MATHEMATICS

S T A N D A R D

MINIMUM LEVEL LEARNING MATERIAL

for

CLASS – X

2019 – 20

Prepared by

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DEDICATED

TO

MY FATHER

LATE SHRI. M. S. MALLAYYA

MINIMUM LEVEL DAILY REVISION SYLLABUS
FOR REMEDIAL STUDENTS
MATHEMATICS (STANDARD): CLASS X

S. NO.	CHAPTER/TOPIC	MARKS COVERED AS PER LATEST CBSE SAMPLE PAPERS
1	Real Numbers – Full Chapter	6
2	Polynomials – Full Chapter	3
3	Arithmetic Progression – Full Chapter	7
4	Triangles Theorem	4
5	Coordinate Geometry – Full Chapter	6
6	Circles – Full Chapter	3
7	Construction – Full Chapter	4
8	Statistics – Full Chapter	8
9	Probability – Full Chapter	3
Total Marks		44

All Remedial Students have to complete the above chapters/topics thoroughly with 100% perfection and then they can also concentrate the below topics for Board Exam:

*Linear Equation in two variables **– Graph Questions, Comparing the ratios of coefficients based questions.**

*Quadratic Equations **– imp questions**

*Triangles **– 1 mark imp questions**

*Trigonometry **– NCERT imp questions**

* Areas related to Circles **–imp questions**

*Surface Areas and Volumes **– NCERT imp questions**

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CHAPTER – 1
REAL NUMBERS

EUCLID'S DIVISION LEMMA

Given positive integers a and b, there exist unique integers q and r satisfying a = bq + r, where $0 \le r < b$.

Here we call 'a' as dividend, 'b' as divisor, 'q' as quotient and 'r' as remainder.

$\therefore$ Dividend = (Divisor x Quotient) + Remainder

If in Euclid's lemma r = 0 then b would be HCF of 'a' and 'b'.

IMPORTANT QUESTIONS

Show that any positive even integer is of the form 6q, or 6q + 2, or 6q + 4, where q is some integer.

Solution: Let x be any positive integer such that x > 6. Then, by Euclid's algorithm,

x = 6q + r for some integer q ≥ 0 and 0 ≤ r < 6.

Therefore, x = 6q or 6q + 1 or 6q + 2 or 6q + 3 or 6q + 4 or 6q + 5

Now, 6q is an even integer being a multiple of 2.

We know that the sum of two even integers are always even integers.

Therefore, 6q + 2 and 6q + 4 are even integers

Hence any positive even integer is of the form 6q, or 6q + 2, or 6q + 4, where q is some integer.

Questions for practice

1. Show that any positive even integer is of the form 4q, or 4q + 2, where q is some integer.

2. Show that any positive odd integer is of the form 4q + 1, or 4q + 3, where q is some integer.

3. Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

6. Use Euclid's division lemma to show that the square of an odd positive integer can be of the form 6q + 1 or 6q + 3 for some integer q.

7. Use Euclid's division lemma to prove that one and only one out of n, n + 2 and n + 4 is divisible by 3, where n is any positive integer.

8. Use Euclid's division lemma to show that the square of an odd positive integer is of the form 8m + 1, for some whole number m.

9. Use Euclid's division lemma to show that the square of any positive integer is either of the form 4q or 4q + 1 for some integer q.

10. Use Euclid's division lemma to show that cube of any positive integer is of the form 4m, 4m + 1 or 4m + 3, for some integer m.

11. Use Euclid's division lemma to show that the square of any positive integer cannot be of the form 5q + 2 or 5q + 3 for any integer q.

12. Use Euclid's division lemma to show that the square of any positive integer cannot be of the form 6m + 2 or 6m + 5 for any integer m.

13. Use Euclid's division lemma to show that the square of any odd integer is of the form 4q + 1, for some integer q.

14. If n is an odd integer, then use Euclid's division lemma to show that $n^2 - 1$ is divisible by 8.

15. Use Euclid's division lemma to prove that if x and y are both odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

16. Use Euclid's division lemma to prove that one of any three consecutive positive integers must be divisible by 3.

17. Use Euclid's division lemma to show that the product of three consecutive natural numbers is divisible by 6.
18. For any positive integer n, use Euclid's division lemma to prove that $n^3 - n$ is divisible by 6.
19. Use Euclid's division lemma to show that one and only one out of n, n + 4, n + 8, n + 12 and n + 16 is divisible by 5, where n is any positive integer.

EUCLID'S DIVISION ALGORITHM

Euclid's division algorithm is a technique to compute the Highest Common Factor (HCF) of two given positive integers. Recall that the HCF of two positive integers *a* and *b* is the largest positive integer *d* that divides both *a* and *b*.

To obtain the HCF of two positive integers, say c and d, with c > d, follow the steps below:

Step 1 : Apply Euclid's division lemma, to *c* and *d*. So, we find whole numbers, *q* and *r* such that $c = dq + r,\ 0 \le r < d$.

Step 2 : If $r = 0$, *d* is the HCF of *c* and *d*. If $r \neq 0$ apply the division lemma to *d* and *r*.

Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

This algorithm works because HCF (*c*, *d*) = HCF (*d*, *r*) where the symbol HCF (*c*, *d*) denotes the HCF of *c* and *d*, etc.

IMPORTANT QUESTIONS

Use Euclid's division algorithm to find the HCF of 867 and 255

Solution: Since 867 > 255, we apply the division lemma to 867 and 255 to obtain

867 = 255 × 3 + 102

Since remainder 102 ≠ 0, we apply the division lemma to 255 and 102 to obtain

255 = 102 × 2 + 51

We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain

102 = 51 × 2 + 0

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51,

Therefore, HCF of 867 and 255 is 51.

Questions for practice

1. Use Euclid's algorithm to find the HCF of 4052 and 12576.
2. Use Euclid's division algorithm to find the HCF of 135 and 225.
3. Use Euclid's division algorithm to find the HCF of 196 and 38220.
4. Use Euclid's division algorithm to find the HCF of 455 and 42.
5. Using Euclid's division algorithm, find which of the following pairs of numbers are co-prime: (i) 231, 396 (ii) 847, 2160
6. If the HCF of 65 and 117 is expressible in the form 65m – 117, then find the value of m.
7. Find the HCF of 81 and 237 and express it as a linear combination of 81 and 237.
8. Find the HCF of 65 and 117 and express it in the form 65m + 117n.
9. If the HCF of 210 and 55 is expressible in the form of 210x5 + 55y, find y.
10. If d is the HCF of 56 and 72, find x, y satisfying d = 56x + 72y. Also show that x and y are not unique.
11. Express the HCF of 468 and 222 as 468x + 222y where x, y are integers in two different ways.
12. Express the HCF of 210 and 55 as 210x + 55y where x, y are integers in two different ways.
13. If the HCF of 408 and 1032 is expressible in the form of 1032m – 408x5, find m.

The Fundamental Theorem of Arithmetic

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

The prime factorisation of a natural number is unique, except for the order of its factors.

- Property of HCF and LCM of two positive integers 'a' and 'b':
  - $HCF(a,b) \times LCM(a,b) = a \times b$
  - $LCM(a,b) = \dfrac{a \times b}{HCF(a,b)}$
  - $HCF(a,b) = \dfrac{a \times b}{LCM(a,b)}$

PRIME FACTORISATION METHOD TO FIND HCF AND LCM

HCF(a, b) = Product of the smallest power of each common prime factor in the numbers.
LCM(a, b) = Product of the greatest power of each prime factor, involved in the numbers.

IMPORTANT QUESTIONS

Find the LCM and HCF of 510 and 92 and verify that LCM × HCF = product of the two numbers

Solution: 510 = 2 x 3 x 5 x 17
92 = 2 x 2 x 23 = 2^2 x 23
HCF = 2
LCM = 2^2 x 3 x 5 x 17 x 23 = 23460
Product of two numbers = 510 x 92 = 46920
HCF x LCM = 2 x 23460 = 46920
Hence, product of two numbers = HCF × LCM

Questions for practice

1. Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.
2. Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.
3. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers: (i) 26 and 91 (ii) 336 and 54
4. Find the LCM and HCF of the following integers by applying the prime factorisation method: (i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25
5. Explain why 3 × 5 × 7 + 7 is a composite number.
6. Can the number 6^n, n being a natural number, end with the digit 5? Give reasons.
7. Can the number 4^n, n being a natural number, end with the digit 0? Give reasons.
8. Given that HCF (306, 657) = 9, find LCM (306, 657).
9. If two positive integers a and b are written as a = x^3y^2 and b = xy^3; x, y are prime numbers, then find the HCF (a, b).
10. If two positive integers p and q can be expressed as p = ab^2 and q = a^3b; a, b being prime numbers, then find the LCM (p, q).
11. Find the largest number which divides 245 and 1029 leaving remainder 5 in each case.
12. Find the largest number which divides 2053 and 967 and leaves a remainder of 5 and 7 respectively.
13. Two tankers contain 850 litres and 680 litres of kerosene oil respectively. Find the maximum capacity of a container which can measure the kerosene oil of both the tankers when used an exact number of times.

14. In a morning walk, three persons step off together. Their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?

15. Find the least number which when divided by 12, 16, 24 and 36 leaves a remainder 7 in each case.

16. The length, breadth and height of a room are 825 cm, 675 cm and 450 cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.

IRRATIONALITY OF NUMBERS

IMPORTANT QUESTIONS

Prove that $\sqrt{5}$ is an irrational number.

Solution: Let $\sqrt{5}$ is a rational number then we have

$\sqrt{5} = \frac{p}{q}$, where p and q are co-primes.

$\Rightarrow p = \sqrt{5}q$

Squaring both sides, we get

$p^2 = 5q^2$

$\Rightarrow$ p^2 is divisible by 5

$\Rightarrow$ p is also divisible by 5

So, assume p = 5m where m is any integer.

Squaring both sides, we get $p^2 = 25m^2$

But $p^2 = 5q^2$

Therefore, $5q^2 = 25m^2$

$\Rightarrow$ $q^2 = 5m^2$

$\Rightarrow$ q^2 is divisible by 5

$\Rightarrow$ q is also divisible by 5

From above we conclude that p and q has one common factor i.e. 5 which contradicts that p and q are co-primes.

Therefore our assumption is wrong.

Hence, $\sqrt{5}$ is an irrational number.

Questions for practice

1. Prove that $\sqrt{2}$ is an irrational number.

2. Prove that $\sqrt{3}$ is an irrational number.

3. Prove that $2+5\sqrt{3}$ is an irrational number.

4. Prove that $3-2\sqrt{5}$ is an irrational number.

5. Prove that $\sqrt{2}+\sqrt{3}$ is an irrational number.

6. Prove that $\sqrt{3}+\sqrt{5}$ is an irrational number.

RATIONAL NUMBERS AND THEIR DECIMAL EXPANSIONS

Let x = $\frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form $2^m.5^n$, where m, n are non-negative integers. Then x has a decimal expansion which terminates.

Let x = $\frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^m.5^n$, where m, n are non-negative integers. Then x has a decimal expansion which is non-terminating repeating (recurring).

IMPORTANT QUESTIONS

Without actually performing the long division, state whether the rational numbers $\frac{987}{10500}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

Solution: Given rational number $\frac{987}{10500}$ is not in the simplest form. Dividing numerator and denominator by 21 we get $\frac{987}{10500} = \frac{987 \div 21}{10500 \div 21} = \frac{47}{500}$ which is in the form of $\frac{p}{q}$

Now q = 500 = 2^2 x 5^3 which is in the form of $2^m.5^n$, where m, n are non-negative integers. Therefore the given rational number has terminating decimal expansion.

Questions for practice

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

$(i)\frac{13}{3125}(ii)\frac{129}{2^2 5^7 7^5}(iii)\frac{77}{210}(iv)\frac{14587}{1250}(v)\frac{833}{2^2 5^5 7^2}$

MCQ QUESTIONS (1 mark)

1. On dividing a positive integer n by 9, we get 7 as a remainder. What will be the remainder if (3n – 1) is divided by 9?
(a) 1 (b) 2 (c) 3 (d) 4

2. Euclid's division lemma states that for two positive integers a and b, there exist unique integers q and r such that a = bq + r, where r must satisfy
(a) $1 < r < b$ (b) $0 < r \leq b$ (c) $0 \leq r < b$ (d) $0 < r < b$

3. Let x = $\frac{7}{20 \times 25}$ be a rational number. Then x has decimal expansion, which terminates:
(a) after four places of decimal (b) after three places of decimal
(c) after two places of decimal (d) after five places of decimal

4. The decimal representation of $\frac{71}{150}$ is
(a) a terminating decimal (b) a non-terminating, repeating decimal
(c) a non-terminating and non-repeating decimal (d) none of these

5. The decimal expansion of $\frac{63}{72 \times 175}$ is
(a) terminating (b) non-terminating
(c) non termination and repeating (d) an irrational number

6. Which of the following has a terminating decimal expansion?
(a) $\frac{32}{91}$ (b) $\frac{19}{80}$ (c) $\frac{23}{45}$ (d) $\frac{25}{42}$

7. If HCF and LCM of two numbers are 4 and 9696, then the product of the two numbers is:
(a) 9696 (b) 24242 (c) 38784 (d) 4848

8. $\left(2+\sqrt{3}+\sqrt{5}\right)$ is :

(a) a rational number (b) a natural number (c) a integer number (d) an irrational number

9. If $\left(\frac{9}{7}\right)^3 \times \left(\frac{49}{81}\right)^{2x-6} = \left(\frac{7}{9}\right)^9$, the value of x is:

(a) 12 (b) 9 (c) 8 (d) 6

10. The number .211 2111 21111….. is a

(a) terminating decimal (b) non-terminating decimal
(c) non termination and non-repeating decimal (d) none of these

11. If $(m)^n = 32$ where m and n are positive integers, then the value of $(n)^{mn}$ is:

(a) 32 (b) 25 (c) 5^{10} (d) 5^{25}

12. The number $0.\overline{57}$ in the $\frac{p}{q}$ form $q \neq 0$ is

(a) $\frac{19}{35}$ (b) $\frac{57}{99}$ (c) $\frac{57}{95}$ (d) $\frac{19}{30}$

13. The number $0.5\overline{7}$ in the $\frac{p}{q}$ form $q \neq 0$ is

(a) $\frac{26}{45}$ (b) $\frac{13}{27}$ (c) $\frac{57}{99}$ (d) $\frac{13}{29}$

14. Any one of the numbers a, a + 2 and a + 4 is a multiple of:

(a) 2 (b) 3 (c) 5 (d) 7

15. If p is a prime number and p divides k^2, then p divides:

(a) $2k^2$ (b) k (c) 3k (d) none of these

16. For some integer *m*, every even integer is of the form

(a) *m* (b) *m* + 1 (c) 2*m* (d) 2*m* + 1

17. For some integer *q*, every odd integer is of the form

(a) *q* (b) *q* + 1 (c) 2*q* (d) 2*q* + 1

18. $n^2 - 1$ is divisible by 8, if *n* is

(a) an integer (b) a natural number
(c) an odd integer (d) an even integer

19. If the HCF of 65 and 117 is expressible in the form 65*m* – 117, then the value of *m* is

(a) 4 (b) 2 (c) 1 (d) 3

20. The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is

(a) 13 (b) 65 (c) 875 (d) 1750

21. If two positive integers *a* and *b* are written as $a = x^3y^2$ and $b = xy^3$; *x*, *y* are prime numbers, then HCF (*a*, *b*) is

(a) *xy* (b) xy^2 (c) x^3y^3 (d) x^2y^2

22. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then LCM (p, q) is

(a) ab (b) a^2b^2 (c) a^3b^2 (d) a^3b^3

23. The product of a non-zero rational and an irrational number is

(a) always irrational (b) always rational

(c) rational or irrational (d) one

24. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is

(a) 10 (b) 100 (c) 504 (d) 2520

25. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after:

(a) one decimal place (b) two decimal places

(c) three decimal places (d) four decimal places

26. The decimal expansion of the rational number $\frac{33}{2^2.5}$ will terminate after

(a) one decimal place (b) two decimal places

(c) three decimal places (d) more than 3 decimal places

CHAPTER – 2
POLYNOMIALS

QUADRATIC POLYNOMIAL

Relationship between zeroes and coefficients

General form of Quadratic polynomial: $ax^2 + bx + c$, $a \neq 0$

Sum of zeroes $(\alpha+\beta) = -\dfrac{\text{Coefficient of x}}{\text{Coefficient of x}^2} = -\dfrac{b}{a}$

Product of zeroes $(\alpha\beta) = \dfrac{\text{Constant term}}{\text{Coefficient of x}^2} = \dfrac{c}{a}$

IMPORTANT QUESTIONS

Find a quadratic polynomial, the sum and product of whose zeroes are – 3 and 2, respectively.

Solution: Here, $\alpha + \beta = -3$ and $\alpha\beta = 2$

We know that quadratic polynomial is given by $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (-3)x + 2 = x^2 + 3x + 2$

Hence, required quadratic polynomial is $x^2 + 3x + 2$

Find a quadratic polynomial, whose zeroes are – 3 and 2.

Solution: Here, $\alpha = -3$ and $\beta = 2$.

Now, $\alpha + \beta = -3 + 2 = -1$ and $\alpha\beta = (-3)(2) = -6$

We know that quadratic polynomial is given by $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (-1)x + (-6) = x^2 + x - 6$

Hence, required quadratic polynomial is $x^2 + x - 6$

Find the zeroes of the quadratic polynomial $x^2 - 2x - 8$ and verify the relationship between the zeroes and the coefficients.

