines and x-axis.
- Solve the following system of linear equations graphically: $3x - 2y - 1 = 0$; $2x - 3y + 6 = 0$.
Shade the region bounded by the lines and x-axis.
- Solve graphically: $x + 4y = 10$, $y - 2 = 0$
- Solve graphically: $2x - 3y = 6$, $x - 6 = 0$
- Solve the following system of equations graphically: $3x - 5y + 1 = 0$, $2x - y + 3 = 0$.
Also find the points where the lines represented by the given equations intersect the x-axis.
- Solve the following system of equations graphically: $x - 5y = 6$, $2x - 10y = 10$
Also find the points where the lines represented by the given equations intersect the x-axis.
- Solve the following pair of linear equations graphically: $x + 3y = 6$; $2x - 3y = 12$
Also find the area of the triangle formed by the lines representing the given equations with y-axis.

CHAPTER – 4 QUADRATIC EQUATIONS

FACTORISATION METHODS TO FIND THE SOLUTION OF QUADRATIC EQUATIONS

Steps to find the solution of given quadratic equation by factorisation

- Firstly, write the given quadratic equation in standard form $ax^2 + bx + c = 0$.
- Find two numbers α and β such that sum of α and β is equal to b and product of α and β is equal to ac .
- Write the middle term bx as $\alpha x + \beta x$ and factorise it by splitting the middle term and let factors are $(x + p)$ and $(x + q)$ i.e. $ax^2 + bx + c = 0 \Rightarrow (x + p)(x + q) = 0$
- Now equate each factor to zero and find the values of x .
- These values of x are the required roots/solutions of the given quadratic equation.

IMPORTANT QUESTIONS

Solve the quadratic equation by using factorization method: $x^2 + 2x - 8 = 0$

Solution: $x^2 + 2x - 8 = 0$

$$\Rightarrow x^2 + 4x - 2x - 8 = 0 \Rightarrow x(x + 4) - 2(x + 4) = 0$$

$$\Rightarrow (x + 4)(x - 2) = 0 \Rightarrow x + 4 = 0, x - 2 = 0 \Rightarrow x = -4, 2$$

Questions for practice

1. Solve the quadratic equation using factorization method: $x^2 + 7x - 18 = 0$
2. Solve the quadratic equation using factorization method: $x^2 + 5x - 6 = 0$
3. Solve the quadratic equation using factorization method: $y^2 - 4y + 3 = 0$
4. Solve the quadratic equation using factorization method: $x^2 - 21x + 108 = 0$
5. Solve the quadratic equation using factorization method: $x^2 - 11x - 80 = 0$
6. Solve the quadratic equation using factorization method: $x^2 - x - 156 = 0$
7. Solve the following for x : $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$.
8. Solve the following for x : $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

NATURE OF ROOTS

The roots of the quadratic equation $ax^2 + bx + c = 0$ by quadratic formula are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

where $D = b^2 - 4ac$ is called discriminant. The nature of roots depends upon the value of discriminant D . There are three cases –

Case – I

When $D > 0$ i.e. $b^2 - 4ac > 0$, then the quadratic equation has two distinct roots.

$$\text{i.e. } x = \frac{-b + \sqrt{D}}{2a} \text{ and } \frac{-b - \sqrt{D}}{2a}$$

Case – II

When $D = 0$, then the quadratic equation has two equal real roots.

$$\text{i.e. } x = \frac{-b}{2a} \text{ and } \frac{-b}{2a}$$

Case – III

When $D < 0$ then there is no real roots exist.

IMPORTANT QUESTIONS

Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its roots.

Solution : The given equation is of the form $ax^2 + bx + c = 0$, where $a = 2$, $b = -4$ and $c = 3$.
Therefore, the discriminant, $D = b^2 - 4ac = (-4)^2 - (4 \times 2 \times 3) = 16 - 24 = -8 < 0$
So, the given equation has no real roots.

Questions for Practice

1. Find the discriminant and the nature of the roots of quadratic equation: $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$.
2. Find discriminant and the nature of the roots of quadratic equation: $4x^2 - 2x + 3 = 0$.
3. Find discriminant and the nature of the roots of quadratic equation: $4x^2 - 12x + 9 = 0$.
4. Find discriminant and the nature of the roots of quadratic equation: $5x^2 + 5x + 6 = 0$.
5. Write the nature of roots of quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$.
6. Write the nature of roots of the quadratic equation $9x^2 - 6x - 2 = 0$.
7. Write the nature of roots of quadratic equation : $4x^2 + 6x + 3 = 0$
8. The roots of $ax^2 + bx + c = 0$, $a \neq 0$ are real and unequal. What is value of D?
9. If $ax^2 + bx + c = 0$ has equal roots, what is the value of c?

QUADRATIC FORMULA METHOD

Steps to find the solution of given quadratic equation by quadratic formula method:

- Firstly, write the given quadratic equation in standard form $ax^2 + bx + c = 0$.
- Write the values of a, b and c by comparing the given equation with standard form.
- Find discriminant $D = b^2 - 4ac$. If value of D is negative, then is no real solution i.e. solution does not exist. If value of $D \geq 0$, then solution exists follow the next step.
- Put the value of a, b and D in quadratic formula $x = \frac{-b \pm \sqrt{D}}{2a}$ and get the required roots/solutions.

IMPORTANT QUESTIONS

Solve the quadratic equation by using quadratic formula: $x^2 + x - 6 = 0$

Solution: Here, $a = 1$, $b = 1$, $c = -6$

$$\Rightarrow D = b^2 - 4ac = 1 - 4(1)(-6) = 1 + 24 = 25 > 0$$

$$\text{Now, } x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{25}}{2(1)} = \frac{-1 \pm 5}{2} \Rightarrow x = \frac{-1-5}{2} \text{ or } \frac{-1+5}{2} \Rightarrow x = \frac{-6}{2} \text{ or } \frac{4}{2} \Rightarrow x = -3 \text{ or } 2$$

Questions for practice

1. Solve the quadratic equation by using quadratic formula: $x^2 - 7x + 18 = 0$
2. Solve the quadratic equation by using quadratic formula: $x^2 - 5x + 6 = 0$
3. Solve the quadratic equation by using quadratic formula: $y^2 + 4y + 3 = 0$
4. Solve the quadratic equation by using quadratic formula: $x^2 + 11x - 80 = 0$
5. Solve the quadratic equation by using quadratic formula: $x^2 + x - 156 = 0$
6. Solve for x by using quadratic formula : $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$.