Solution: Here, $p(x) = x^2 - 2x - 8 = 0$

$x^2 - 4x + 2x - 8 = 0 \Rightarrow x(x-4) + 2(x-4) = 0 \Rightarrow (x-4)(x+2) = 0$

$\Rightarrow x = 4, -2$

Now, $a = 1$, $b = -2$, $c = -8$, $\alpha = 4$, $\beta = -2$

Sum of zeroes, $\alpha + \beta = 4 + (-2) = 2$ and $\dfrac{-b}{a} = \dfrac{-(-2)}{1} = 2$ $\quad \therefore \alpha + \beta = \dfrac{-b}{a}$

Product of zeroes, $\alpha\beta = 4(-2) = -8$ and $\dfrac{c}{a} = \dfrac{-8}{1} = -8$ $\quad \therefore \alpha\beta = \dfrac{c}{a}$.

Hence verified.

Questions for practice

1. Find a quadratic polynomial, the sum and product of whose zeroes are – 5 and 3, respectively.
2. Find a quadratic polynomial, whose zeroes are – 4 and 1, respectively.
3. Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ and $-\dfrac{3}{2}$, respectively. Also find its zeroes.
4. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomials by factorisation.

   $(i)\ \dfrac{-8}{3}, \dfrac{4}{3}$ $\quad (ii)\ \dfrac{21}{8}, \dfrac{5}{16}$ $\quad (iii)\ -2\sqrt{3}, -9$ $\quad (iv)\ \dfrac{-3}{2\sqrt{5}}, -\dfrac{1}{2}$
5. Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.

6. Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.

7. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.

8. Find the zeroes of the quadratic polynomial $3x^2 - x - 4$ and verify the relationship between the zeroes and the coefficients.

9. Find the zeroes of the polynomial $x^2 + \frac{1}{6}x - 2$, and verify the relation between the coefficients and the zeroes of the polynomial.

10. Find the zeroes of the quadratic polynomial $4x^2 - 4x + 1$ and verify the relationship between the zeroes and the coefficients.

11. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 3x - 2$, then find a quadratic polynomial whose zeroes are $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$.

12. If α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, then find a quadratic polynomial whose zeroes are $2\alpha+3\beta$ and $2\beta+3\alpha$.

13. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 1$, then find a quadratic polynomial whose zeroes are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$.

14. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 2x + 3$, then find a quadratic polynomial whose zeroes are $\alpha+2$ and $\beta+2$

15. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 2x + 3$, then find a quadratic polynomial whose zeroes are $\frac{\alpha-1}{\alpha+1}$ and $\frac{\beta-1}{\beta+1}$.

16. If α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 + 5x + k$ such that $\alpha^2+\beta^2+\alpha\beta = \frac{21}{4}$, find the value of k.

17. If α and β are the zeroes of the quadratic polynomial $f(x) = kx^2 + 4x + 4$ such that $\alpha^2+\beta^2 = 24$, find the value of k.

18. If α and β are the zeroes of the quadratic polynomial $f(x) = 4x^2 - 5x - 1$, then find the value of

(i) $\alpha-\beta$ (ii) $\alpha^2+\beta^2$ (iii) $\alpha^4+\beta^4$ (iv) $\alpha\beta^2+\alpha^2\beta$

(v) $\frac{1}{\alpha}+\frac{1}{\beta}$ (vi) $\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta$ (vii) $\frac{1}{\alpha}-\frac{1}{\beta}$ (viii) $\alpha^3+\beta^3$

(ix) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ (x) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ (xi) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta$

(xii) $\alpha^4\beta^3+\alpha^3\beta^4$ (xiii) $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$ (xiv) $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}$

DIVISION ALGORITHM FOR POLYNOMIALS

If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x) \times q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$.

- If r(x) = 0, then g(x) is a factor of p(x).
- Dividend = Divisor × Quotient + Remainder

IMPORTANT QUESTIONS

Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$, and verify the division algorithm.

Solution:

$$
\begin{array}{r|l}
 & x - 2 \\
\hline
-x^2 + x - 1 & -x^3 + 3x^2 - 3x + 5 \\
 & -x^3 + x^2 - x \\
 & + \quad - \quad + \\
\hline
 & 2x^2 - 2x + 5 \\
 & 2x^2 - 2x + 2 \\
 & - \quad + \quad - \\
\hline
 & 3
\end{array}
$$

So, quotient = x – 2, remainder = 3.

Now, Divisor × Quotient + Remainder

$= (-x^2 + x - 1)(x - 2) + 3$

$= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$

$= -x^3 + 3x^2 - 3x + 5$

= Dividend

Hence, the division algorithm is verified.

Questions for Practice

1. Divide $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$.

2. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following :

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

3. Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$

4. Obtain all other zeroes of $3x^4 + 6x^3 - 2x2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

5. Find all the zeroes of the polynomial $x^3 + 3x^2 - 5x - 15$, if two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

6. Find all the zeroes of the polynomial $x^3 - 4x^2 - 3x + 12$, if two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

7. Find all the zeroes of the polynomial $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

8. Find all the zeroes of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

9. If two zeroes of the polynomial $x^4 + 3x^3 - 20x^2 - 6x + 36$ are $\sqrt{2}$ and $-\sqrt{2}$, find the other zeroes of the polynomial.

10. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were x – 2 and –2x + 4, respectively. Find g(x).

11. If the remainder on division of $x^3 + 2x^2 + kx + 3$ by x – 3 is 21, find the quotient and the value of k. Hence, find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$.

12. Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeroes of the two polynomials.

13. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

14. If the polynomial $6x^4 + 8x^3 - 5x^2 + ax + b$ is exactly divisible by the polynomial $2x^2 - 5$, then find the values of a and b.

15. If the polynomial $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by another polynomial $x^2 + 5$, the remainder comes out to be $px + q$, find the value of p and q.

16. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $(ax + b)$, find a and b.

17. Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3 + \sqrt{2}\,x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.

18. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, $a + b$, $a + 2b$ for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial.

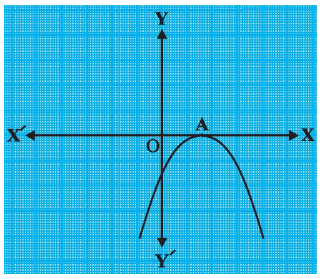
19. Given that $x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}\,x + 13x - 3\sqrt{5}$, find all the zeroes of the polynomial.

20. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$.

MCQ QUESTIONS (1 mark)

1. The value of k for which (–4) is a zero of the polynomial $x^2 - x - (2k + 2)$ is
(a) 3 (b) 9 (c) 6 (d) –1

2. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal, then
(a) c and a have opposite sign (b) c and b have opposite sign
(c) c and a have the same sign (d) c and b have the same sign



3. The number of zeroes of the polynomial from the graph is
(a) 0 (b) 1 (c) 2 (d) 3

4. If one of the zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is
(a) 10 (b) –10 (c) 5 (d) –5

5. A quadratic polynomial whose zeroes are –3 and 4 is
(a) $x^2 - x + 12$ (b) $x^2 + x + 12$ (c) $2x^2 + 2x - 24$. (d) none of the above.

6. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then
(a) a = –7, b = –1 (b) a = 5, b = –1 (c) a = 2, b = – 6 (d) a = 0, b = – 6

7. The number of polynomials having zeroes as –2 and 5 is
(a) 1 (b) 2 (c) 3 (d) more than 3

8. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is
(a) $-\frac{c}{a}$ (b) $\frac{c}{a}$ (c) 0 (d) $-\frac{b}{a}$

9. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then the product of the other two zeroes is
(a) b – a + 1 (b) b – a – 1 (c) a – b + 1 (d) a – b –1

10. The relationship between the zeroes and coefficients of the quadratic polynomial $ax^2 + bx + c$ is

(a) $\alpha + \beta = \frac{c}{a}$ (b) $\alpha + \beta = \frac{-b}{a}$ (c) $\alpha + \beta = \frac{-c}{a}$ (d) $\alpha + \beta = \frac{b}{a}$

11. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are
(a) both positive (b) both negative (c) one positive and one negative (d) both equal

12. The zeroes of the quadratic polynomial $x^2 + kx + k$, $k \neq 0$,
(a) cannot both be positive (b) cannot both be negative
(c) are always unequal (d) are always equal

13. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal, then
(a) c and a have opposite signs (b) c and b have opposite signs
(c) c and a have the same sign (d) c and b have the same sign

14. If one of the zeroes of a quadratic polynomial of the form $x^2+ax + b$ is the negative of the other, then it
(a) has no linear term and the constant term is negative.
(b) has no linear term and the constant term is positive.
(c) can have a linear term but the constant term is negative.
(d) can have a linear term but the constant term is positive.

15. The zeroes of the polynomial $x^2 + 7x + 10$ are
(a) 2 and 5 (b) –2 and 5 (c) –2 and –5 (d) 2 and –5

16. The relationship between the zeroes and coefficients of the quadratic polynomial $ax^2 + bx + c$ is (a) $\alpha.\beta = \frac{c}{a}$ (b) $\alpha .\beta = \frac{-b}{a}$ (c) $\alpha .\beta = \frac{-c}{a}$ (d) $\alpha .\beta = \frac{b}{a}$

17. The zeroes of the polynomial $x^2 – 3$ are
(a) 2 and 5 (b) –2 and 5 (c) –2 and –5 (d) none of the above

18. A quadratic polynomial whose sum and product of zeroes are –3 and 2 is
(a) $x^2 – 3x +2$ (b) $x^2 + 3x + 2$ (c) $x^2 + 2x – 3$. (d) $x^2 + 2x + 3$.

19. If one of the zeroes of the quadratic polynomial $(k–1) x^2 + k x + 1$ is –3, then the value of k is
(a) $\frac{4}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

20. The zeroes of the quadratic polynomial $x^2 + kx + k$, $k \neq 0$,
(a) cannot both be positive (b) cannot both be negative
(c) are always unequal (d) are always equal

CHAPTER – 5
ARITHMETIC PROGRESSION

<u>*n*th Term of an ARITHMETIC PROGRESSION (AP)</u>

*n*th term a_n of the AP with first term a and common difference d is given by

$$\boxed{a_n = a + (n - 1)\, d.}$$

IMPORTANT QUESTIONS

Find the 15th term of the 21, 24, 27, . . .

Solution: Here, a = 21, d = 24 – 21 = 3

We know that $a_n = a + (n - 1)d$

So, $a_{15} = a + 14d = 21 + 14(3) = 21 + 42 = 63$

Which term of the AP : 3, 9, 15, 21, . . . , is 99?

Solution: Here, a = 3, d = 9 – 3 = 6

We know that $a_n = a + (n - 1)d$

Let an = 99 $\Rightarrow$ a + (n – 1)d = 99

$\Rightarrow 3 + (n - 1)6 = 99 \Rightarrow (n - 1)6 = 99 - 3 = 96$

$\Rightarrow n - 1 = \frac{96}{6} = 16 \Rightarrow n = 16 + 1 = 17$

Hence, 17th term of the given AP is 99

Determine the AP whose 3rd term is 5 and the 7th term is 9.

Solution: We have $a_3 = a + (3 - 1)\, d = a + 2d = 5$ ..... (1)

and $a_7 = a + (7 - 1)\, d = a + 6d = 9$ ..... (2)

Solving the pair of linear equations (1) and (2), we get a = 3, d = 1

Hence, the required AP is 3, 4, 5, 6, 7, . . .

Questions for practice

1. Find the 10th term of the AP : 2, 7, 12, . . .
2. Which term of the AP : 21, 18, 15, . . . is – 81?
3. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?
4. How many two-digit numbers are divisible by 3?
5. How many three-digit numbers are divisible by 7?
6. How many multiples of 4 lie between 10 and 250?
7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.
8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.
9. If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero?
10. Which term of the AP : 3, 15, 27, 39, . . . will be 132 more than its 54th term?
11. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.
12. The sum of 4th term and 8th term of an AP is 24 and the sum of 6th and 10th terms is 44. Find the AP.
13. The sum of 5th term and 9th term of an AP is 72 and the sum of 7th and 12th terms is 97. Find the AP.
14. If the numbers n – 2, 4n – 1 and 5n + 2 are in AP, find the value of n.
15. Find the value of the middle most term (s) of the AP : –11, –7, –3,..., 49.
16. The sum of the first three terms of an AP is 33. If the product of the first and the third term exceeds the second term by 29, find the AP.
17. The sum of the 5th and the 7th terms of an AP is 52 and the 10th term is 46. Find the AP.

18. Find the 20th term of the AP whose 7th term is 24 less than the 11th term, first term being 12.
19. If the 9th term of an AP is zero, prove that its 29th term is twice its 19th term.
20. The 26th, 11th and the last term of an AP are 0, 3 and –1/5 , respectively. Find the common difference and the number of terms.
21. Find whether 55 is a term of the AP: 7, 10, 13,--- or not. If yes, find which term it is.
22. Determine k so that k2+ 4k + 8, 2k2 + 3k + 6, 3k2 + 4k + 4 are three consecutive terms of an AP.
23. Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.
24. The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.
25. If the nth terms of the two APs: 9, 7, 5, ... and 24, 21, 18,... are the same, find the value of n. Also find that term.
26. If sum of the 3rd and the 8th terms of an AP is 7 and the sum of the 7th and the 14th terms is –3, find the 10th term.
27. Which term of the AP: 53, 48, 43,... is the first negative term?
28. A sum of Rs 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.
29. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

***n*th Term from the end of an ARITHMETIC PROGRESSION (AP)**

Let the last term of an AP be '*l*' and the common difference of an AP is 'd' then the nth term from the end of an AP is given by

$$\boxed{l_n = l - (n - 1)\, d.}$$

IMPORTANT QUESTIONS

Find the 11th term from the last term (towards the first term) of the AP : 10, 7, 4, . . ., – 62.

Solution : Here, a = 10, d = 7 – 10 = – 3, l = – 62,

We know that nth term from the last is given by $l_n = l - (n - 1)\, d$.

$\therefore l_{11} = l - 10d = -62 - 10(-3) = -62 + 30 = -32$

Questions for practice

1. Find the 20th term from the last term of the AP : 3, 8, 13, . . ., 253.
2. Find the 10th term from the last term of the AP : 4, 9 , 14, . . ., 254.
3. Find the 6th term from the end of the AP 17, 14, 11, …… (–40).
4. Find the 8th term from the end of the AP 7, 10, 13, …… 184.
5. Find the 10th term from the last term of the AP : 8, 10, 12, . . ., 126.
6. Find the 12th term from the end of the AP: –2, –4, –6,..., –100.

Sum of First *n* Terms of an ARITHMETIC PROGRESSION (AP)

The sum of the first *n* terms of an AP is given by

$$\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$$

where a = first term, d = common difference and n = number of terms.

or

$$\boxed{S_n = \frac{n}{2}[a + l]}$$

where *l* = last term

IMPORTANT QUESTIONS

Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .

Solution : Here, a = 8, d = 3 – 8 = –5, n = 22.

We know that $S = \frac{n}{2}[2a + (n-1)d]$

$\therefore S = \frac{22}{2}[16 + (22-1) \times (-5)]$ = 11(16 – 105) = 11(–89) = – 979

So, the sum of the first 22 terms of the AP is – 979.

Questions for practice

1. If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.
2. How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?
3. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?
4. Find the sum of first 24 terms of the list of numbers whose nth term is given by an = 3 + 2n
5. Find the sum of the first 40 positive integers divisible by 6.
6. Find the sum of the first 15 multiples of 8.
7. Find the sum of the odd numbers between 0 and 50.
8. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.
9. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.
10. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.
11. If $a_n = 3 - 4n$, show that $a_1, a_2, a_3, \ldots$ form an AP. Also find S_{20} .
12. In an AP, if $S_n = n(4n + 1)$, find the AP.
13. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.
14. If S_n denotes the sum of first n terms of an AP, prove that $S_{12} = 3(S_8 - S_4)$
15. Find the sum of first 17 terms of an AP whose 4th and 9th terms are –15 and –30 respectively.
16. If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.
17. Find the sum of all the 11 terms of an AP whose middle most term is 30.
18. Find the sum of last ten terms of the AP: 8, 10, 12,---, 126.
19. How many terms of the AP: –15, –13, –11,--- are needed to make the sum –55? Explain the reason for double answer.
20. The first term of an AP is –5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.
21. Which term of the AP: –2, –7, –12,... will be –77? Find the sum of this AP upto the term –77.
22. Find the sum of first seven numbers which are multiples of 2 as well as of 9.
23. The sum of the first n terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first 2n terms of another AP whose first term is – 30 and the common difference is 8. Find n.
24. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last terms to the product of the two middle terms is 7 : 15. Find the numbers.
25. The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.
26. Find the sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.
27. Find the sum of those integers from 1 to 500 which are multiples of 2 as well as of 5 .
28. Find the sum of those integers from 1 to 500 which are multiples of 2 or 5.
29. The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15th term.
30. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.
31. Find the sum of the integers between 100 and 200 that are (i) divisible by 9 (ii) not divisible by 9

32. The ratio of the 11th term to the 18th term of an AP is 2 : 3. Find the ratio of the 5th term to the 21st term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms.

33. Solve the equation : 1 + 4 + 7 + 10 +...+ x =287

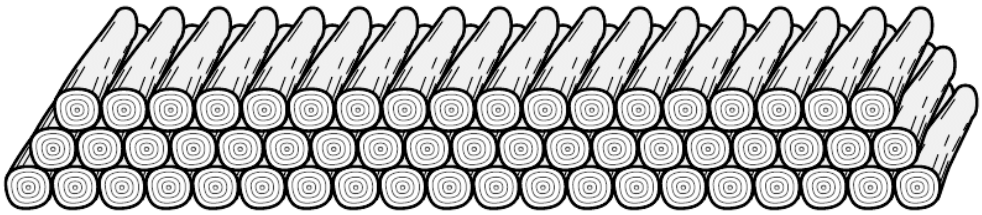
34. Solve the equation – 4 + (–1) + 2 +...+ x = 437

35. Jaspal Singh repays his total loan of Rs 118000 by paying every month starting with the first installment of Rs 1000. If he increases the installment by Rs 100 every month, what amount will be paid by him in the 30th installment? What amount of loan does he still have to pay after the 30th installment?

36. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

37. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find : (i) the production in the 1st year (ii) the production in the 10th year (iii) the total production in first 7 years

38. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how may rows are the 200 logs placed and how many logs are in the top row?

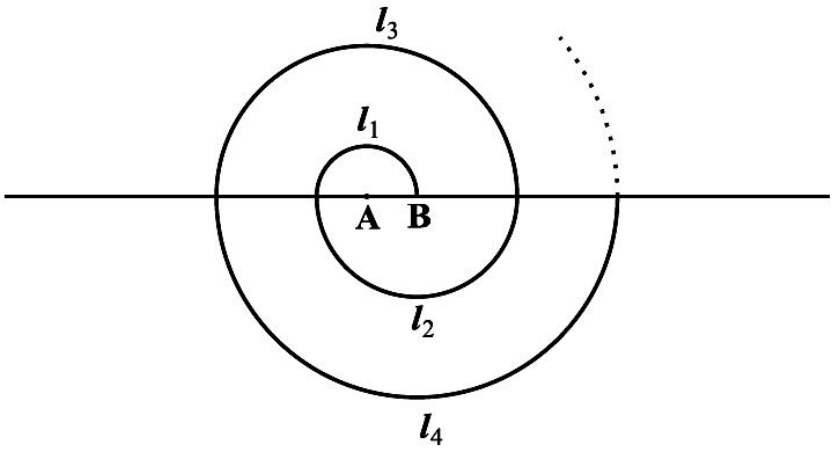


39. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

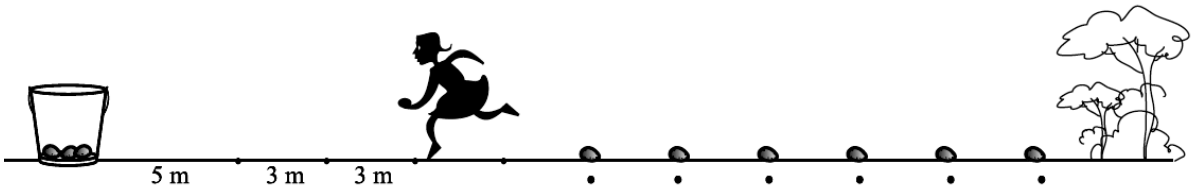
40. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

41. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

42. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... What is the total length of such a spiral made up of thirteen consecutive semicircles?



43. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?



44. The students of a school decided to beautify the school on the Annual Day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time. How much distance did she cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying a flag?

45. Show that the sum of an AP whose first term is a, the second term b and the last term c, is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$

MCQ QUESTIONS (1 mark)

1. The 10th term of the AP: 5, 8, 11, 14, ... is
(a) 32 (b) 35 (c) 38 (d) 185

2. In an AP if a = –7.2, d = 3.6, an = 7.2, then n is
(a) 1 (b) 3 (c) 4 (d) 5

3. In an AP, if d = –4, n = 7, an = 4, then a is
(a) 6 (b) 7 (c) 20 (d) 28

4. In an AP, if a = 3.5, d = 0, n = 101, then an will be
(a) 0 (b) 3.5 (c) 103.5 (d) 104.5

5. The list of numbers – 10, – 6, – 2, 2,... is
(a) an AP with d = – 16 (b) an AP with d = 4
(c) an AP with d = – 4 (d) not an AP

6. The 11th term of the AP: –5, –5/2 , 0, 5/2 , ...is
(a) –20 (b) 20 (c) –30 (d) 30

7. The first four terms of an AP, whose first term is –2 and the common difference is –2, are
(a) – 2, 0, 2, 4 (b) – 2, 4, – 8, 16 (c) – 2, – 4, – 6, – 8 (d) – 2, – 4, – 8, –16

8. The 21st term of the AP whose first two terms are –3 and 4 is
(a) 17 (b) 137 (c) 143 (d) –143

9. If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7th term?
(a) 30 (b) 33 (c) 37 (d) 38

10. Which term of the AP: 21, 42, 63, 84,... is 210?
(a) 9th (b) 10th (c) 11th (d) 12th

11. If the common difference of an AP is 5, then what is a18 – a13 ?
(a) 5 (b) 20 (c) 25 (d) 30

12. What is the common difference of an AP in which a18 – a14 = 32?
(a) 8 (b) – 8 (c) – 4 (d) 4

13. Two APs have the same common difference. The first term of one of these is –1 and that of the other is – 8. Then the difference between their 4th terms is
(a) –1 (b) – 8 (c) 7 (d) –9

14. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18th term will be
(a) 7 (b) 11 (c) 18 (d) 0

15. The 4th term from the end of the AP: –11, –8, –5, ..., 49 is
(a) 37 (b) 40 (c) 43 (d) 58

16. If the first term of an AP is –5 and the common difference is 2, then the sum of the first 6 terms is (a) 0 (b) 5 (c) 6 (d) 15

17. The sum of first 16 terms of the AP: 10, 6, 2,... is
(a) –320 (b) 320 (c) –352 (d) –400

18. In an AP if a = 1, an = 20 and Sn = 399, then n is
(a) 19 (b) 21 (c) 38 (d) 42

19. The sum of first 100 multiples of 3 is
(a) 30300 (b) 15150 (c) 300 (d) none of these

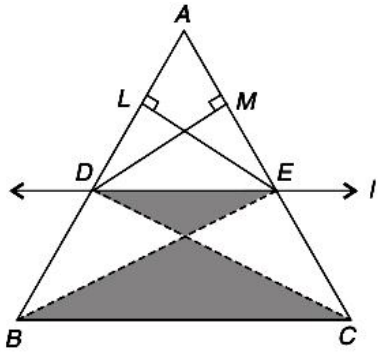
20. The sum of first five multiples of 3 is
(a) 45 (b) 55 (c) 65 (d) 75

CHAPTER – 6
TRIANGLES

IMPORTANT THEOREMS
BASIC PROPORTIONALITY THEOREM OR THALES THEOREM

If a straight line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

GIVEN: A ΔABC and line 'l' parallel to BC intersect AB at D and AC at E.



TO PROVE : $$\frac{AD}{DB} = \frac{AE}{EC}$$

CONSTRUCTION : Join BE and CD. Draw $EL \perp$ to AB and $DM \perp AC$.

PROOF: We know that areas of the triangles on the same base and between same parallel lines are equal, hence we have :

$$\text{area } (\Delta BDE) = \text{area } (\Delta CDE) \quad \text{...(i)}$$

Now, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2} \times AD \times EL}{\frac{1}{2} \times DB \times EL} = \frac{AD}{DB} \quad \text{...(ii)}$$

Again, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \text{...(iii)}$$

Put value form (i) in (ii), we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{AD}{DB} \quad \text{...(iv)}$$

On comparing equation (ii) and (iii), we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$ **Hence Proved.**

COROLLARY :

(i) $\frac{AB}{DB} = \frac{AC}{EC}$

(ii) $\frac{DB}{AD} = \frac{EC}{AE}$

(iii) $\frac{AB}{AD} = \frac{AC}{AE}$

(iv) $\frac{DB}{AB} = \frac{EC}{AC}$

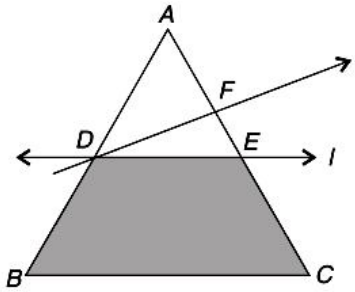
(v) $\frac{AD}{AB} = \frac{AE}{AC}$

CONVERSE OF BASIC PROPORTIONALITY THEOREM
(CONVERSE OF THALES THEOREM)

If a straight line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

GIVEN : A Δ ABC and line 'l' intersecting the sides AB at D and AC at E such that :

$$\frac{AD}{DB} = \frac{AE}{EC}$$



TO PROVE : $l \parallel BC$.

PROOF : Let us suppose that the line l is not parallel to BC.

Then through D, there must be any other line which must be parallel to BC.

Let $DF \parallel BC$, such that $E \neq F$.

Since, $DF \parallel BC$ (by supposition)

$$\frac{AD}{DB} = \frac{AF}{FC} \quad \text{...(i) (Basic Proportionality Theorem)}$$

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{...(ii) (Given)}$$

Comparing (i) and (ii), we get

$$\frac{AF}{FC} = \frac{AE}{EC}$$

Adding 1 to both sides, we get

$$\frac{AF}{FC} + 1 = \frac{AE}{EC} + 1$$

$$\Rightarrow \quad \frac{AF + FC}{FC} = \frac{AE + EC}{EC}$$

$$\Rightarrow \quad \frac{AC}{FC} = \frac{AC}{EC}$$

$$\Rightarrow \quad \frac{1}{FC} = \frac{1}{EC}$$

$$\Rightarrow \quad FC = EC$$

This shows that E and F must coincide, but it contradicts our supposition that $E \neq F$ and $DF \parallel BC$.

Hence, there is one and only line, $DE \parallel BC$, *i.e.*

$$\boxed{l \parallel BC}$$

Hence Proved.

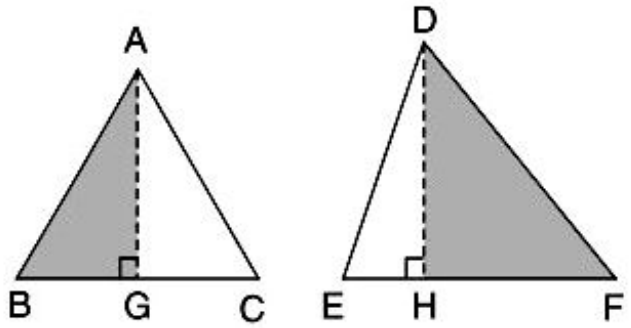
AREAS OF SIMILAR TRIANGLES THEOREM

The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

GIVEN : $\Delta ABC \sim \Delta DEF$

TO PROVE : $\dfrac{\text{Area of } (\Delta ABC)}{\text{Area of } (\Delta DEF)} = \dfrac{BC^2}{EF^2} = \dfrac{AB^2}{DE^2} = \dfrac{AC^2}{DF^2}$

CONSTRUCTION : Draw $AG \perp BC$ and $DH \perp EF$.



PROOF :

$$\frac{\text{Area } (\Delta ABC)}{\text{Area } (\Delta DEF)} = \frac{\frac{1}{2} \times BC \times AG}{\frac{1}{2} \times EF \times DH} = \frac{BC}{EF} \times \frac{AG}{DH} \quad \text{...(i)}$$

(area of Δ = ½ × base × height)

Now in triangle ABG and DEH, we have

$\angle B = \angle E$ (since, $\Delta ABC \sim \Delta DEF$)

$\angle AGB = \angle DHE$ (each 90°)

Therefore, $\Delta ABG \sim \Delta DEH$ (by AA criterion)

Hence, $\frac{AB}{DE} = \frac{AG}{DH}$ ...(ii) (Using property of similar triangles)

$\Rightarrow$ $\frac{AB}{DE} = \frac{BC}{EF}$ ...(iii) (since, $\Delta ABC \sim \Delta DEF$)

Comparing (ii) and (iii), we get

$$\frac{AG}{DH} = \frac{BC}{EF} \quad \text{...(iv)}$$

Using (i) and (iv), we get

$$\frac{\text{Area } (\Delta ABC)}{\text{Area } (\Delta DEF)} = \frac{BC}{EF} \times \frac{BC}{EF} = \frac{BC^2}{EF^2} \quad \text{...(v)}$$

$$\frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF} \quad \text{...(vi) (since, } \Delta ABC \sim \Delta DEF)$$

Using (v) and (vi), we get

$$\boxed{\frac{\text{Area of } (\Delta ABC)}{\text{Area of } (\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}}$$

Hence Proved.

PYTHAGORAS THEOREM

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

GIVEN : ΔABC is right angled at B.

TO PROVE : $AC^2 = AB^2 + BC^2$.

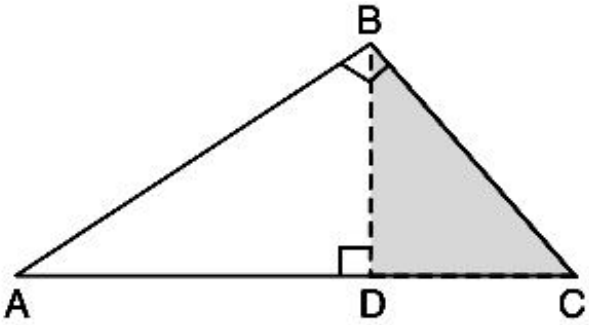
CONSTRUCTION : Draw $BD \perp AC$.

PROOF : Taking ΔADB and ΔABC

$\angle B = \angle ADB$ (each 90°)

$\angle A = \angle A$ (common)

Therefore, $\Delta ADB \sim \Delta ABC$ (by AA criterion)



Hence, $\frac{AD}{AB} = \frac{AB}{AC}$

$\Rightarrow$ $AB^2 = AD \times AC$ ...(i)

Now taking ΔCDB and ΔCBA

$\angle B = \angle BDC$ (each 90°)

$\angle C = \angle C$ (common)

Therefore, $\Delta CDB \sim \Delta CBA$ (by AA criterion)

Hence, $\frac{CD}{BC} = \frac{BC}{AC}$

$\Rightarrow$ $BC^2 = CD \times AC$ ...(ii)

Adding (i) and (ii), we get

$\Rightarrow$ $AB^2 + BC^2 = AD \times AC + CD \times AC$

$\Rightarrow$ $AB^2 + BC^2 = AC \times (AD + CD)$

$\Rightarrow$ $AB^2 + BC^2 = AC \times AC$

$\boxed{AC^2 = AB^2 + BC^2}$

Hence Proved.

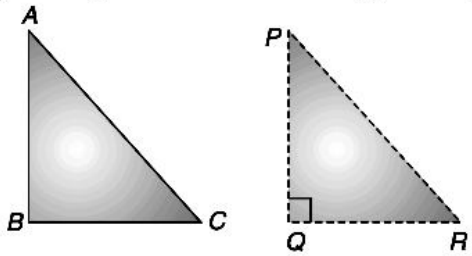
Converse of Pythagoras theorem

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

GIVEN : $AC^2 = AB^2 + BC^2$. ...(i)

TO PROVE : ΔABC is right angled at B.

CONSTRUCTION : Draw right ΔPQR such that $AB = PQ$, $BC = QR$ and $\angle Q = 90°$.



PROOF : Using Pythagoras theorem in ΔPQR, we get

$PR^2 = PQ^2 + QR^2$ ...(ii)

By construction, $AB = PQ$

$BC = QR$, substituting these values in (ii), we get

$PR^2 = AB^2 + BC^2$ ...(iii)

Comparing (i) and (iii), we get

$AC^2 = PR^2$

$\Rightarrow$ $AC = PR$ ...(iv)

In ΔABC and ΔPQR

	$AB = PQ$	(by construction)
	$BC = QR$	(by construction)
	$AC = PR$	(proved above in (iv))
$\Rightarrow$	$\Delta ABC = \Delta PQR$	(by SSS congruence rule)
$\Rightarrow$	$\angle B = \angle Q$	(by cpct)
But	$\angle Q = 90^\circ$	(by construction)
Hence,	$\angle B = 90^\circ$	
	$\boxed{\Delta ABC \text{ is right angled at } B.}$	

Hence Proved.

CHAPTER – 7
COORDINATE GEOMETRY

DISTANCE FORMULA

The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$or \quad AB = \sqrt{(\text{difference of abscissae})^2 + (\text{difference of ordinates})^2}$$

Distance of a point from origin

The distance of a point P(x, y) from origin O is given by OP = $\sqrt{x^2 + y^2}$

Problems based on geometrical figure

To show that a given figure is a

- Parallelogram – prove that the opposite sides are equal
- Rectangle – prove that the opposite sides are equal and the diagonals are equal.
- Parallelogram but not rectangle – prove that the opposite sides are equal and the diagonals are not equal.
- Rhombus – prove that the four sides are equal
- Square – prove that the four sides are equal and the diagonals are equal.
- Rhombus but not square – prove that the four sides are equal and the diagonals are not equal.
- Isosceles triangle – prove any two sides are equal.
- Equilateral triangle – prove that all three sides are equal.
- Right triangle – prove that sides of triangle satisfies Pythagoras theorem.

IMPORTANT QUESTIONS

Show that the points (1, 7), (4, 2), (–1, –1) and (– 4, 4) are the vertices of a square.

Solution : Let A(1, 7), B(4, 2), C(–1, –1) and D(– 4, 4) be the given points.

$$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$$

$$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$$

$$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$$

$$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$$

$$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$$

$$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$$

Since, AB = BC = CD = DA and AC = BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Therefore, ABCD is a square.

Find a point on the y-axis which is equidistant from the points A(6, 5) and B(– 4, 3).

Solution : We know that a point on the y-axis is of the form (0, y). So, let the point P(0, y) be equidistant from A and B. Then $AP^2 = BP^2$

$\Rightarrow (6 - 0)^2 + (5 - y)^2 = (-4 - 0)^2 + (3 - y)^2$

$\Rightarrow 36 + 25 + y^2 - 10y = 16 + 9 + y^2 - 6y \Rightarrow 4y = 36 \Rightarrow y = 9$

So, the required point is (0, 9).