WORD PROBLEMS

IMPORTANT QUESTIONS

A motor boat whose speed is 18 km/h in still water takes 1 hr. more to go 24 km upstream than to return downstream to the same spot. Find the speed of stream.

Solution: Let the speed of the stream be x km/h.

Therefore, the speed of the boat upstream = $(18 - x)$ km/h and the speed of the boat downstream = $(18 + x)$ km/h.

$$\text{The time taken to go upstream} = \frac{\text{distance}}{\text{speed}} = \frac{24}{18-x}$$

$$\text{Similarly, the time taken to go downstream} = \frac{24}{18+x}$$

$$\text{According to the question, } \frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$\Rightarrow 24(18+x) - 24(18-x) = (18-x)(18+x)$$

$$\Rightarrow x^2 + 48x - 324 = 0$$

$$\Rightarrow (x-6)(x+54) = 0 \text{ (using factorisation)}$$

$$\Rightarrow x = 6, -54$$

Since x is the speed of the stream, it cannot be negative. So, we ignore the root $x = -54$. Therefore, $x = 6$ gives the speed of the stream as 6 km/h.

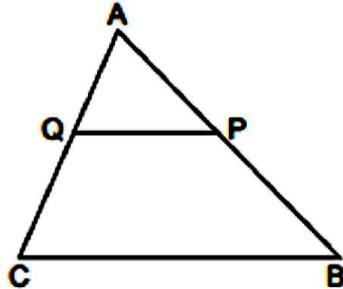
Questions for Practice

1. In a class test, the sum of the marks obtained by P in Mathematics and Science is 28. Had he got 3 more marks in maths and 4 marks less in science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained in the two subjects separately.
2. A peacock is sitting on the top of a pillar which is 9 m high. From a point 27 m away from the bottom of the pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake the peacock pounces on it. If their speeds are equal at what distance from the hole is the snake caught?
3. Some students planned a picnic. The total budget for food was Rs. 2,000. But 5 students failed to attend the picnic and thus the cost of food for each member increased by Rs. 20. How many students attended the picnic and how much did each student pay for the food?
4. In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and time increased by 30 minutes. Find the original duration of the flight.
5. A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work.
6. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.
7. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.
8. Two water taps together can fill a tank in 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.
9. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.
10. A man bought a certain number of toys for 180, he kept one for his own use and sold the rest for one rupee each more than he gave for them, besides getting his own toy for nothing he made a profit of 10. Find the number of toys.
11. Nine times the side of one square exceeds a perimeter of a second square by one metre and six times the area of the second square exceeds twenty nine times the area of the first by one square metre. Find the side of each square.
12. One-fourth of a herd of camels was seen in a forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.
13. One pipe can fill a cistern in $(x+2)$ hours and the other pipe can fill the same cistern in $(x+7)$ hours. If both the pipes, when opened together take 6 hours to fill the empty cistern, find the value of x .

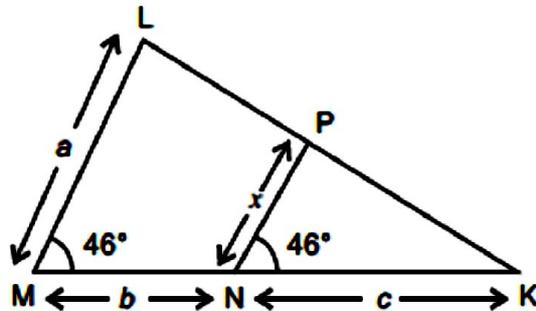
CHAPTER – 6 TRIANGLES

IMPORTANT 1 MARK QUESTIONS

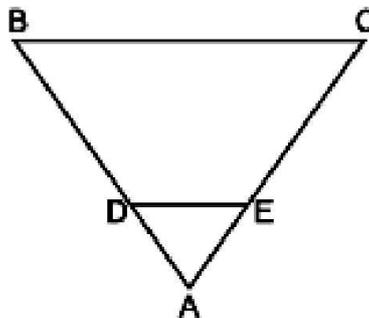
- In $\triangle ABC$, D and E are points on sides AB and AC respectively such that $DE \parallel BC$ and $AD : DB = 3 : 1$. If $EA = 6.6$ cm then find AC.
- In the fig., P and Q are points on the sides AB and AC respectively of $\triangle ABC$ such that $AP = 3.5$ cm, $PB = 7$ cm, $AQ = 3$ cm and $QC = 6$ cm. If $PQ = 4.5$ cm, find BC.



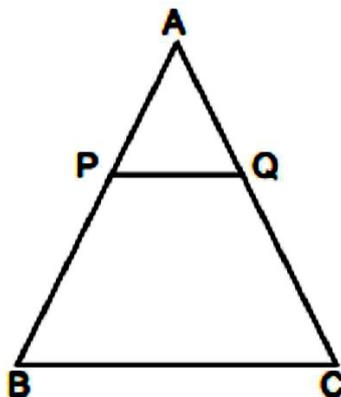
- The perimeter of two similar triangles ABC and LMN are 60 cm and 48 cm respectively. If $LM = 8$ cm, then what is the length of AB ?
- In fig. $\angle M = \angle N = 46^\circ$, express x in terms of a, b and c, where a, b and c are lengths of LM, MN and NK respectively.



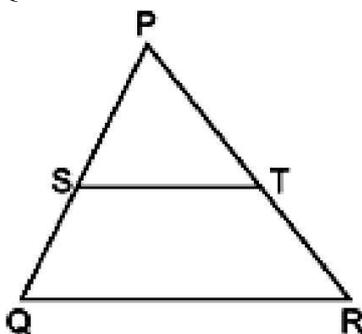
- In figure, $DE \parallel BC$ in $\triangle ABC$ such that $BC = 8$ cm, $AB = 6$ cm and $DA = 1.5$ cm. Find DE.



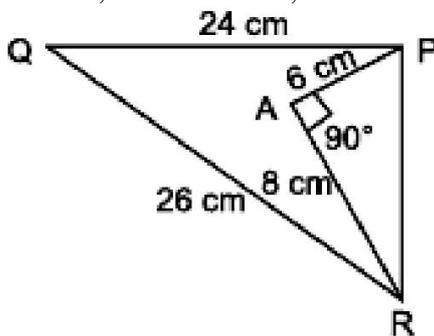
- In the fig., $PQ \parallel BC$ and $AP : PB = 1 : 2$. Find $\frac{ar(\triangle APQ)}{ar(\triangle ABC)}$



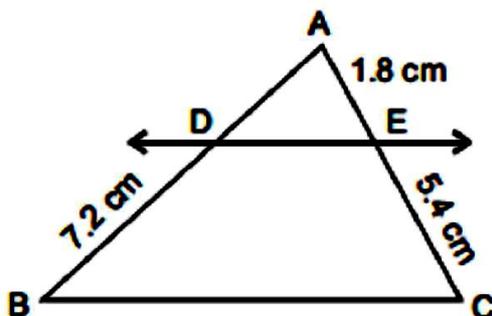
7. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time a tower casts the shadow 40 m long on the ground. Determine the height of the tower.
8. If $\triangle ABC$ and $\triangle DEF$ are similar triangles such that $\angle A = 57^\circ$ and $\angle E = 83^\circ$. Find C.
9. If the areas of two similar triangles are in ratio 25 : 64, write the ratio of their corresponding sides.
10. In figure, S and T are points on the sides PQ and PR, respectively of $\triangle PQR$, such that $PT = 2$ cm, $TR = 4$ cm and ST is parallel to QR . Find the ratio of the areas of $\triangle PST$ and $\triangle PQR$.



11. In the fig., $PQ = 24$ cm, $QR = 26$ cm, $\angle PAR = 90^\circ$, $PA = 6$ cm and $AR = 8$ cm. Find $\angle QPR$.



12. The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus.
13. In the given figure, $DE \parallel BC$. Find AD.



14. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm., what is the corresponding side of the other triangle ?

CHAPTER – 7

COORDINATE GEOMETRY

DISTANCE FORMULA

The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or $AB = \sqrt{(\text{difference of abscissae})^2 + (\text{difference of ordinates})^2}$

Distance of a point from origin

The distance of a point $P(x, y)$ from origin O is given by $OP = \sqrt{x^2 + y^2}$

Problems based on geometrical figure

To show that a given figure is a

- ☞ Parallelogram – prove that the opposite sides are equal
- ☞ Rectangle – prove that the opposite sides are equal and the diagonals are equal.
- ☞ Parallelogram but not rectangle – prove that the opposite sides are equal and the diagonals are not equal.
- ☞ Rhombus – prove that the four sides are equal
- ☞ Square – prove that the four sides are equal and the diagonals are equal.
- ☞ Rhombus but not square – prove that the four sides are equal and the diagonals are not equal.
- ☞ Isosceles triangle – prove any two sides are equal.
- ☞ Equilateral triangle – prove that all three sides are equal.
- ☞ Right triangle – prove that sides of triangle satisfies Pythagoras theorem.

IMPORTANT QUESTIONS

Show that the points (1, 7), (4, 2), (-1, -1) and (-4, 4) are the vertices of a square.

Solution : Let $A(1, 7)$, $B(4, 2)$, $C(-1, -1)$ and $D(-4, 4)$ be the given points.

$$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$$

$$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$$

$$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$$

$$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$$

$$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$$

$$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$$

Since, $AB = BC = CD = DA$ and $AC = BD$, all the four sides of the quadrilateral $ABCD$ are equal and its diagonals AC and BD are also equal. Therefore, $ABCD$ is a square.

Find a point on the y-axis which is equidistant from the points $A(6, 5)$ and $B(-4, 3)$.

Solution : We know that a point on the y-axis is of the form $(0, y)$. So, let the point $P(0, y)$ be equidistant from A and B . Then $AP^2 = BP^2$

$$\Rightarrow (6-0)^2 + (5-y)^2 = (-4-0)^2 + (3-y)^2$$

$$\Rightarrow 36 + 25 + y^2 - 10y = 16 + 9 + y^2 - 6y \Rightarrow 4y = 36 \Rightarrow y = 9$$

So, the required point is $(0, 9)$.

Questions for practice

1. Show that the points A(1, 2), B(5, 4), C(3, 8) and D(-1, 6) are vertices of a square.
2. Show that the points A(5, 6), B(1, 5), C(2, 1) and D(6, 2) are vertices of a square.
3. Show that the points A(1, -3), B(13, 9), C(10, 12) and D(-2, 0) are vertices of a rectangle.
4. Show that the points A(1, 0), B(5, 3), C(2, 7) and D(-2, 4) are vertices of a rhombus.
5. Prove that the points A(-2, -1), B(1, 0), C(4, 3) and D(1, 2) are vertices of a parallelogram.
6. Find the point on x-axis which is equidistant from (7, 6) and (-3, 4).
7. Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9).
8. Find a point on the y-axis which is equidistant from the points A(5, 2) and B(-4, 3).
9. Find a point on the y-axis which is equidistant from the points A(5, -2) and B(-3, 2).
10. Find the values of y for which the distance between the points P(2, -3) and Q(10, y) is 10 units.
11. Find the value of a, if the distance between the points A(-3, -14) and B(a, -5) is 9 units.
12. If the point A(2, -4) is equidistant from P(3, 8) and Q(-10, y), find the values of y. Also find distance PQ.