Questions for practice

1. Show that the points A(1, 2), B(5, 4), C(3, 8) and D(–1, 6) are vertices of a square.
2. Show that the points A(5, 6), B(1, 5), C(2, 1) and D(6, 2) are vertices of a square.
3. Show that the points A(1, –3), B(13, 9), C(10, 12) and D(–2, 0) are vertices of a rectangle.
4. Show that the points A(1, 0), B(5, 3), C(2, 7) and D(–2, 4) are vertices of a rhombus.
5. Prove that the points A(–2, –1), B(1, 0), C(4, 3) and D(1, 2) are vertices of a parallelogram.
6. Find the point on x-axis which is equidistant from (7, 6) and (–3, 4).
7. Find the point on the x-axis which is equidistant from (2, –5) and (–2, 9).
8. Find a point on the y-axis which is equidistant from the points A(5, 2) and B(– 4, 3).
9. Find a point on the y-axis which is equidistant from the points A(5, – 2) and B(– 3, 2).
10. Find the values of y for which the distance between the points P(2, – 3) and Q(10, y) is 10 units.
11. Find the value of a , if the distance between the points A (–3, –14) and B (a, –5) is 9 units.
12. If the point A (2, – 4) is equidistant from P (3, 8) and Q (–10, y), find the values of y. Also find distance PQ.

Section formula

The coordinates of the point P(x, y) which divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2), internally, in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

Mid-point formula

The coordinates of the point P(x, y) which is the midpoint of the line segment joining the points A(x_1, y_1) and B(x_2, y_2), are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

IMPORTANT QUESTIONS

Find the coordinates of the point which divides the line segment joining the points (4, – 3) and (8, 5) in the ratio 3 : 1 internally.

Solution : Let P(x, y) be the required point.

Using the section formula, $x=\frac{m_2x_1+m_1x_2}{m_1+m_2}, y=\frac{m_2y_1+m_1y_2}{m_1+m_2}$ we get

$$x=\frac{3(8)+1(4)}{3+1}=7, y=\frac{3(5)+1(-3)}{3+1}=3$$

Therefore, (7, 3) is the required point.

In what ratio does the point (– 4, 6) divide the line segment joining the points A(– 6, 10) and B(3, – 8)?

Solution : Let (– 4, 6) divide AB internally in the ratio k : 1.

Using the section formula, $x=\frac{m_2x_1+m_1x_2}{m_1+m_2}, y=\frac{m_2y_1+m_1y_2}{m_1+m_2}$ we get

$$y=\frac{k(-8)+1(10)}{k+1}=6$$

$$\Rightarrow -8k+10=6k+6 \Rightarrow -8k-6k=6-10$$

$$\Rightarrow -14k=-4 \Rightarrow k=\frac{4}{14}=\frac{2}{7}$$

Therefore, the point (– 4, 6) divides the line segment joining the points A(– 6, 10) and B(3, – 8) in the ratio 2 : 7.

Questions for practice

1. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.
2. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).
3. Find the coordinates of the points of trisection (i.e., points dividing in three equal parts) of the line segment joining the points A(2, – 2) and B(– 7, 4).
4. Find the ratio in which the y-axis divides the line segment joining the points (5, – 6) and (–1, – 4). Also find the point of intersection.
5. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).
6. Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.
7. Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.
8. If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram, taken in order, find the value of p.
9. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.
10. In what ratio does the x–axis divide the line segment joining the points (– 4, – 6) and (–1, 7)? Find the coordinates of the point of division.
11. If P (9a – 2, –b) divides line segment joining A (3a + 1, –3) and B (8a, 5) in the ratio 3 : 1, find the values of a and b.
12. If (a, b) is the mid-point of the line segment joining the points A (10, –6) and B (k, 4) and a – 2b = 18, find the value of k and the distance AB.
13. The centre of a circle is (2a, a – 7). Find the values of a if the circle passes through the point (11, –9) and has diameter $10\sqrt{2}$ units.
14. The line segment joining the points A (3, 2) and B (5,1) is divided at the point P in the ratio 1:2 and it lies on the line 3x – 18y + k = 0. Find the value of k.
15. Find the coordinates of the point R on the line segment joining the points P (–1, 3) and Q (2, 5) such that PR = $\frac{3}{5}$PQ.
16. Find the values of k if the points A (k + 1, 2k), B (3k, 2k + 3) and C (5k – 1, 5k) are collinear.
17. Find the ratio in which the line 2x + 3y – 5 = 0 divides the line segment joining the points (8, –9) and (2, 1). Also find the coordinates of the point of division.
18. The mid-points D, E, F of the sides of a triangle ABC are (3, 4), (8, 9) and (6, 7). Find the coordinates of the vertices of the triangle.

Area of a Triangle

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a ΔABC, then the area of ΔABC is given by

$$Area\ \ of\ \ \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Trick to remember the formula

The formula of area of a triangle can be learn with the help of following arrow diagram:

$$\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

Find the sum of products of numbers at the ends of the lines pointing downwards and then subtract the sum of products of numbers at the ends of the line pointing upwards, multiply the difference by $\frac{1}{2}$. i.e. $Area\ \ of\ \ \Delta ABC = \frac{1}{2}[(x_1y_2 + x_2y_3 + x_3y_1) - (x_1y_3 + x_3y_2 + x_2y_1]$

IMPORTANT QUESTIONS

Find the area of a triangle whose vertices are (1, –1), (– 4, 6) and (–3, –5).

Solution: Here, A(1, –1), B(– 4, 6) and C (–3, –5).

Using the formula

$$\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

we get

$$\Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & -1 \\ -4 & 6 \\ -3 & -5 \\ 1 & -1 \end{vmatrix}$$

$\Delta ABC = \frac{1}{2}[(6 + 20 + 3) - (-5 - 18 + 4)] = \frac{1}{2}[29 - (-19)] = \frac{1}{2}(29 + 19) = \frac{1}{2} \times 48 = 24$ sq. units

So, the area of the triangle is 24 square units.

Questions for practice

1. Find the area of a triangle formed by the points A(5, 2), B(4, 7) and C (7, – 4).

2. Find the area of the triangle formed by the points P(–1.5, 3), Q(6, –2) and R(–3, 4).

3. Find the value of k if the points A(2, 3), B(4, k) and C(6, –3) are collinear.

4. If A(–5, 7), B(– 4, –5), C(–1, –6) and D(4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.

5. Find the area of the triangle whose vertices are : (i) (2, 3), (–1, 0), (2, – 4) (ii) (–5, –1), (3, –5), (5, 2)

6. In each of the following find the value of 'k', for which the points are collinear. (i) (7, –2), (5, 1), (3, k) (ii) (8, 1), (k, – 4), (2, –5)

7. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

8. Find the area of the quadrilateral whose vertices, taken in order, are (– 4, – 2), (– 3, – 5), (3, – 2) and (2, 3).

9. Find the value of m if the points (5, 1), (–2, –3) and (8, 2m) are collinear.

10. Find the area of the triangle whose vertices are (–8, 4), (–6, 6) and (–3, 9).

11. A (6, 1), B (8, 2) and C (9, 4) are three vertices of a parallelogram ABCD. If E is the midpoint of DC, find the area of Δ ADE.

12. The points A (2, 9), B (a, 5) and C (5, 5) are the vertices of a triangle ABC right angled at B. Find the values of a and hence the area of ΔABC.

13. If the points A (1, –2), B (2, 3) C (a, 2) and D (– 4, –3) form a parallelogram, find the value of a and height of the parallelogram taking AB as base.

MCQ QUESTIONS (1 mark)

1. If the distance between the points (2, –2) and (–1, x) is 5, one of the values of x is
(a) –2 (b) 2 (c) –1 (d) 1

2. The mid-point of the line segment joining the points A (–2, 8) and B (– 6, – 4) is
(a) (– 4, – 6) (b) (2, 6) (c) (– 4, 2) (d) (4, 2)

3. The distance of the point P (2, 3) from the x-axis is
(a) 2 (b) 3 (c) 1 (d) 5

4. The distance between the points A (0, 6) and B (0, –2) is
(a) 6 (b) 8 (c) 4 (d) 2

5. The distance of the point P (–6, 8) from the origin is
(a) 8 (b) $2\sqrt{7}$ (c) 10 (d) 6

6. The distance between the points (0, 5) and (–5, 0) is
(a) 5 (b) $5\sqrt{2}$ (c) $2\sqrt{5}$ (d) 10

7. AOBC is a rectangle whose three vertices are vertices A (0, 3), O (0, 0) and B (5, 0). The length of its diagonal is
(a) 5 (b) 3 (c) $\sqrt{34}$ (d) 4

8. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is
(a) 5 (b) 12 (c) 11 (d) $7 + \sqrt{5}$

9. The area of a triangle with vertices A (3, 0), B (7, 0) and C (8, 4) is
(a) 14 (b) 28 (c) 8 (d) 6

10. The points (–4, 0), (4, 0), (0, 3) are the vertices of a
(a) right triangle (b) isosceles triangle
(c) equilateral triangle (d) scalene triangle

11. The point which divides the line segment joining the points (7, –6) and (3, 4) in ratio 1 : 2 internally lies in the
(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

12. The point which lies on the perpendicular bisector of the line segment joining the points A (–2, –5) and B (2, 5) is
(a) (0, 0) (b) (0, 2) (c) (2, 0) (d) (–2, 0)

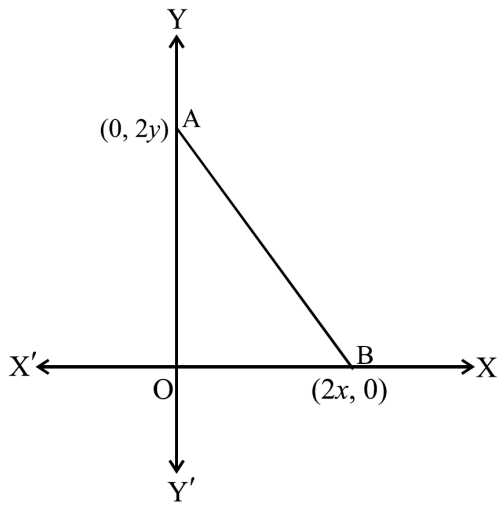
13. The fourth vertex D of a parallelogram ABCD whose three vertices are A (–2, 3), B (6, 7) and C (8, 3) is
(a) (0, 1) (b) (0, –1) (c) (–1, 0) (d) (1, 0)

14. If the point P (2, 1) lies on the line segment joining points A (4, 2) and B (8, 4), then
(a) AP = $\frac{1}{3}$AB (b) AP = PB (c) PB = $\frac{1}{3}$AB (d) AP = $\frac{1}{2}$AB

15. If $P\left(-\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points Q (– 6, 5) and R (– 2, 3), then the value of a is
(a) – 4 (b) – 12 (c) 12 (d) – 6

16. The perpendicular bisector of the line segment joining the points A (1, 5) and B (4, 6) cuts the y-axis at
(a) (0, 13) (b) (0, –13)
(c) (0, 12) (d) (13, 0)

17. The coordinates of the point which is equidistant from the three vertices of the Δ AOB as shown in the Fig. 7.1 is
(a) (x, y) (b) (y, x)
(c) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (d) $\left(\frac{y}{2}, \frac{x}{2}\right)$



18. A circle drawn with origin as the centre passes through (13/2 ,0). The point which does not lie in the interior of the circle is
(a) $\left(\frac{-3}{4}, 1\right)$ (b) $\left(2, \frac{7}{3}\right)$ (c) $\left(5, \frac{-1}{2}\right)$ (d) $\left(-6, \frac{5}{2}\right)$

19. A line intersects the y-axis and x-axis at the points P and Q, respectively. If (2, –5) is the mid-point of PQ, then the coordinates of P and Q are, respectively
(a) (0, – 5) and (2, 0) (b) (0, 10) and (– 4, 0)
(c) (0, 4) and (– 10, 0) (d) (0, – 10) and (4, 0)

20. The area of a triangle with vertices (a, b + c), (b, c + a) and (c, a + b) is
(a) (a + b + c)2 (b) 0 (c) a + b + c (d) abc

21. If the distance between the points (4, p) and (1, 0) is 5, then the value of p is
(a) 4 only (b) ± 4 (c) – 4 only (d) 0

22. If the points A (1, 2), O (0, 0) and C (a, b) are collinear, then
(a) a = b (b) a = 2b (c) 2a = b (d) a = –b

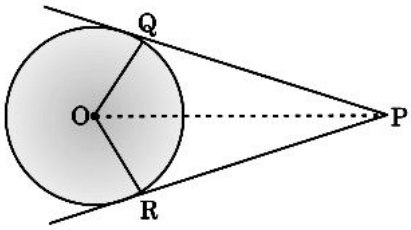
CHAPTER – 10
CIRCLES

THEOREMS

1) The tangent to a circle is perpendicular to the radius through the point of contact.
2) The lengths of tangents drawn from an external point to a circle are equal.

Given : A circle $C(O, r)$ and two tangents say PQ and PR from an external point P.

To prove : $PQ = PR$.



Construction : Join OQ, OR and OP.

Proof : In ΔOQP and ΔORP

$OQ = OR$ (radii of the same circle)

$OP = OP$ (Common)

$\angle Q = \angle R$ = each 90° (The tangent at any point of a circle is perpendicular to the radius through the point of contact)

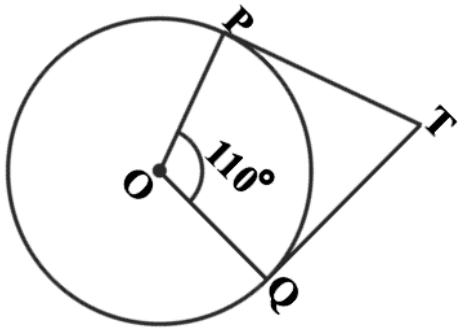
Hence $\Delta OQP \cong \Delta ORP$ (By RHS Criterion)

$\therefore$ $PQ = PR$

(By CPCT) **Hence Proved.**

IMPORTANT QUESTIONS

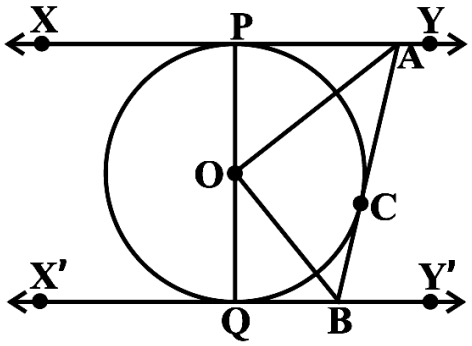
1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle
2. In the below figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle$POQ = 110°, then find $\angle$PTQ.



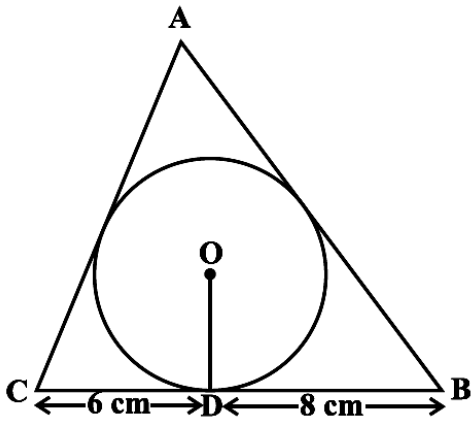
3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then find $\angle$POA
4. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.
5. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
6. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC
7. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.
8. Prove that the parallelogram circumscribing a circle is a rhombus.
9. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

10. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

11. XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that $\angle AOB = 90°$.



12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.



13. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\ \angle OPQ$.

14. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.

15. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

16. If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that $\angle DBC = 120°$, prove that BC + BD = BO, i.e., BO = 2BC.

17. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

18. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.

19. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. If PA = 10 cm, find the the perimeter of the triangle PCD.

20. In a right triangle ABC in which $\angle B = 90°$, a circle is drawn with AB as diameter intersecting the hypotenuse AC and P. Prove that the tangent to the circle at P bisects BC.

21. If d_1, d_2 ($d_2 > d_1$) be the diameters of two concentric circles and c be the length of a chord of a circle which is tangent to the other circle, prove that $d_2^{\,2} = c^2 + d_1^{\,2}$

22. If a, b, c are the sides of a right triangle where c is the hypotenuse, prove that the radius r of the circle which touches the sides of the triangle is given by $r = \dfrac{a+b+c}{2}$

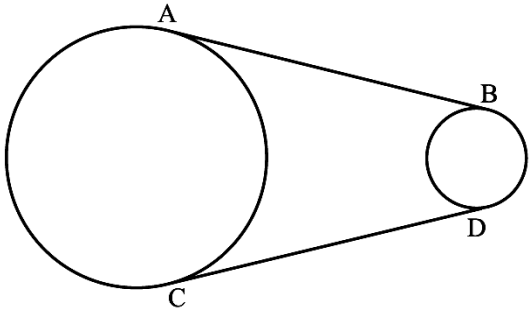
23. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

24. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

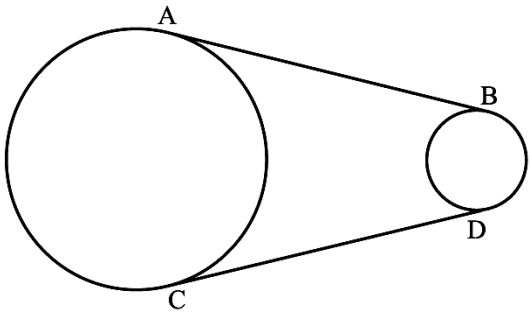
25. If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that $\angle DBC = 120°$, prove that BC + BD = BO, i.e., BO = 2BC.

26. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

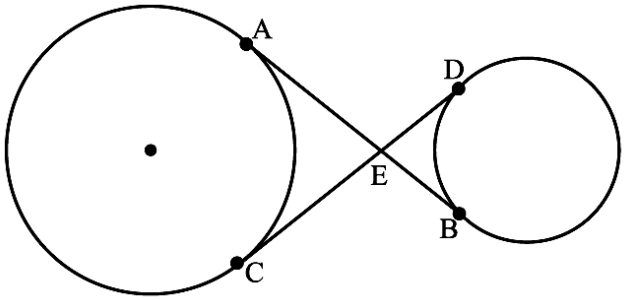
27. In below figure, AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD.



28. In below figure, AB and CD are common tangents to two circles, if radii of the two circles are equal, prove that AB = CD.



29. In below figure, common tangents AB and CD to two circles intersect at E. Prove that AB = CD.

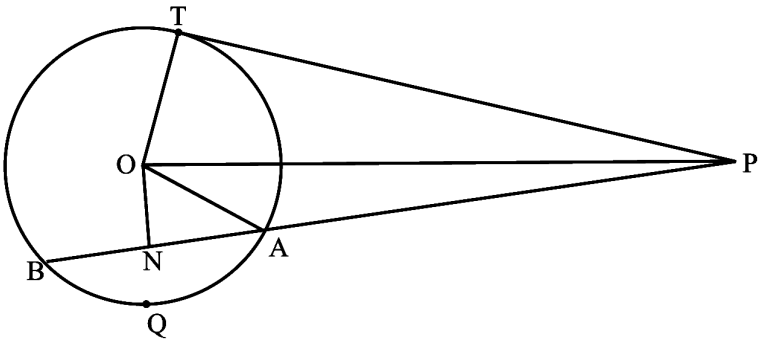


30. In below figure, from an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that :

(i) $PA \cdot PB = PN^2 - AN^2$

(ii) $PN^2 - AN^2 = OP^2 - OT^2$

(iii) $PA.PB = PT^2$



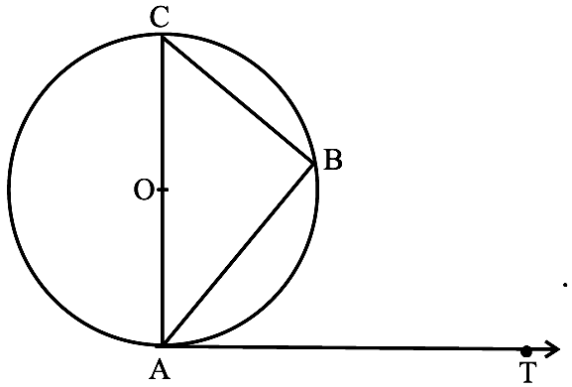
31. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that AQ = $\frac{1}{2}$(BC + CA + AB)

32. If a hexagon ABCDEF circumscribe a circle, prove that AB + CD + EF = BC + DE + FA.

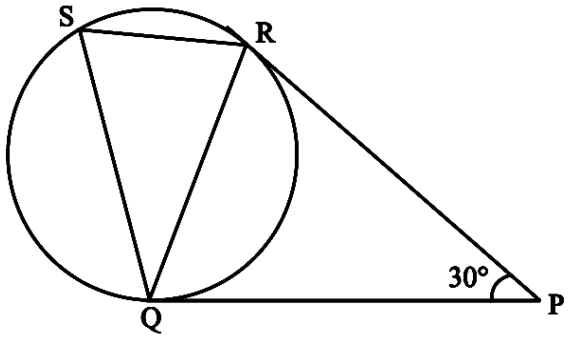
33. Let s denote the semi-perimeter of a triangle ABC in which BC = a, CA = b, AB = c. If a circle touches the sides BC, CA, AB at D, E, F, respectively, prove that BD = s – b.

34. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of the triangle PCD.

35. If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in below figure. Prove that ∠BAT = ∠ACB



36. In below figure, tangents PQ and PR are drawn to a circle such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find the ∠RQS.

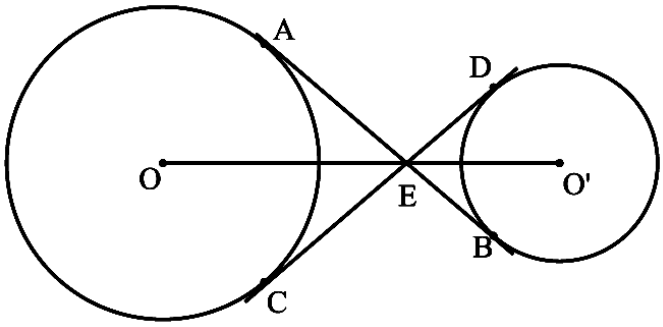


37. AB is a diameter and AC is a chord of a circle with centre O such that ∠BAC = 30°. The tangent at C intersects extended AB at a point D. Prove that BC = BD.

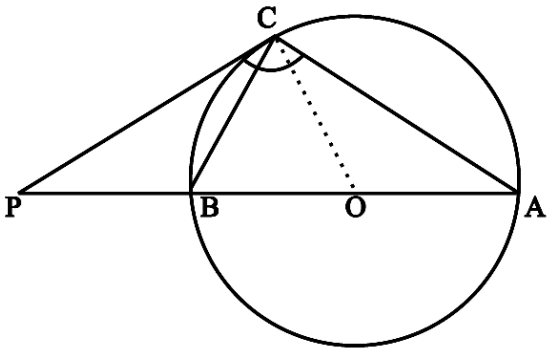
38. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

39. A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.

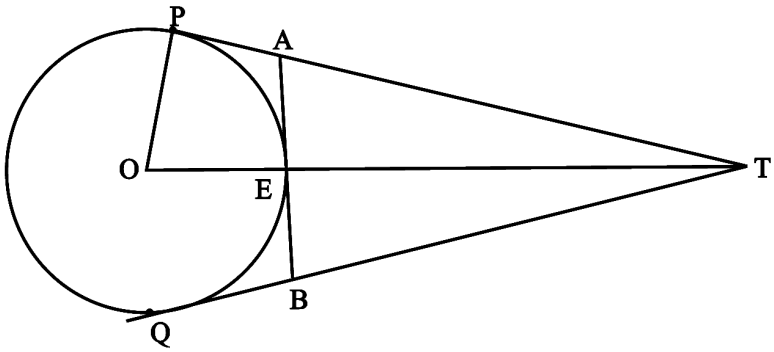
40. In below figure, the common tangent, AB and CD to two circles with centres O and O' intersect at E. Prove that the points O, E, O' are collinear.



41. The tangent at a point C of a circle and a diameter AB when extended intersect at P. If ∠PCA =110º , find ∠CBA



42. In below figure. O is the centre of a circle of radius 5 cm, T is a point such that OT = 13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E, find the length of AB.



43. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

44. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.

45. If an isosceles triangle ABC, in which AB = AC = 6 cm, is inscribed in a circle of radius 9 cm, find the area of the triangle.

46. Two circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.

47. In a right triangle ABC in which $\angle B = 90°$, a circle is drawn with AB as diameter intersecting the hypotenuse AC and P. Prove that the tangent to the circle at P bisects BC.

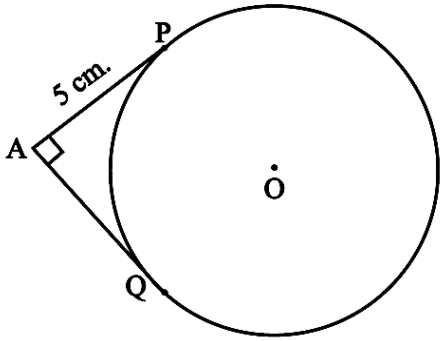
48. A is a point at a distance 13 cm from the centre O of a circle of radius 5 cm. AP and AQ are the tangents to the circle at P and Q. If a tangent BC is drawn at a point R lying on the minor arc PQ to intersect AP at B and AQ at C, find the perimeter of the ΔABC.

MCQ QUESTIONS (1 mark)

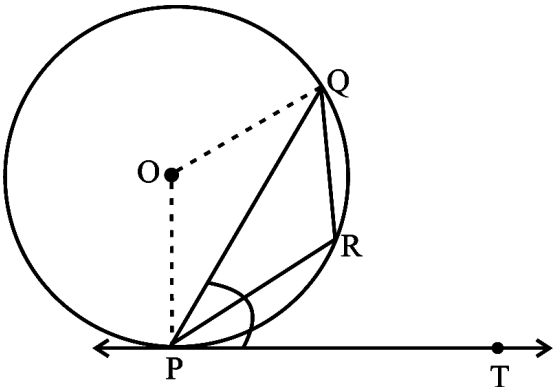
1. If angle between two radii of a circle is 130º, the angle between the tangents at the ends of the radii is :
(a) 90º (b) 50º (c) 70º (d) 40º

2. If radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other circle is
(a) 3 cm (b) 6 cm (c) 9 cm (d) 1 cm

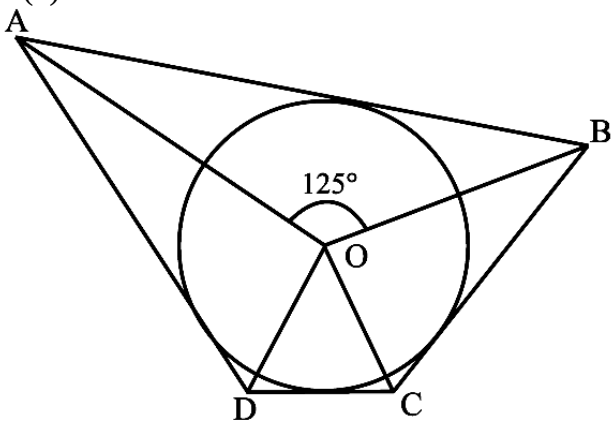
3. In the below figure, the pair of tangents AP and AQ drawn from an external point A to a circle with centre O are perpendicular to each other and length of each tangent is 5 cm. Then the radius of the circle is
(a) 10 cm (b) 7.5 cm (c) 5 cm (d) 2.5 cm



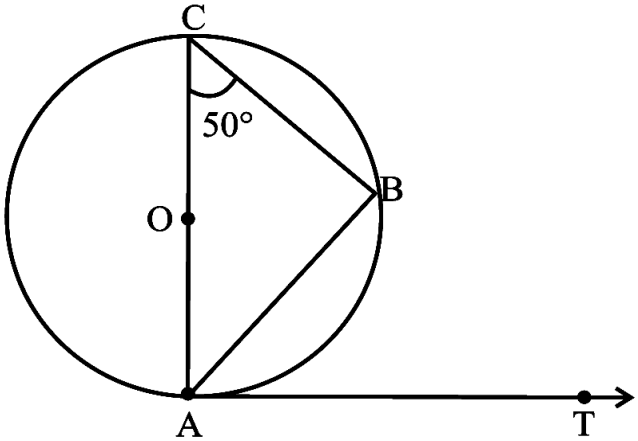
4. In below figure, PQ is a chord of a circle and PT is the tangent at P such that ∠QPT = 60°. Then ∠PRQ is equal to
(a) 135° (b) 150° (c) 120° (d) 110°



5. In the below figure, if ∠AOB = 125°, then ∠COD is equal to
(a) 62.5° (b) 45° (c) 35° (d) 55°



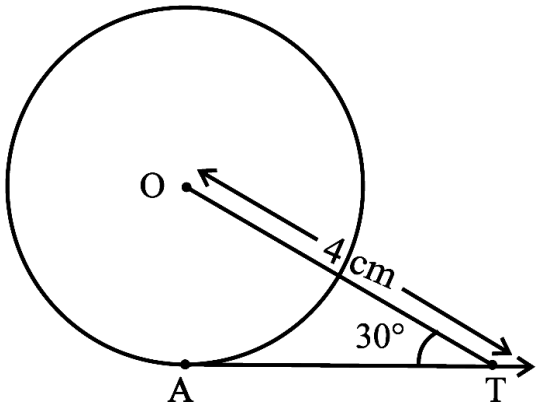
6. In the below figure, AB is a chord of the circle and AOC is its diameter such that ∠ACB = 50°. If AT is the tangent to the circle at the point A, then ∠BAT is equal to
(a) 65° (b) 60° (c) 50° (d) 40°



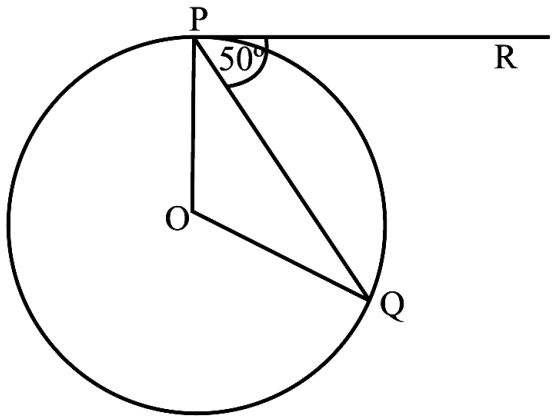
7. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is
(a) 60 cm^2 (b) 65 cm^2 (c) 30 cm^2 (d) 32.5 cm^2

8. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is
(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

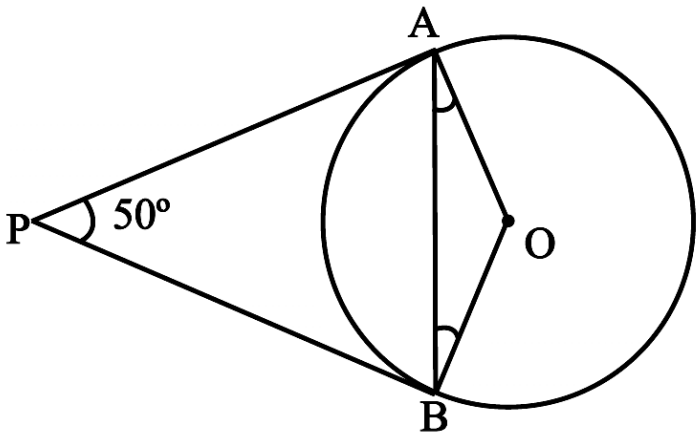
9. In below figure, AT is a tangent to the circle with centre O such that OT = 4 cm and ∠OTA = 30°. Then AT is equal to
(a) 4 cm (b) 2 cm (c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm



10. In below figure, if O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then ∠POQ is equal to
(a) 100° (b) 80° (c) 90° (d) 75°



11. In below figure, if PA and PB are tangents to the circle with centre O such that ∠APB = 50°, then ∠OAB is equal to
(a) 25° (b) 30° (c) 40° (d) 50°

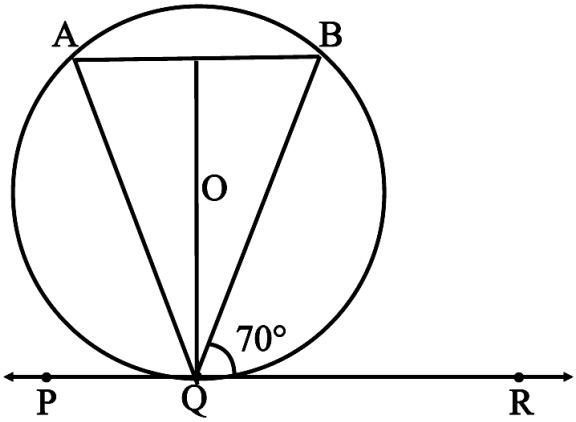


12. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then length of each tangent is equal to

(a) $\frac{3}{2}\sqrt{3}$ cm (b) 6 cm (c) 3 cm (d) $3\sqrt{3}$ cm

13. In below figure, if PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and ∠BQR = 70°, then ∠AQB is equal to

(a) 20° (b) 40° (c) 35° (d) 45°



CHAPTER – 11
CONSTRUCTIONS

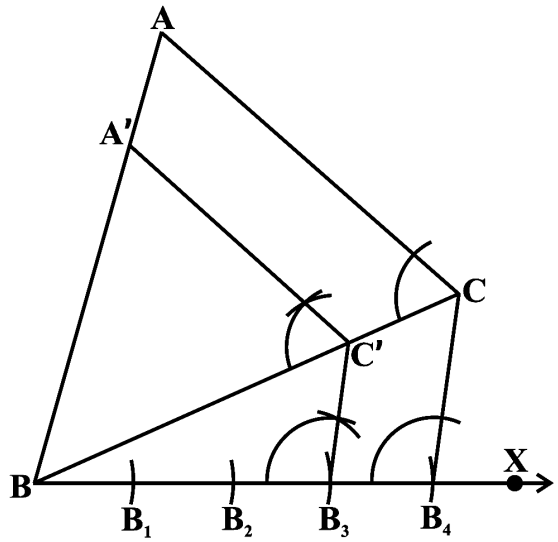
CONSTRUCTION OF SIMILAR TRIANGLE

Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{3}{4}$ of the corresponding sides of the triangle ABC (i.e., of scale factor $\frac{3}{4}$).

Steps of Construction :

- Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
- Locate 4 (the greater of 3 and 4 in $\frac{3}{4}$) points B_1, B_2, B_3 and B_4 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
- Join B_4C and draw a line through B3 (the 3rd point, 3 being smaller of 3 and 4 in $\frac{3}{4}$) parallel to B_4C to intersect BC at C′.
- Draw a line through C′ parallel to the line CA to intersect BA at A′ (see below figure).

Then, Δ A′BC′ is the required triangle.



Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{5}{3}$ of the corresponding sides of the triangle ABC (i.e., of scale factor $\frac{5}{3}$).

Steps of Construction :

- Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
- Locate 5 points (the greater of 5 and 3 in $\frac{5}{3}$) B1, B2, B3, B4 and B5 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
- Join B_3(the 3rd point, 3 being smaller of 3 and 5 in $\frac{5}{3}$) to C and draw a line through B_5 parallel to B_3C, intersecting the extended line segment BC at C′.
- Draw a line through C′ parallel to CA intersecting the extended line segment BA at A′ (see the below figure).
  Then A′BC′ is the required triangle.