Section formula

The coordinates of the point P(x, y) which divides the line segment joining the points A(x₁, y₁) and B(x₂, y₂), internally, in the ratio m₁ : m₂ are

$$\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$$

Mid-point formula

The coordinates of the point P(x, y) which is the midpoint of the line segment joining the points

A(x₁, y₁) and B(x₂, y₂), are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

IMPORTANT QUESTIONS

Find the coordinates of the point which divides the line segment joining the points (4, -3) and (8, 5) in the ratio 3 : 1 internally.

Solution : Let P(x, y) be the required point.

Using the section formula, $x = \frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}$, $y = \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}$ we get

$$x = \frac{3(8) + 1(4)}{3 + 1} = 7, y = \frac{3(5) + 1(-3)}{3 + 1} = 3$$

Therefore, (7, 3) is the required point.

In what ratio does the point (-4, 6) divide the line segment joining the points A(-6, 10) and B(3, -8)?

Solution : Let (-4, 6) divide AB internally in the ratio k : 1.

Using the section formula, $x = \frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}$, $y = \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}$ we get

$$y = \frac{k(-8) + 1(10)}{k + 1} = 6$$

$$\Rightarrow -8k + 10 = 6k + 6 \Rightarrow -8k - 6k = 6 - 10$$

$$\Rightarrow -14k = -4 \Rightarrow k = \frac{4}{14} = \frac{2}{7}$$

Therefore, the point (-4, 6) divides the line segment joining the points A(-6, 10) and B(3, -8) in the ratio 2 : 7.

Questions for practice

1. Find the coordinates of the point which divides the join of $(-1, 7)$ and $(4, -3)$ in the ratio $2 : 3$.
2. Find the coordinates of the points of trisection of the line segment joining $(4, -1)$ and $(-2, -3)$.
3. Find the coordinates of the points of trisection (i.e., points dividing in three equal parts) of the line segment joining the points $A(2, -2)$ and $B(-7, 4)$.
4. Find the ratio in which the y-axis divides the line segment joining the points $(5, -6)$ and $(-1, -4)$. Also find the point of intersection.
5. Find the ratio in which the line segment joining the points $(-3, 10)$ and $(6, -8)$ is divided by $(-1, 6)$.
6. Find the ratio in which the line segment joining $A(1, -5)$ and $B(-4, 5)$ is divided by the x-axis. Also find the coordinates of the point of division.
7. Find the coordinates of the points which divide the line segment joining $A(-2, 2)$ and $B(2, 8)$ into four equal parts.
8. If the points $A(6, 1)$, $B(8, 2)$, $C(9, 4)$ and $D(p, 3)$ are the vertices of a parallelogram, taken in order, find the value of p .
9. If $(1, 2)$, $(4, y)$, $(x, 6)$ and $(3, 5)$ are the vertices of a parallelogram taken in order, find x and y .
10. In what ratio does the x-axis divide the line segment joining the points $(-4, -6)$ and $(-1, 7)$? Find the coordinates of the point of division.
11. If $P(9a - 2, -b)$ divides line segment joining $A(3a + 1, -3)$ and $B(8a, 5)$ in the ratio $3 : 1$, find the values of a and b .
12. If (a, b) is the mid-point of the line segment joining the points $A(10, -6)$ and $B(k, 4)$ and $a - 2b = 18$, find the value of k and the distance AB .
13. The centre of a circle is $(2a, a - 7)$. Find the values of a if the circle passes through the point $(11, -9)$ and has diameter $10\sqrt{2}$ units.
14. The line segment joining the points $A(3, 2)$ and $B(5, 1)$ is divided at the point P in the ratio $1 : 2$ and it lies on the line $3x - 18y + k = 0$. Find the value of k .
15. Find the coordinates of the point R on the line segment joining the points $P(-1, 3)$ and $Q(2, 5)$ such that $PR = \frac{3}{5}PQ$.
16. Find the values of k if the points $A(k + 1, 2k)$, $B(3k, 2k + 3)$ and $C(5k - 1, 5k)$ are collinear.
17. Find the ratio in which the line $2x + 3y - 5 = 0$ divides the line segment joining the points $(8, -9)$ and $(2, 1)$. Also find the coordinates of the point of division.
18. The mid-points D, E, F of the sides of a triangle ABC are $(3, 4)$, $(8, 9)$ and $(6, 7)$. Find the coordinates of the vertices of the triangle.

Area of a Triangle

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a ΔABC , then the area of ΔABC is given by

$$\text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Trick to remember the formula

The formula of area of a triangle can be learn with the help of following arrow diagram:

$$\Delta ABC = \frac{1}{2} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

Find the sum of products of numbers at the ends of the lines pointing downwards and then subtract the sum of products of numbers at the ends of the line pointing upwards, multiply the difference by $\frac{1}{2}$. i.e. $Area\ of\ \Delta ABC = \frac{1}{2}[(x_1y_2 + x_2y_3 + x_3y_1) - (x_1y_3 + x_3y_2 + x_2y_1)]$

IMPORTANT QUESTIONS

Find the area of a triangle whose vertices are (1, -1), (-4, 6) and (-3, -5).

Solution: Here, A(1, -1), B(-4, 6) and C(-3, -5).

Using the formula

$$\Delta ABC = \frac{1}{2} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

we get

$$\Delta ABC = \frac{1}{2} \begin{vmatrix} 1 & -1 \\ -4 & 6 \\ -3 & -5 \\ 1 & -1 \end{vmatrix}$$

$$\Delta ABC = \frac{1}{2} [(6 + 20 + 3) - (-5 - 18 + 4)] = \frac{1}{2} [29 - (-19)] = \frac{1}{2} (29 + 19) = \frac{1}{2} \times 48 = 24\ sq.\ units$$

So, the area of the triangle is 24 square units.

Questions for practice

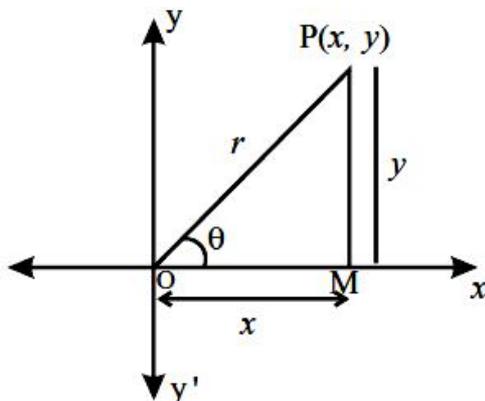
1. Find the area of a triangle formed by the points A(5, 2), B(4, 7) and C (7, -4).
2. Find the area of the triangle formed by the points P(-1.5, 3), Q(6, -2) and R(-3, 4).
3. Find the value of k if the points A(2, 3), B(4, k) and C(6, -3) are collinear.
4. If A(-5, 7), B(-4, -5), C(-1, -6) and D(4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.
5. Find the area of the triangle whose vertices are : (i) (2, 3), (-1, 0), (2, -4) (ii) (-5, -1), (3, -5), (5, 2)
6. In each of the following find the value of 'k', for which the points are collinear. (i) (7, -2), (5, 1), (3, k) (ii) (8, 1), (k, -4), (2, -5)
7. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.
8. Find the area of the quadrilateral whose vertices, taken in order, are (-4, -2), (-3, -5), (3, -2) and (2, 3).
9. Find the value of m if the points (5, 1), (-2, -3) and (8, 2m) are collinear.
10. Find the area of the triangle whose vertices are (-8, 4), (-6, 6) and (-3, 9).
11. A (6, 1), B (8, 2) and C (9, 4) are three vertices of a parallelogram ABCD. If E is the midpoint of DC, find the area of ΔADE .
12. The points A (2, 9), B (a, 5) and C (5, 5) are the vertices of a triangle ABC right angled at B. Find the values of a and hence the area of ΔABC .
13. If the points A (1, -2), B (2, 3) C (a, 2) and D (-4, -3) form a parallelogram, find the value of a and height of the parallelogram taking AB as base.

CHAPTER – 8 & 9 TRIGONOMETRY

Trigonometric Ratios (T - Ratios) of an acute angle of a right triangle

In XOY-plane, let a revolving line OP starting from OX, trace out $\angle XOP = \theta$. From P (x, y) draw $PM \perp$ to OX.

In right angled triangle OMP. OM = x (Adjacent side); PM = y (opposite side); OP = r (hypotenuse).



$$\sin \theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{y}{r}, \quad \cos \theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{x}{r}, \quad \tan \theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{y}{x}$$

$$\operatorname{cosec} \theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{r}{y}, \quad \sec \theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{r}{x}, \quad \cot \theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{x}{y}$$

Reciprocal Relations

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}, \quad \sec \theta = \frac{1}{\cos \theta} \quad \text{and} \quad \cot \theta = \frac{1}{\tan \theta}$$

Quotient Relations

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \text{and} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

IMPORTANT QUESTIONS

If $\tan A = \frac{4}{3}$, find the value of all T-ratios of θ .

Solution: Given that, In right ΔABC , $\tan A = \frac{BC}{AB} = \frac{4}{3}$

Therefore, if $BC = 4k$, then $AB = 3k$, where k is a positive number.

Now, by using the Pythagoras Theorem, we have

$$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2 = 25k^2$$

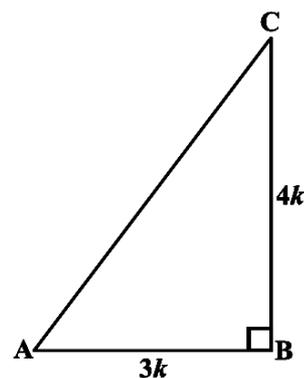
So, $AC = 5k$

Now, we can write all the trigonometric ratios using their definitions

$$\sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}, \quad \cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\text{and } \cot A = \frac{1}{\tan A} = \frac{3}{4},$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{4},$$



$$\sec A = \frac{1}{\cos A} = \frac{5}{3}$$

Questions for Practice

1. If $\sin \theta = \frac{5}{13}$, find the value of all T-ratios of θ .
2. If $\cos \theta = \frac{7}{25}$, find the value of all T-ratios of θ .
3. If $\tan \theta = \frac{15}{8}$, find the value of all T-ratios of θ .
4. If $\cot \theta = 2$, find the value of all T-ratios of θ .
5. If $\operatorname{cosec} \theta = \sqrt{10}$, find the value of all T-ratios of θ .
6. In ΔOPQ , right-angled at P, $OP = 7$ cm and $OQ - PQ = 1$ cm. Determine the values of $\sin Q$ and $\cos Q$.
7. In ΔPQR , right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

Trigonometric ratios for angle of measure.

$0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° in tabular form.

$\angle A$	0°	30°	45°	60°	90°
sinA	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosA	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanA	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosecA	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
secA	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cotA	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

IMPORTANT QUESTIONS

If $\cos(A - B) = \frac{\sqrt{3}}{2}$ and $\sin(A + B) = 1$, then find the value of A and B.

Solution: Given that $\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30^\circ$

$$\Rightarrow A - B = 30^\circ \dots\dots\dots (1)$$

and $\sin(A + B) = 1 = \sin 90^\circ$

$$\Rightarrow A + B = 90^\circ \dots\dots\dots (2)$$

Solving equations (1) and (2), we get $A = 60^\circ$ and $B = 30^\circ$.

Questions for Practice

Evaluate each of the following:

- $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$
- $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$
- $\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$
- $\sin 60^\circ \sin 45^\circ - \cos 60^\circ \cos 45^\circ$
- $(\sin^2 30^\circ + 4 \cot^2 45^\circ - \sec^2 60^\circ)(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)$
- If $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$, then find the value of A and B.
- If $\tan(A - B) = \frac{1}{\sqrt{3}}$ and $\tan(A + B) = \sqrt{3}$, then find the value of A and B.

Trigonometric ratios of Complementary angles.

$$\sin(90 - \theta) = \cos \theta$$

$$\cos(90 - \theta) = \sin \theta$$

$$\tan(90 - \theta) = \cot \theta$$

$$\cot(90 - \theta) = \tan \theta$$

$$\sec(90 - \theta) = \operatorname{cosec} \theta$$

$$\operatorname{cosec}(90 - \theta) = \sec \theta.$$

IMPORTANT QUESTIONS

If $\sin 3A = \cos(A - 26^\circ)$, where $3A$ is an acute angle, find the value of A.