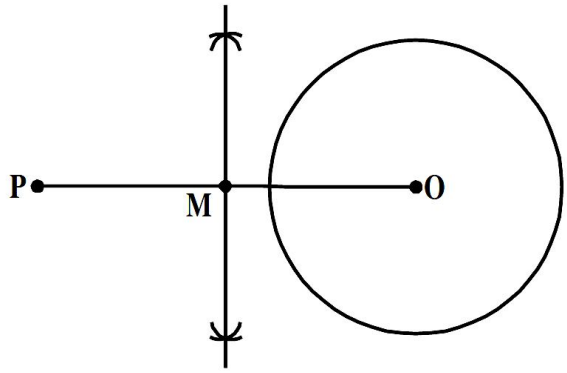
To construct the tangents to a circle from a point outside it.

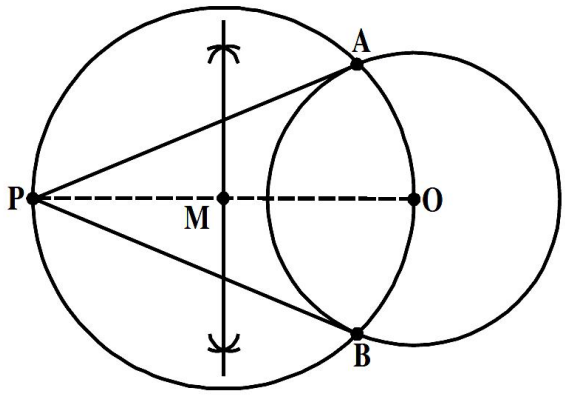
Given : We are given a circle with centre 'O' and a point P outside it. We have to construct two tangents from P to the circle.

Steps of construction :

- ☞ Join PO and draw a perpendicular bisector of it. Let M be the midpoint of PO.
- ☞ Taking M as centre and PM or MO as radius, draw a circle. Let it intersect the given circle at the points A and B.
- ☞ Join PA and PB.

Then PA and PB are the required two tangents.





IMPORTANT QUESTIONS FOR PRACTICE

1. Construct an isosceles triangle whose base is 7 cm and altitude 4 cm and then construct another similar triangle whose sides are $\frac{3}{2}$ time the corresponding sides of the isosceles triangle.

2. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

3. Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

4. Draw a triangle ABC with side BC = 7 cm, $\angle B = 45°$, $\angle A = 105°$. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC.

5. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.

6. Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

7. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

8. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

9. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

10. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

CHAPTER – 14
STATISTICS

MEAN OF GROUPED DATA

Direct method

Mean, $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

Assume mean method or Short-cut method

Mean, $\bar{x} = A + \dfrac{\sum f_i d_i}{\sum f_i}$ where $d_i = x_i - A$

Step Deviation method

Mean, $\bar{x} = A + \dfrac{\sum f_i u_i}{\sum f_i} \times h$ where $u = \dfrac{x_i - A}{h}$

IMPORTANT QUESTIONS

The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95
Number of cities	3	10	11	8	3

Solution:

Literacy rate (in %)	**Number of Cities 'f'**	**Class mark 'x'**	$u = \dfrac{x - A}{h}$	**fu**
45 – 55	3	50	–2	–6
55 – 65	10	60	–1	–10
65 – 75	11	70	0	0
75 – 85	8	80	1	8
85 – 95	3	90	2	6
Total	35			–2

Here, $\sum fu = -2, \sum f = 35$, A = 70, h = 10

Mean, $\bar{x} = A + \dfrac{\sum fu}{\sum f} \times h \;=\; \Rightarrow \bar{x} = 70 + \dfrac{-2}{35} \times 10 = 70 - \dfrac{20}{35} = 70 - \dfrac{4}{7} = 70 - 0.57 \;\Rightarrow \bar{x} = 69.43$

Questions for Practice

1. Find the mean of the following data:

Class Interval	10 – 25	25 – 40	40 – 55	55 – 70	70 – 85	85 – 100
Frequency	2	3	7	6	6	6

2. Find the mean percentage of female teachers of the following data:

Percentage of female teachers	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75	75 – 85
Number of States/U.T	6	11	7	4	4	2	1

3. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
Number of houses	1	2	1	5	6	2	3

4. Find the mean daily wages of the workers of the factory by using an appropriate method for the following data:

Daily wages (in Rs)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

5. Find the mean number of mangoes kept in a packing box for the following data:

Number of mangoes	50 – 52	53 – 55	56 – 58	59 – 61	62 – 64
Number of boxes	15	110	135	115	25

6. Find the mean daily expenditure on food for the following data:

Daily expenditure (in Rs.)	100 – 150	150 – 200	200 – 250	250 – 300	300 – 350
Number of households	4	5	12	2	2

MODE OF GROUPED DATA

$$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where l = lower limit of the modal class,
h = size of the class interval (assuming all class sizes to be equal),
f_1 = frequency of the modal class,
f_0 = frequency of the class preceding the modal class,
f_2 = frequency of the class succeeding the modal class.

IMPORTANT QUESTIONS

Find the mean, mode and median for the following frequency distribution.

Class	0-10	10-20	20-30	30-40	40-50	Total
Frequency	8	16	36	34	6	100

Solution:

Here, highest frequency is 36 which belongs to class 20 – 30. So, modal class is 20 – 30,
l = 20, f_0 = 16, f_1 = 36, f_2 = 34, h = 10

We know that $Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

$$\Rightarrow Mode = 20 + \frac{36-16}{2(36)-16-34} \times 10$$

$$\Rightarrow Mode = 20 + \frac{20}{72-50} \times 10 = 20 + \frac{200}{22} = 20 + 9.09 = 29.09$$

Questions for Practice

1. The frequency distribution table of agriculture holdings in a village is given below:

Area of land(in ha)	1-3	3-5	5-7	79	9-11	11-13
No. of families	20	45	80	55	40	12

Find the modal agriculture holdings of the village.

2. Find the mode age of the patients from the following distribution :

Age(in years)	6-15	16-25	26-35	36-45	46-55	56-65
No. of patients	6	11	21	23	14	5

3. Find the mode of the following frequency distribution:

Class	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

4. Find the modal height of maximum number of students from the following distribution:

Height(in cm)	160-162	163-165	166-168	169-171	172-174
No. of students	15	118	142	127	18

5. A survey regarding the heights (in cms) of 50 girls of a class was conducted and the following data was obtained.

Height(in cm)	120-130	130-140	140-150	150-160	160-170	**Total**
No. of girls	2	8	12	20	8	**50**

Find the mode of the above data.

- **Cumulative Frequency:** The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceeding the given class.

MEDIAN OF GROUPED DATA

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

where l = lower limit of median class,
n = number of observations,
cf = cumulative frequency of class preceding the median class,
f = frequency of median class,
h = class size (assuming class size to be equal).

EMPIRICAL FORMULA

3Median = Mode + 2 Mean

IMPORTANT QUESTIONS

Find the median of the following frequency distribution:

Class	75-84	85-94	95-104	105-114	115-124	125-134	135-144
Frequency	8	11	26	31	18	4	2

Solution:

Class	**True Class limits**	**Frequency**	**cf**
75-84	74.5 – 84.5	8	8
85-94	84.5 – 94.5	11	19
95-104	94.5 – 104.5	26	45
105-114	104.5 – 114.5	31	76
115-124	114.5 – 124.5	18	94
125-134	124.5 – 134.5	4	98
135-144	134.5 – 144.5	2	100
Total		**100**	

Here, n = 100 $\Rightarrow \frac{n}{2} = 50$ which belongs to 104.5 – 114.5

So, l = 104.5, cf = 45, f = 31, h = 10

We know that $Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$

$$\Rightarrow Median = 104.5 + \frac{50-45}{31} \times 10 \Rightarrow Median = 104.5 + \frac{50}{31} = 104.5 + 1.61 = 106.11$$

Questions for Practice

1. The percentage of marks obtained by 100 students in an examination are given below:

Marks	30-35	35-40	40-45	45-50	50-55	55-60	60-65
No. of Students	14	16	18	23	18	8	3

Determine the median percentage of marks.

2. Weekly income of 600 families is as under:

Income(in Rs.)	0-1000	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000
No. of Families	250	190	100	40	15	5

Compute the median income.

3. Find the median of the following frequency distribution:

Marks	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40
Number of students	8	12	20	12	18	13	10	7

4. The following table gives the distribution of the life time of 500 neon lamps:

Life time (in hrs)	1500 – 2000	2000 – 2500	2500 – 3000	3000 – 3500	3500 – 4000	4000 – 4500	4500 – 5000
Number of Lamps	24	86	90	115	95	72	18

Find the median life time of a lamp.

5. Find the median marks for the following distribution:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of Students	6	15	29	41	60	70

6. Find the median wages for the following frequency distribution:

Wages per day	61-70	71-80	81-90	91-100	101-110	111-120
No. of workers	5	15	20	30	10	8

7. Find the median marks for the following distribution:

Marks	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
No. of Students	2	3	6	7	14	12	4	2

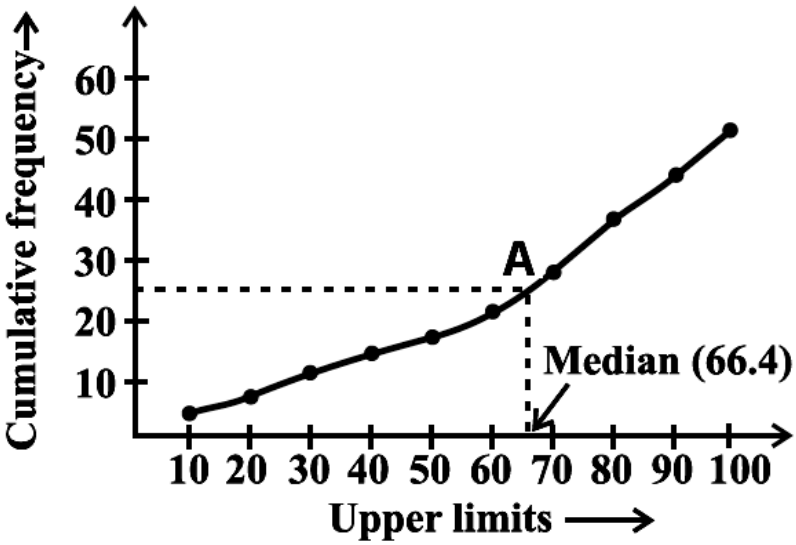
❖ CUMULATIVE FREQUENCY CURVE IS ALSO KNOWN AS ‘OGIVE’.

There are three methods of drawing ogive:

1. LESS THAN METHOD

Steps involved in calculating median using less than Ogive approach-

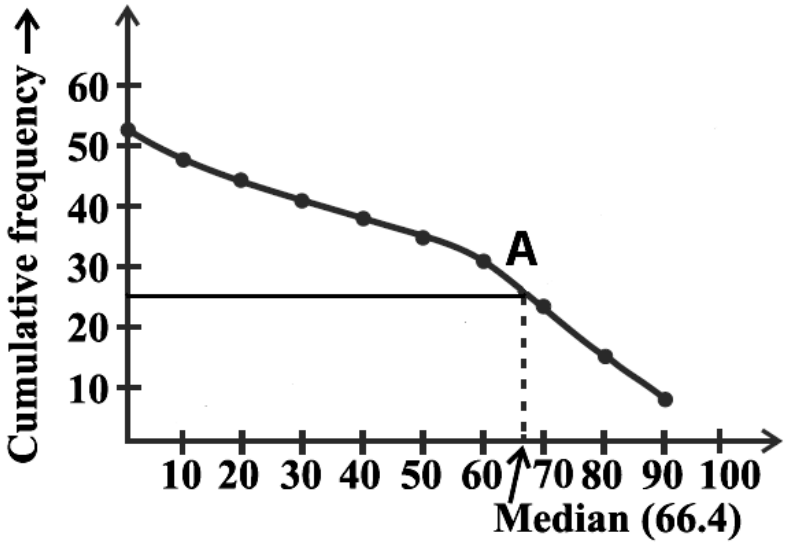
- Convert the series into a 'less than ' cumulative frequency distribution.
- Let N be the total number of students who's data is given. N will also be the cumulative frequency of the last interval. Find the $(N/2)^{th}$ itemand mark it on the y-axis.
- Draw a perpendicular from that point to the right to cut the Ogive curve at point A.
- From point A where the Ogive curve is cut, draw a perpendicular on the x-axis. The point at which it touches the x-axis will be the median value of the series as shown in the graph.



2. MORE THAN METHOD

Steps involved in calculating median using more than Ogive approach-

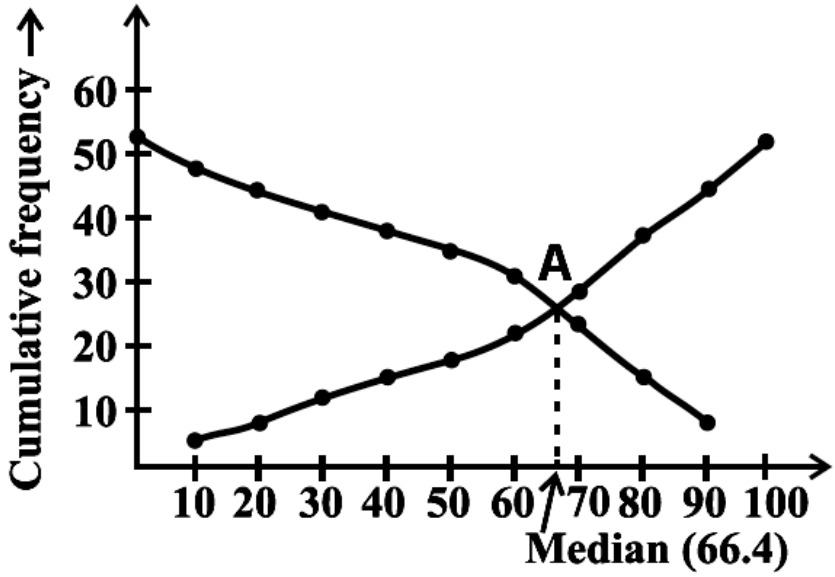
- Convert the series into a 'more than ' cumulative frequency distribution.
- Let N be the total number of students who's data is given. N will also be the cumulative frequency of the last interval. Find the $(N/2)^{th}$ item and mark it on the y-axis.
- Draw a perpendicular from that point to the right to cut the Ogive curve at point A.
- From point A where the Ogive curve is cut, draw a perpendicular on the x-axis. The point at which it touches the x-axis will be the median value of the series as shown in the graph.



3. LESS THAN AND MORE THAN OGIVE METHOD

Another way of graphical determination of median is through simultaneous graphic presentation of both the less than and more than Ogives.

- Mark the point A where the Ogive curves cut each other.
- Draw a perpendicular from A on the x-axis. The corresponding value on the x-axis would be the median value.



❖ The median of grouped data can be obtained graphically as the *x*-coordinate of the point of intersection of the two ogives for this data.

IMPORTANT QUESTIONS

The following distribution gives the daily income of 50 workers of a factory.

Daily income (in Rs)	100 – 120	120 – 140	140 –160	160 –180	180 –200
Number of workers	12	14	8	6	10

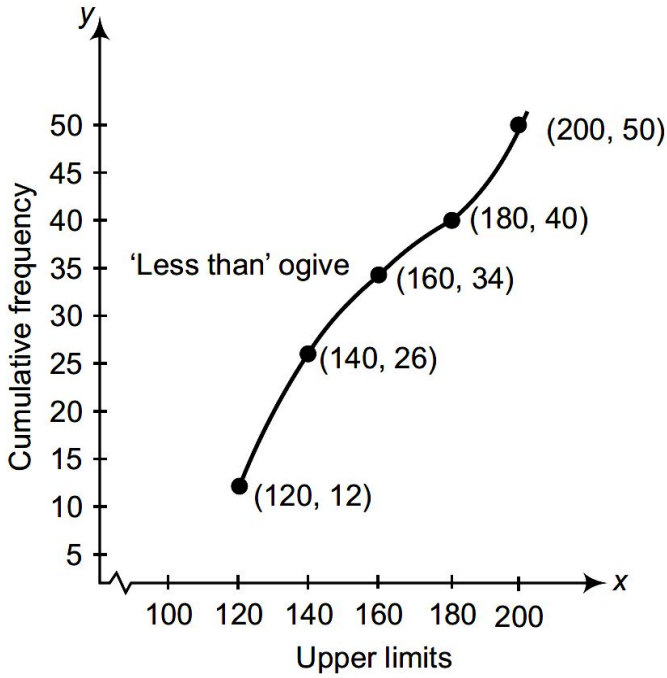
Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

Solution:

Cumulative frequency less than type

Daily income (in Rs)	**Less than type cf**
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

On the graph, we will plot the points (120, 12), (140, 26), (160, 34), (180, 40) and (200, 50).



Questions for Practice

1. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (in kg/ha)	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution, and draw its ogive.

2. For the following distribution, draw the cumulative frequency curve more than type and hence obtain the median from the graph.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

3. Draw less than ogive for the following frequency distribution:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	5	8	6	10	6	6

Also find the median from the graph and verify that by using the formula.

4. The table given below shows the frequency distribution of the cores obtained by 200 candidates in a BCA examination.

Score	200-250	250-300	300-350	350-400	400-450	450-500	500-550	550-600
No. of students	30	15	45	20	25	40	10	15

Draw cumulative frequency curves by using (i) less than type and (ii) more than type. Hence find median

5. Draw less than and more than ogive for the following frequency distribution:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	8	5	10	6	6	6

Also find the median from the graph and verify that by using the formula.