Solution: Given that $\sin 3A = \cos(A - 26^\circ)$. (1)

Since $\sin 3A = \cos(90^\circ - 3A)$, we can write (1) as

$$\cos(90^\circ - 3A) = \cos(A - 26^\circ)$$

Since $90^\circ - 3A$ and $A - 26^\circ$ are both acute angles, therefore comparing both sides we get,

$$90^\circ - 3A = A - 26^\circ \text{ which gives } A = 29^\circ$$

Questions for Practice

- Express $\cot 85^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45° .
- Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45° .
- If $\tan 2A = \cot(A - 18^\circ)$, where $2A$ is an acute angle, find the value of A.
- If $\tan A = \cot B$, prove that $A + B = 90^\circ$.
- If $\sec 4A = \operatorname{cosec}(A - 20^\circ)$, where $4A$ is an acute angle, find the value of A.
- If A, B and C are interior angles of a triangle ABC, then show that

TRIGONOMETRIC IDENTITIES

An equation involving trigonometric ratios of an angle is said to be a trigonometric identity if it is satisfied for all values of θ for which the given trigonometric ratios are defined.

Identity (1): $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \sin^2\theta = 1 - \cos^2\theta \text{ and } \cos^2\theta = 1 - \sin^2\theta.$$

Identity (2): $\sec^2\theta = 1 + \tan^2\theta$

$$\Rightarrow \sec^2\theta - \tan^2\theta = 1 \text{ and } \tan^2\theta = \sec^2\theta - 1.$$

Identity (3): $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$$\Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta = 1 \text{ and } \cot^2\theta = \operatorname{cosec}^2\theta - 1.$$

IMPORTANT QUESTIONS

Prove that: $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

$$\text{Solution: LHS} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

(Dividing Numerator and Denominator by $\sin A$, we get)

$$\begin{aligned} &= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}} = \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} \quad \left[\because \cot A = \frac{\cos A}{\sin A}, \operatorname{cosec} A = \frac{1}{\sin A} \right] \\ &= \frac{\cot A + \operatorname{cosec} A - 1}{\cot A + 1 - \operatorname{cosec} A} = \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cosec} A} \quad [\because \operatorname{cosec}^2 A - \cot^2 A = 1] \\ &= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{\cot A + 1 - \operatorname{cosec} A} \\ &= \frac{(\operatorname{cosec} A + \cot A)(1 - \operatorname{cosec} A + \cot A)}{\cot A + 1 - \operatorname{cosec} A} = \operatorname{cosec} A + \cot A = \text{RHS} \end{aligned}$$

Questions for Practice

Prove the following identities:

1. $\sec A (1 - \sin A)(\sec A + \tan A) = 1.$

2. $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

3. $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$

4. $(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$

5. $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$

6. $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$

7. $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$

8. $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

9. $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$

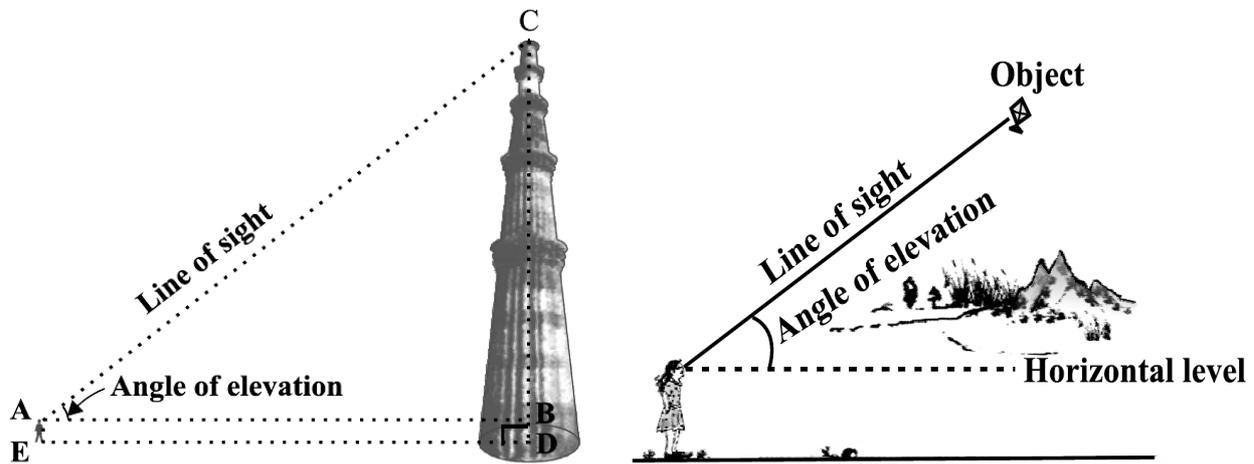
10. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

11. $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

12. $\left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A$

ANGLE OF ELEVATION

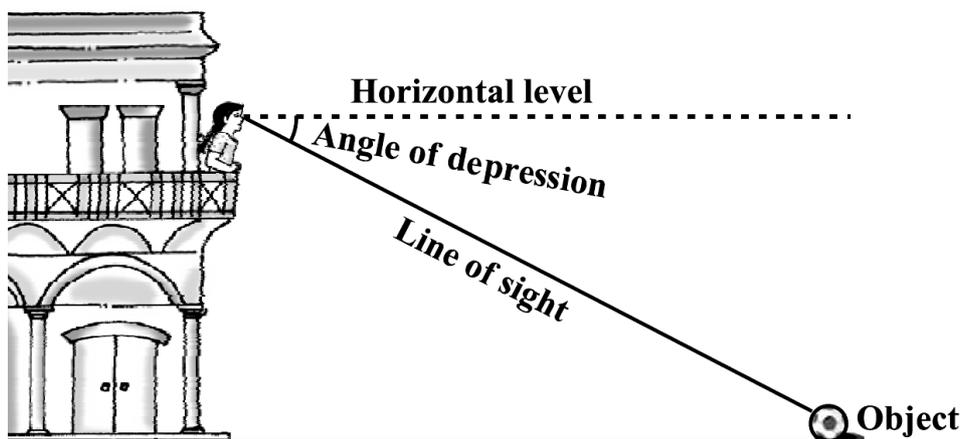
In the below figure, the line AC drawn from the eye of the student to the top of the minar is called the *line of sight*. The student is looking at the top of the minar. The angle BAC, so formed by the line of sight with the horizontal, is called the *angle of elevation* of the top of the minar from the eye of the student. Thus, the **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.



The **angle of elevation** of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object

ANGLE OF DEPRESSION

In the below figure, the girl sitting on the balcony is *looking down* at a flower pot placed on a stair of the temple. In this case, the line of sight is *below* the horizontal level. The angle so formed by the line of sight with the horizontal is called the *angle of depression*. Thus, the **angle of depression** of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed



IMPORTANT QUESTIONS

The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° , respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

Solution : Let $PC = h$ m be the height of multistoreyed building and AB denotes the 8 m tall building.

$BD = AC = x$ m, $PC = h = PD + DC = PD + AB = PD + 8$ m

So, $PD = h - 8$ m

Now, $\angle QPB = \angle PBD = 30^\circ$

Similarly, $\angle QPA = \angle PAC = 45^\circ$.

In right ΔPBD , $\tan 30^\circ = \frac{PD}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h-8}{x}$

$\Rightarrow x = (h-8)\sqrt{3}$ m (1)

Also, In right ΔPAC , $\tan 45^\circ = \frac{PC}{AC} \Rightarrow 1 = \frac{h}{x}$

$$\Rightarrow x = h \text{ m} \dots\dots\dots (2)$$

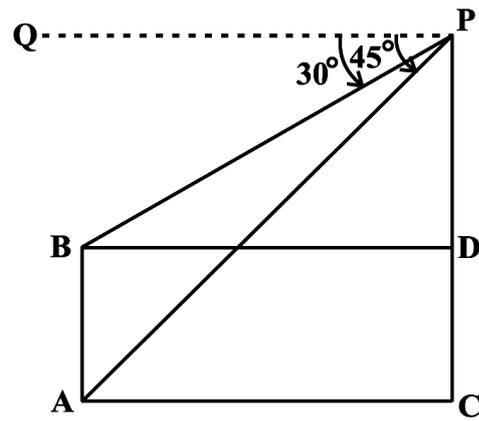
From equations (1) and (2), we get $h = (h-8)\sqrt{3}$

$$\Rightarrow h = h\sqrt{3} - 8\sqrt{3} \Rightarrow h\sqrt{3} - h = 8\sqrt{3}$$

$$\Rightarrow h(\sqrt{3}-1) = 8\sqrt{3} \Rightarrow h = \frac{8\sqrt{3}}{\sqrt{3}-1}$$

$$\Rightarrow h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{8\sqrt{3}(\sqrt{3}+1)}{3-1}$$

$$\Rightarrow h = \frac{8(3+\sqrt{3})}{2} = 4(3+\sqrt{3})m$$



Hence, the height of the multi-storeyed building is $4(3+\sqrt{3})m$ and the distance between the two buildings is also $4(3+\sqrt{3})m$.

From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° , respectively. If the bridge is at a height of 3 m from the banks, find the width of the river.

Solution: Let A and B represent points on the bank on opposite sides of the river, so that AB is the width of the river. P is a point on the bridge at a height of 3 m, i.e., $DP = 3$ m.

Now, $AB = AD + DB$

In right ΔAPD , $\tan 30^\circ = \frac{PD}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AD}$

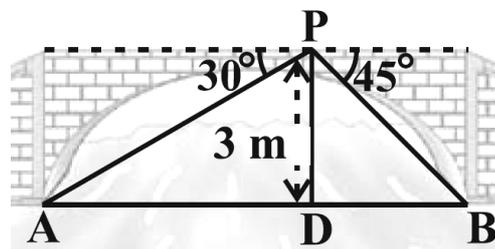
$$\Rightarrow AD = 3\sqrt{3} \text{ m}$$

Also, in right ΔPBD , $\tan 45^\circ = \frac{PD}{BD} \Rightarrow 1 = \frac{3}{BD}$

$$\Rightarrow BD = 3 \text{ m}$$

Now, $AB = BD + AD = 3 + 3\sqrt{3} = 3(1 + \sqrt{3}) \text{ m}$

Therefore, the width of the river is $3(1 + \sqrt{3}) \text{ m}$



Questions for Practice

1. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30° . Find the height of the tower.
2. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60° . Find the length of the string, assuming that there is no slack in the string.
3. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.
4. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.
5. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal.
6. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° . If the tower is 50 m high, find the height of the building.

7. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° , respectively. Find the height of the poles and the distances of the point from the poles.
8. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° . Find the height of the tower and the width of the canal.
9. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower.
10. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.
11. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60° . After some time, the angle of elevation reduces to 30° . Find the distance travelled by the balloon during the interval.
12. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30° , which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60° . Find the time taken by the car to reach the foot of the tower from this point.
13. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

CHAPTER – 13 SURFACE AREAS AND VOLUMES

IMPORTANT FORMULAE

Name of the Solid	Curved Surface Area	Total Surface Area	Volume
Cuboid	$2h(l+b)$	$2(lb+bh+hl)$	lbh
Cube	$4a^2$	$6a^2$	a^3
Right Circular Cylinder	$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$
Right Circular Cone	πrl	$2\pi r(r+l)$	$\frac{1}{3}\pi r^2 h$
Sphere	–	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Hemisphere	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
Frustum of a Cone	$\pi(r_1+r_2)l$ where $l = \sqrt{h^2 + (r_1 - r_2)^2}$	$\pi(r_1+r_2)l$ + $\pi r_1^2 + \pi r_2^2$	$\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

COMBINATIONAL FIGURE BASED QUESTIONS

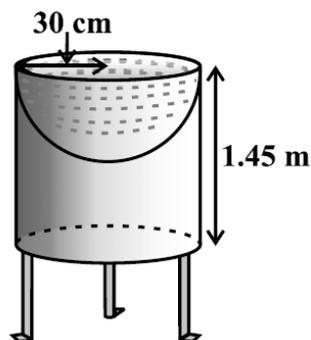
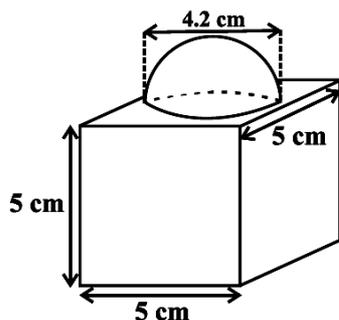
IMPORTANT QUESTIONS

The decorative block is shown in below left figure made of two solids — a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block.