MCQ QUESTIONS (1 mark)

1. Construction of a cumulative frequency table is useful in determining the
(a) mean (b) median (c) mode (d) all the above three measures

2. In the following distribution :

Monthly income range (in Rs)	**Number of families**
Income more than Rs 10000	100
Income more than Rs 13000	85
Income more than Rs 16000	69
Income more than Rs 19000	50
Income more than Rs 22000	33
Income more than Rs 25000	15

the number of families having income range (in Rs) 16000 – 19000 is
(a) 15 (b) 16 (c) 17 (d) 19

3. Consider the following frequency distribution of the heights of 60 students of a class :

Height (in cm)	**Number of students**
150-155	15
155-160	13
160-165	10
165-170	8
170-175	9
175-180	5

The sum of the lower limit of the modal class and upper limit of the median class is
(a) 310 (b) 315 (c) 320 (d) 330

4. In the formula $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$, for finding the mean of grouped data d_i's are deviations from a of

(a) lower limits of the classes
(b) upper limits of the classes
(c) mid points of the classes
(d) frequencies of the class marks

5. While computing mean of grouped data, we assume that the frequencies are
(a) evenly distributed over all the classes (b) centred at the classmarks of the classes
(c) centred at the upper limits of the classes (d) centred at the lower limits of the classes

6. The abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives its
(a) mean (b) median (c) mode (d) all the three above

7. For the following distribution :

Class	Frequency
0-5	10
5-10	15
10-15	12
15-20	20
20-25	9

the sum of lower limits of the median class and modal class is
(a) 15 (b) 25 (c) 30 (d) 35

8. Consider the following frequency distribution :

Class	Frequency
0-5	13
6-11	10
12-17	15
18-23	8
24-29	11

The upper limit of the median class is
(a) 17 (b) 17.5 (c) 18 (d) 18.5

9. For the following distribution :

Marks	Number of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

the modal class is
(a) 10-20 (b) 20-30 (c) 30-40 (d) 50-60

10. Consider the data :

Class	Frequency
65-85	4
85-105	5
105-125	13
125-145	20
145-165	14
165-185	7
185-205	4

The difference of the upper limit of the median class and the lower limit of the modal class is
(a) 0 (b) 19 (c) 20 (d) 38

11. The times, in seconds, taken by 150 atheletes to run a 110 m hurdle race are tabulated below :

Class	Frequency
13.8-14	2
14-14.2	4
14.2-14.4	5
14.4-14.6	71
14.6-14.8	48
14.8-15	20

The number of atheletes who completed the race in less then 14.6 seconds is :
(a) 11 (b) 71 (c) 82 (d) 130

12. Consider the following distribution :

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

the frequency of the class 30-40 is
(a) 3 (b) 4 (c) 48 (d) 51

13. If x_i's are the mid points of the class intervals of grouped data, f_i's are the corresponding frequencies and $\bar{x}$ is the mean, then $\sum(f_i x_i - \bar{x})$ is equal to
(a) 0 (b) –1 (c) 1 (d) 2

14. In the formula $\bar{x} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$, for finding the mean of grouped frequency distribution, u_i =

(a) $\frac{x_i + a}{h}$ (b) $h(x_i - a)$ (c) $\frac{x_i - a}{h}$ (d) $\frac{a - x_i}{h}$

CHAPTER – 15
PROBABILITY

PROBABILITY

The theoretical probability (also called classical probability) of an event A, written as P(A), is defined as

$$P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Number of all possible outcomes of the experiment}}$$

COMPLIMENTARY EVENTS AND PROBABILITY

We denote the event 'not E' by E . This is called the **complement** event of event E.
So, P(E) + P(not E) = 1
i.e., $P(E) + P(\overline{E}) = 1$, which gives us $P(\overline{E}) = 1 - P(E)$.

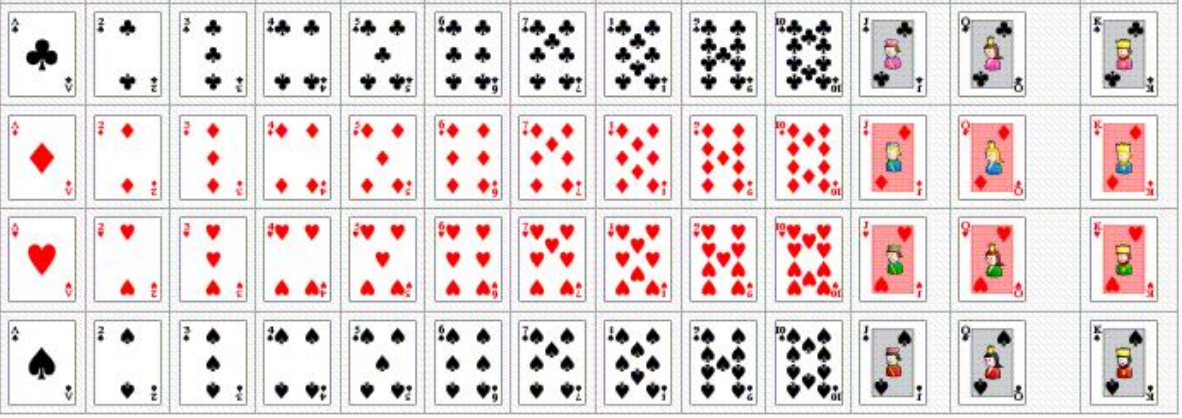
- ☞ The probability of an event which is impossible to occur is 0. Such an event is called an **impossible event**.
- ☞ The probability of an event which is sure (or certain) to occur is 1. Such an event is called a **sure event** or a **certain event**.
- ☞ The probability of an event E is a number P(E) such that $0 \le P(E) \le 1$
- ☞ An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.

DECK OF CARDS AND PROBABILITY

A deck of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each. They are black spades (♠) red hearts (♥), red diamonds (♦) and black clubs (♣).

The cards in each suit are Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2. Kings, Queens and Jacks are called face cards.

Example set of 52 poker playing cards

Suit	Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
Clubs													
Diamonds													
Hearts													
Spades													

IMPORTANT QUESTIONS

Two dice are thrown together. Find the probability that the sum of the numbers on the top of the dice is (i) 9 (ii) 10

Solution:

Here, total number of outcomes, n(s) = 36

(i) Let A be the event of getting the sum of the numbers on the top of the dice is 9 then we have n(A) = 4 i.e. (3, 6), (4, 5), (5, 4), (6, 3)

Therefore, Probability of getting the sum of the numbers on the top of the dice is 9, $P(A) = \frac{n(A)}{n(S)}$

$\Rightarrow P(A) = \frac{4}{36} = \frac{1}{9}$

(ii) Let B be the event of getting the sum of the numbers on the top of the dice is 10 then we have n(B) = 3 i.e. (4, 6), (5, 5), (6, 4)

Therefore, Probability of getting the sum of the numbers on the top of the dice is 10, $P(B) = \frac{n(B)}{n(S)}$

$\Rightarrow P(B) = \frac{3}{36} = \frac{1}{12}$

One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) red colour ace card (ii) a face card or a spade card (iii) a black face card

Solution:

Here, total number of outcomes, n(s) = 52

(i) Let A be the event of getting red colour ace card and we know that the number of red ace card is 2 then we have, n(A) = 2

Therefore, Probability of getting red colour ace card, $P(A) = \frac{n(A)}{n(S)}$

$\Rightarrow P(A) = \frac{2}{52} = \frac{1}{26}$

(ii) Let B be the event of getting a face card or a spade card and we know that there are 12 face cards, 13 spade cards and 3 face cards are spade then we have, n(B) = 12 + 13 – 3 = 22

Therefore, Probability of getting a face card or a spade card, $P(B) = \frac{n(B)}{n(S)}$

$\Rightarrow P(B) = \frac{22}{52} = \frac{11}{26}$

(ii) Let B be the event of getting a black face card and we know that there are 6 face cards are black then we have, n(C) = 6

Therefore, Probability of getting a black face card, $P(C) = \frac{n(C)}{n(S)}$

$\Rightarrow P(C) = \frac{6}{52} = \frac{3}{26}$

Questions for Practice

1. Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is (i) 6 (ii) 12 (iii) 7
2. A die is thrown twice. What is the probability that (i) 5 will not come up either time? (ii) 5 will come up at least once?
3. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that (i) She will buy it ? (ii) She will not buy it ?

4. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour (ii) a face card (iii) a red face card (iv) the jack of hearts (v) a spade (vi) the queen of diamonds

5. Five cards—the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random. (i) What is the probability that the card is the queen? (ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

6. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

7. A piggy bank contains hundred 50p coins, fifty Re 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 p coin ? (ii) will not be a Rs 5 coin?

8. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red ? (ii) white ? (iii) not green?

9. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?
(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective ?

10. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

11. A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Jimmy, a trader, will only accept the shirts which are good, but Sujatha, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that (i) it is acceptable to Jimmy? (ii) it is acceptable to Sujatha?

12. Two customers are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on (i) the same day? (ii) consecutive days? (iii) different days?

13. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is *double* that of a red ball, determine the number of blue balls in the bag.

14. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball? If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x.

15. A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue marbles in the jar.

MCQ QUESTIONS (1 mark)

1. Which of the following can be the probability of an event?
(a) – 0.04 (b) 1.004 (c) $\frac{18}{23}$ (d) $\frac{8}{7}$

2. A card is selected at random from a well shuffled deck of 52 playing cards. The probability of its being a face card is
(a) $\frac{3}{13}$ (b) $\frac{4}{13}$ (c) $\frac{6}{13}$ (d) $\frac{9}{13}$

3. A bag contains 3 red balls, 5 white balls and 7 black balls. What is the probability that a ball drawn from the bag at random will be neither red nor black?

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{7}{15}$ (d) $\frac{8}{15}$

4. If an event cannot occur, then its probability is

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) 0

5. Which of the following cannot be the probability of an event?

(a) $\frac{1}{3}$ (b) 0.1 (c) 3% (d) $\frac{17}{16}$

6. An event is very unlikely to happen. Its probability is closest to
(a) 0.0001 (b) 0.001 (c) 0.01 (d) 0.1

7. If the probability of an event is p, the probability of its complementary event will be

(a) p – 1 (b) p (c) 1 – p (d) $1-\frac{1}{p}$

8. The probability expressed as a percentage of a particular occurrence can never be
(a) less than 100 (b) less than 0 (c) greater than 1 (d) anything but a whole number

9. If P(A) denotes the probability of an event A, then
(a) $P(A) < 0$ (b) $P(A) > 1$ (c) $0 \le P(A) \le 1$ (d) $-1 \le P(A) \le 1$

10. A card is selected from a deck of 52 cards. The probability of its being a red face card is

(a) $\frac{3}{26}$ (b) $\frac{3}{13}$ (c) $\frac{2}{13}$ (d) $\frac{1}{2}$

11. The probability that a non leap year selected at random will contain 53 sundays is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{5}{7}$

12. When a die is thrown, the probability of getting an odd number less than 3 is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 0

13. A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is
(a) 4 (b) 13 (c) 48 (d) 51

14. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is
(a) 7 (b) 14 (c) 21 (d) 28

15. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought?
(a) 40 (b) 240 (C) 480 (D) 750

16. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{1}{3}$

17. Someone is asked to take a number from 1 to 100. The probability that it is a prime is

(a) $\frac{1}{5}$ (b) $\frac{6}{25}$ (c) $\frac{1}{4}$ (d) $\frac{13}{50}$

18. A school has five houses A, B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and rest from house E. A single student is selected at random to be the class monitor. The probability that the selected student is not from A, B and C is

(a) $\frac{4}{23}$ (b) $\frac{6}{23}$ (c) $\frac{8}{23}$ (d) $\frac{17}{23}$

Cards are marked with numbers 1 to 25 are placed in the box and mixed thoroughly. One card is drawn at random from the box. Answer the following questions (Q19-Q28)

19. What is the probability of getting a number 5?

(a) 1 (b) 0 (c) $\frac{1}{25}$ (d) $\frac{1}{5}$

20. What is the probability of getting a number less than 11?

(a) 1 (b) 0 (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

21. What is the probability of getting a number greater than 25?

(a) 1 (b) 0 (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

22. What is the probability of getting a multiple of 5?

(a) 1 (b) 0 (c) $\frac{1}{25}$ (d) $\frac{1}{5}$

23. What is the probability of getting an even number?

(a) 1 (b) 0 (c) $\frac{12}{25}$ (d) $\frac{13}{25}$

24. What is the probability of getting an odd number?

(a) 1 (b) 0 (c) $\frac{12}{25}$ (d) $\frac{13}{25}$

25. What is the probability of getting a prime number?

(a) $\frac{8}{25}$ (b) $\frac{9}{25}$ (c) $\frac{12}{25}$ (d) $\frac{13}{25}$

26. What is the probability of getting a number divisible by 3?

(a) $\frac{8}{25}$ (b) $\frac{9}{25}$ (c) $\frac{12}{25}$ (d) $\frac{13}{25}$

27. What is the probability of getting a number divisible by 4?

(a) $\frac{8}{25}$ (b) $\frac{9}{25}$ (c) $\frac{6}{25}$ (d) $\frac{3}{25}$

28. What is the probability of getting a number divisible by 7?

(a) $\frac{8}{25}$ (b) $\frac{9}{25}$ (c) $\frac{6}{25}$ (d) $\frac{3}{25}$

29. A bag has 4 red balls and 2 yellow balls. A ball is drawn from the bag without looking into the bag. What is probability of getting a red ball?

(a) $\frac{1}{6}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) 1

30. A bag has 4 red balls and 2 yellow balls. A ball is drawn from the bag without looking into the bag. What is probability of getting a yellow ball?

(a) $\frac{1}{6}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) 1

31. There are 6 marbles in a box with number 1 to 6 marked on each of them . What is the probability of drawing a marble with number 2 ?

(a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{1}{3}$ (d) 1

32. A coin is flipped to decide which team starts the game . What is the probability of your team will start ?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 0

33. A die is thrown once . What will be the probability of getting a prime number ?

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) 1 (d) 0

CHAPTER – 3
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

ALGEBRAIC INTERPRETATION OF PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

The pair of linear equations represented by these lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

1. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then the pair of linear equations has exactly one solution.

2. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the pair of linear equations has infinitely many solutions.

3. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then the pair of linear equations has no solution.

S. No.	Pair of lines	Compare the ratios	Graphical representation	Algebraic interpretation
1	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique solution (Exactly one solution)
2	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
3	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

IMPORTANT QUESTIONS

1. On comparing the ratios $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:
(i) $5x – 4y + 8 = 0$ and $7x + 6y – 9 = 0$ (ii) $9x + 3y + 12 = 0$ and $18x + 6y + 24 = 0$
(iii) $6x – 3y + 10 = 0$ and $2x – y + 9 = 0$.

2. On comparing the ratios $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.
(i) $3x + 2y = 5$; $2x – 3y = 7$ (ii) $2x – 3y = 8$; $4x – 6y = 9$
(iii) $5x – 3y = 11$; $– 10x + 6y = –22$

3. Find the number of solutions of the following pair of linear equations:
$x + 2y – 8 = 0$
$2x + 4y = 16$

4. Write whether the following pair of linear equations is consistent or not.
$x + y = 14$, $x – y = 4$

5. Given the linear equation $3x + 4y – 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is parallel lines.

6. Find the value of k so that the following system of equations has no solution:
$3x – y – 5 = 0$, $6x – 2y + k = 0$

7. Find the value of k so that the following system of equation has infinite solutions:
$3x – y – 5 = 0$, $6x – 2y + k = 0$

8. For which values of p, does the pair of equations given below has unique solution?
$4x + py + 8 = 0$ and $2x + 2y + 2 = 0$

9. Determine k for which the system of equations has infinite solutions:
$4x + y = 3$ and $8x + 2y = 5k$

10. Find whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident:
$2x – 3y + 6 = 0$; $4x – 5y + 2 = 0$

11. Find the value of k for which the system $3x + ky = 7$, $2x – 5y = 1$ will have infinitely many solutions.

12. For what value of k, the system of equations $2x – ky + 3 = 0$, $4x + 6y – 5 = 0$ is consistent?

13. For what value of k, the system of equations $kx – 3y + 6 = 0$, $4x – 6y + 15 = 0$ represents parallel lines?

14. For what value of p, the pair of linear equations $5x + 7y = 10$, $2x + 3y = p$ has a unique solution.

15. Find the value of m for which the pair of linear equations has infinitely many solutions.
$2x + 3y – 7 = 0$ and $(m – 1)x + (m + 1)y = (3m – 1)$

16. For what value of p will the following pair of linear equations have infinitely many solutions?
$(p – 3)x + 3y = p$; $px + py = 12$

17. For what value of k will the system of linear equations has infinite number of solutions?
$kx + 4y = k – 4$, $16x + ky = k$

18. Find the values of a and b for which the following system of linear equations has infinite number of solutions:
$2x – 3y = 7$, $(a + b) x – (a + b – 3) y = 4a + b$

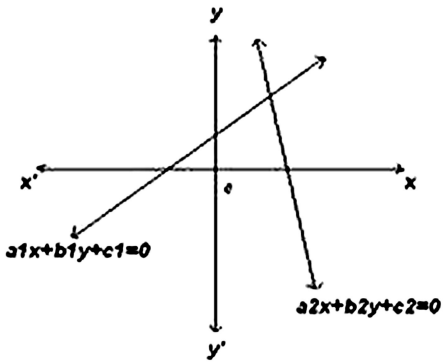
19. For what value of k will the equations $x + 2y + 7 = 0$, $2x + ky + 14 = 0$ represent coincident lines?