Solution: The total surface area of the cube = $6 \times (\text{edge})^2 = 6 \times 5 \times 5 \text{ cm}^2 = 150 \text{ cm}^2$.

So, the surface area of the block = TSA of cube – base area of hemisphere + CSA of hemisphere
 $= 150 - \pi r^2 + 2\pi r^2 = (150 + \pi r^2) \text{ cm}^2$

$$= 150 + \left(\frac{22}{7} \times \frac{4.2}{2} \times \frac{4.2}{2} \right) \text{ cm}^2 = 150 + 13.86 \text{ cm}^2 = 163.86 \text{ cm}^2$$



Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath.

Solution : Let h be height of the cylinder, and r the common radius of the cylinder and hemisphere. (See above right sided figure)

Total surface area of the bird-bath = CSA of cylinder + CSA of hemisphere

$$= 2\pi rh + 2\pi r^2 = 2\pi r (h + r) = 2 \times \frac{22}{7} \times 30(145 + 30) = 2 \times \frac{22}{7} \times 30 \times 175 = 33000 \text{ cm}^2 = 3.3 \text{ m}^2$$

A juice seller was serving his customers using glasses as shown in below figure. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. (Use $\pi = 3.14$.)

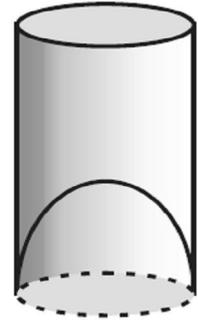
Solution: Here, inner diameter = 5 cm. height, $h = 10$ cm

So, radius, $r = \frac{5}{2}$ cm

Apparent capacity of the glass = Volume of cylinder – Volume of hemisphere

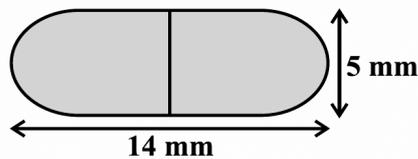
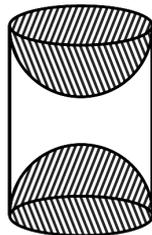
$$= \pi r^2 h - \frac{2}{3} \pi r^3 = \pi r^2 \left(h - \frac{2}{3} r \right) = 3.14 \times \frac{5}{2} \times \frac{5}{2} \times \left(10 - \frac{2}{3} \times \frac{5}{2} \right)$$

$$= 3.14 \times \frac{25}{4} \times \frac{25}{3} = \frac{19625}{12} = 163.54 \text{ cm}^3$$



Questions for Practice

1. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder (see below left figure). If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.



2. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see above right sided figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.
3. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2 .
4. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2 .
5. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.
6. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take $\pi = 3.14$)
7. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm

8. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately 8g mass. (Use $\pi = 3.14$)
9. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

CONVERSION BASED QUESTIONS

IMPORTANT QUESTIONS

A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.

Solution: Here, radius of cone, $r = 6$ cm, height of cone, $h = 24$ cm

Let the radius of the sphere be R cm, then we have

Volume of Sphere = Volume of cone

$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h \Rightarrow 4R^3 = r^2 h \Rightarrow R^3 = \frac{r^2 h}{4} = \frac{6 \times 6 \times 24}{4} = 6 \times 6 \times 6 \Rightarrow R = 6 \text{ cm}$$

Therefore, the radius of the sphere is 6 cm.

Questions for Practice

1. A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire.
2. A hemispherical tank full of water is emptied by a pipe at the rate of litres per second. How much time will it take to empty half the tank, if it is 3m in diameter?
3. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.
4. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.
5. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?
6. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

FRUSTUM OF A CONE BASED QUESTIONS

IMPORTANT QUESTIONS

A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Solution: Here, height of frustum of cone, $h = 14$ cm, diameters of its two circular ends are 4 cm and 2 cm

So, radii of its two circular ends are $R = 2$ cm and $r = 1$ cm

Now, Capacity of the glass = Volume of a frustum of a cone

$$= \frac{\pi h}{3}(R^2 + r^2 + Rr) = \frac{22}{7} \times \frac{14}{3} (2^2 + 1^2 + 2 \times 1) = \frac{44}{3} (4 + 1 + 2) = \frac{44}{3} \times 7 = \frac{308}{3} = 102.67 \text{ cm}^3$$

The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Solution: Here, slant height of a frustum of a cone, $l = 4$ cm,

Circumference of upper end = $2\pi r = 6$ cm

So, $\pi r = 3$ cm

and Circumference of upper end = $2\pi R = 18$ cm

So, $\pi R = 9$ cm

Now, curved surface area of the frustum = $\pi l(R + r) = l \times (\pi R + \pi r)$
 $= 4 \times (9 + 3) = 4 \times 12 = 48\text{cm}^2$

Questions for Practice

1. The radii of the ends of a frustum of a cone 45 cm high are 28 cm and 7 cm. Find its volume, the curved surface area and the total surface area
2. An open metal bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metallic sheet. The diameters of the two circular ends of the bucket are 45 cm and 25 cm, the total vertical height of the bucket is 40 cm and that of the cylindrical base is 6 cm. Find the area of the metallic sheet used to make the bucket, where we do not take into account the handle of the bucket. Also, find the volume of water the bucket can hold.
3. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm². (Take $\pi = 3.14$)
4. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{4}$ cm, find the length of the wire.



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