20. For what value of k, the following system of equations $2x + ky = 1$, $3x – 5y = 7$ has (i) a unique solution (ii) no solution

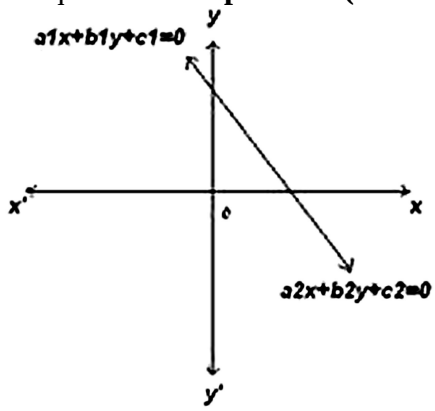
GRAPHICAL METHOD OF SOLUTION OF A PAIR OF LINEAR EQUATIONS

The graph of a pair of linear equations in two variables is represented by two lines.

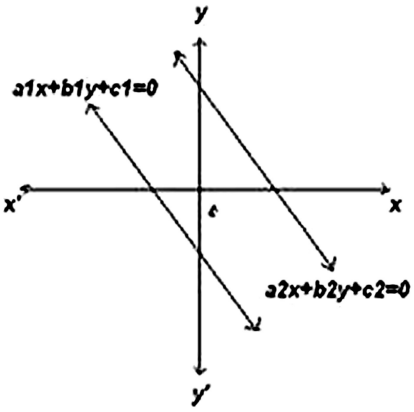
1. If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is **consistent**.



2. If the lines coincide, then there are infinitely many solutions — each point on the line being a solution. In this case, the pair of equations is **dependent (consistent)**.



3. If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is **inconsistent**.



IMPORTANT QUESTIONS

Solve the equation graphically: x + 3y = 6 and 2x – 3y = 12.

Solution: Given that

$$x+3y=6 \Rightarrow 3y=6-x \Rightarrow y=\frac{6-x}{3}$$

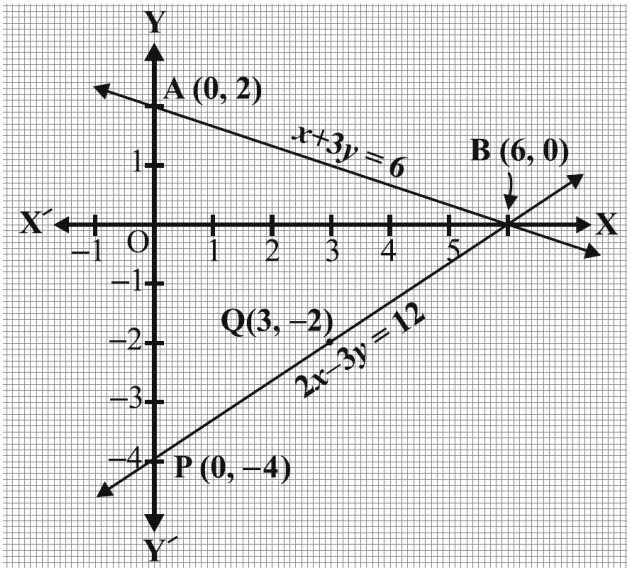
x	0	3	6
y	2	1	0

and $2x-3y=12 \Rightarrow 3y=2x-12 \Rightarrow y=\frac{2x-12}{3}$

x	0	3	6
y	-4	-2	0

Now plot the points and join the points to form the lines AB and PQ as shown in graph

Since point B(6, 0) common to both the lines AB and PQ. Therefore, the solution of the pair of linear equations is x = 6 and y = 0



Questions for Practice

1. Determine by drawing graphs, whether the following pair of linear equations has a unique solution or not: 3x + 4y = 12; y = 2
2. Determine by drawing graphs, whether the following pair of linear equations has a unique solution or not: 2x – 5 = 0, y + 4 = 0.
3. Draw the graphs of the equations 4x – y – 8 = 0 and 2x – 3y + 6 = 0.
   Also, determine the vertices of the triangle formed by the lines and x-axis.
4. Solve the following system of linear equations graphically:3x – 2y – 1 = 0; 2x – 3y + 6 = 0.
   Shade the region bounded by the lines and x-axis.
5. Solve graphically: x + 4y = 10, y – 2 = 0
6. Solve graphically: 2x – 3y = 6, x – 6 = 0
7. Solve the following system of equations graphically: 3x – 5y + 1 = 0, 2x – y + 3 = 0.
   Also find the points where the lines represented by the given equations intersect the x-axis.
8. Solve the following system of equations graphically: x – 5y = 6, 2x – 10y = 10
   Also find the points where the lines represented by the given equations intersect the x-axis.
9. Solve the following pair of linear equations graphically: x + 3y = 6; 2x – 3y = 12
   Also find the area of the triangle formed by the lines representing the given equations with y-axis.

MCQ QUESTIONS (1 mark)

1. The pair of equations y = 0 and y = –7 has
(a) one solution (b) two solution (c) infinitely many solutions (d) no solution

2. The pair of equations x = a and y = b graphically represents the lines which are
(a) parallel (b) intersecting at (a, b)
(c) coincident (d) intersecting at (b, a)

3. The value of c for which the pair of equations cx – y = 2 and 6x – 2y = 3 will have no solution is
(a) 3 (b) – 3 (c) – 12 (d) no value

4. The pair of equations 5x – 15y = 8 and 3x – 9y = 24/5 has
(a) infinite number of solutions (b) unique solution
(c) no solution (d) one solution

5. The pair of equations x + 2y + 5 = 0 and –3x – 6y + 1 = 0 have
(a) infinite number of solutions (b) unique solution
(c) no solution (d) one solution

6. The sum of the digits of a two digit number is 9. If 27 is added to it, the digits of the numbers get reversed. The number is
(a) 36 (b) 72 (c) 63 (d) 25

7. The pair of equations 3x + 4y = 18 and 4x + $\frac{16}{3}$y = 24 has
(a) infinite number of solutions (b) unique solution
(c) no solution (d) cannot say anything

8. If the pair of equations 2x + 3y = 7 and kx + $\frac{9}{2}$y = 12 have no solution, then the value of k is:
(a) $\frac{2}{3}$ (b) – 3 (c) 3 (d) $\frac{3}{2}$

9. If bx + ay = $a^2 + b^2$ and ax – by = 0, then the value of x – y equals:
(a) a – b (b) b – a (c) $a^2 - b^2$ (d) $b^2 + a^2$.

10. If 2x + 3y = 0 and 4x – 3y = 0, then x + y equals:
(a) 0 (b) – 1 (c) 1 (d) 2

11. If $\sqrt{a}x - \sqrt{b}y = b - a$ and $\sqrt{b}x - \sqrt{a}y = 0$, then the value of x, y is:
(a) a + b (b) a – b (c) $\sqrt{ab}$ (d) $-\sqrt{ab}$

12. If 31x + 43y = 117 and 43 + 31y = 105, then value of x – y is:
(a) $\frac{1}{3}$ (b) – 3 (c) 3 (d) $-\frac{1}{3}$

13. If 19x – 17y = 55 and 17x – 19y = 53, then the value of x – y is:

(a) $\frac{1}{3}$ (b) – 3 (c) 3 (d) 5

14. If (6, k) is a solution of the equation 3x + y – 22 = 0, then the value of k is:
(a) 4 (b) –4 (c) 3 (d) –3

15. If 3x –5y = 1, $\frac{2x}{x-y} = 4$, then the value of x + y is

(a) $\frac{1}{3}$ (b) – 3 (c) 3 (d) $-\frac{1}{3}$

16. If 3x + 2y = 13 and 3x – 2y = 5, then the value of x + y is:
(a) 5 (b) 3 (c) 7 (d) none of these

17. If the pair of equations 2x + 3y = 5 and 5x + $\frac{15}{2}$y = k represent two coincident lines, then the value of k is:

(a) –5 (b) $\frac{-25}{2}$ (c) $\frac{25}{2}$ (d) $\frac{-5}{2}$

18. If x = a, y = b is the solution of the equations x – y = 2 and x + y = 4, then the values of a and b are, respectively
(a) 3 and 5 (b) 5 and 3 (c) 3 and 1 (d) –1 and –3

19. Aruna has only Re 1 and Rs 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs 75, then the number of Re 1 and Rs 2 coins are, respectively
(a) 35 and 15 (b) 35 and 20 (c) 15 and 35 (d) 25 and 25

20. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, in years, of the son and the father are, respectively
(a) 4 and 24 (b) 5 and 30 (c) 6 and 36 (d) 3 and 24

CHAPTER – 4
QUADRATIC EQUATIONS

FACTORISATION METHODS TO FIND THE SOLUTION OF QUADRATIC EQUATIONS

Steps to find the solution of given quadratic equation by factorisation

- Firstly, write the given quadratic equation in standard form $ax^2 + bx + c = 0$.
- Find two numbers α and β such that sum of α and β is equal to b and product of α and β is equal to ac.
- Write the middle term bx as $\alpha x + \beta x$ and factorise it by splitting the middle term and let factors are (x + p) and (x + q) i.e. $ax^2 + bx + c = 0 \Rightarrow (x + p)(x + q) = 0$
- Now equate reach factor to zero and find the values of x.
- These values of x are the required roots/solutions of the given quadratic equation.

IMPORTANT QUESTIONS

Solve the quadratic equation by using factorization method: $x^2 + 2x - 8 = 0$

Solution: $x^2 + 2x - 8 = 0$

$\Rightarrow x^2 + 4x - 2x - 8 = 0 \Rightarrow x(x + 4) - 2(x + 4) = 0$

$\Rightarrow (x + 4)(x - 2) = 0 \Rightarrow x + 4 = 0, x - 2 = 0 \Rightarrow x = -4, 2$

Questions for practice

1. Solve the quadratic equation using factorization method: $x^2 + 7x - 18 = 0$

2. Solve the quadratic equation using factorization method: $x^2 + 5x - 6 = 0$

3. Solve the quadratic equation using factorization method: $y^2 - 4y + 3 = 0$

4. Solve the quadratic equation using factorization method: $x^2 - 21x + 108 = 0$

5. Solve the quadratic equation using factorization method: $x^2 - 11x - 80 = 0$

6. Solve the quadratic equation using factorization method: $x^2 - x - 156 = 0$

7. Solve the following for x : $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$.

8. Solve the following for x : $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

NATURE OF ROOTS

The roots of the quadratic equation $ax^2 + bx + c = 0$ by quadratic formula are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

where D = $b^2 - 4ac$ is called discriminant. The nature of roots depends upon the value of discriminant D. There are three cases –

Case – I

When D > 0 i.e. $b^2 - 4ac > 0$, then the quadratic equation has two distinct roots.

i.e. $x = \frac{-b + \sqrt{D}}{2a}$ *and* $\frac{-b - \sqrt{D}}{2a}$

Case – II

When D = 0, then the quadratic equation has two equal real roots.

i.e. $x = \frac{-b}{2a}$ *and* $\frac{-b}{2a}$

Case – III

When D < 0 then there is no real roots exist.

IMPORTANT QUESTIONS

Find the discriminant of the quadratic equation 2x2 – 4x + 3 = 0, and hence find the nature of its roots.

Solution : The given equation is of the form $ax^2 + bx + c = 0$, where a = 2, b = – 4 and c = 3.
Therefore, the discriminant, $D = b^2 – 4ac = (– 4)^2 – (4 \times 2 \times 3) = 16 – 24 = – 8 < 0$
So, the given equation has no real roots.

Questions for Practice

1. Find the discriminant and the nature of the roots of quadratic equation: $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$.
2. Find discriminant and the nature of the roots of quadratic equation: $4x^2 – 2x2 + 3 = 0$.
3. Find discriminant and the nature of the roots of quadratic equation: $4x^2 – 12x + 9 = 0$.
4. Find discriminant and the nature of the roots of quadratic equation: $5x^2 + 5x + 6 = 0$.
5. Write the nature of roots of quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$.
6. Write the nature of roots of the quadratic equation $9x^2 – 6x – 2 = 0$.
7. Write the nature of roots of quadratic equation : $4x^2 + 6x + 3 = 0$
8. The roots of $ax^2 + bx + c = 0$, $a \neq 0$ are real and unequal. What is value of D?
9. If $ax^2 + bx + c = 0$ has equal roots, what is the value of c?

QUADRATIC FORMULA METHOD

Steps to find the solution of given quadratic equation by quadratic formula method:

- Firstly, write the given quadratic equation in standard form $ax^2 + bx + c = 0$.
- Write the values of a, b and c by comparing the given equation with standard form.
- Find discriminant $D = b^2 – 4ac$. If value of D is negative, then is no real solution i.e. solution does not exist. If value of $D \geq 0$, then solution exists follow the next step.
- Put the value of a, b and D in quadratic formula $x = \frac{-b \pm \sqrt{D}}{2a}$ and get the required roots/solutions.

IMPORTANT QUESTIONS

Solve the quadratic equation by using quadratic formula: $x^2 + x – 6 = 0$

Solution: Here, a = 1, b = 1, c = –6

$\Rightarrow D = b^2 – 4ac = 1 – 4(1)(–6) = 1 + 24 = 25 > 0$

Now, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{25}}{2(1)} = \frac{-1 \pm 5}{2} \Rightarrow x = \frac{-1-5}{2} or \frac{-1+5}{2} \Rightarrow x = \frac{-6}{2} or \frac{4}{2} \Rightarrow x = -3 \ or \ 2$

Questions for practice

1. Solve the quadratic equation by using quadratic formula: $x^2 – 7x + 18 = 0$
2. Solve the quadratic equation by using quadratic formula: $x^2 – 5x + 6 = 0$
3. Solve the quadratic equation by using quadratic formula: $y^2 + 4y + 3 = 0$
4. Solve the quadratic equation by using quadratic formula: $x^2 + 11x – 80 = 0$
5. Solve the quadratic equation by using quadratic formula: $x^2 + x – 156 = 0$
6. Solve for x by using quadratic formula : $9x^2 – 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$.

WORD PROBLEMS

IMPORTANT QUESTIONS

A motor boat whose speed is 18 km/h in still water takes 1 hr. more to go 24 km upstream than to return downstream to the same spot. Find the speed of stream.

Solution: Let the speed of the stream be x km/h.
Therefore, the speed of the boat upstream = (18 – x) km/h and the speed of the boat downstream = (18 + x) km/h.

The time taken to go upstream = $\frac{\text{distance}}{\text{speed}} = \frac{24}{18-x}$

Similarly, the time taken to go downstream = $\frac{24}{18+x}$

According to the question, $\frac{24}{18-x} - \frac{24}{18+x} = 1$

$\Rightarrow$ 24(18 + x) – 24(18 – x) = (18 – x) (18 + x)

$\Rightarrow$ $x^2 + 48x - 324 = 0$

$\Rightarrow$ (x – 6)(x + 54) = 0 (using factorisation)

$\Rightarrow$ x = 6, –54

Since x is the speed of the stream, it cannot be negative. So, we ignore the root x = – 54. Therefore, x = 6 gives the speed of the stream as 6 km/h.

Questions for Practice

1. In a class test, the sum of the marks obtained by P in Mathematics and Science is 28. Had he got 3 more marks in maths and 4 marks less in science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained in the two subjects separately.

2. A peacock is sitting on the top of a pillar which is 9 m high. From a point 27 m away from the bottom of the pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake the peacock pounces on it. If their speeds are equal at what distance from the hole is the snake caught?

3. Some students planned a picnic. The total budget for food was Rs. 2,000. But 5 students failed to attend the picnic and thus the cost of food for each member increased by Rs. 20. How many students attended the picnic and how much did each student pay for the food?

4. In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and time increased by 30 minutes. Find the original duration of the flight.

5. A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work.

6. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

7. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

8. Two water taps together can fill a tank is 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

9. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.

10. A man bought a certain number of toys for 180, he kept one for his own use and sold the rest for one rupee each more than he gave for them, besides getting his own toy for nothing he made a profit of 10. Find the number of toys.

11. Nine times the side of one square exceeds a perimeter of a second square by one metre and six times the area of the second square exceeds twenty nine times the area of the first by one square metre. Find the side of each square.

12. One-fourth of a herd of camels was seen in a forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

13. One pipe can fill a cistern in (x + 2) hours and the other pipe can fill the same cistern in (x + 7) hours. If both the pipes, when opened together take 6 hours to fill the empty cistern, find the value of x.

MCQ QUESTIONS (1 mark)

1. Which of the following is a quadratic equation?

(a) $x^2 + 2x + 1 = (4 - x)^2 + 3$ (b) $-2x^2 = (5 - x)(2x - \frac{2}{5})$

(c) $(k + 1)x^2 + \frac{3}{2}x = 7$, where $k \neq 1$ (d) $x^3 - x^2 = (x - 1)^3$.

2. Which of the following is not a quadratic equation?

(a) $2(x - 1)^2 = 4x^2 - 2x + 1$ (b) $2x - x^2 = x^2 + 5$

(c) $(\sqrt{2}x - \sqrt{3})^2 + x^2 = 3x^2 - 5x$ (d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$.

3. If is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

(a) 2 (b) – 2 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

4. Which of the following equations has the sum of its roots as 3?

(a) $2x^2 - 3x + 6 = 0$ (b) $-x^2 + 3x - 3 = 0$

(c) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ (d) $3x^2 - 3x + 3 = 0$

5. Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

(a) 0 only (b) 4 (c) 8 only (d) 0, 8

6. Which constant must be added and subtracted to solve the quadratic equation

$9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square?

(a) $\frac{1}{8}$ (b) $\frac{1}{64}$ (c) $\frac{1}{4}$ (d) $\frac{9}{64}$

7. The quadratic equation $2x^2 - 5x + 1 = 0$ has

(a) two distinct real roots (b) two equal real roots

(c) no real roots (d) more than 2 real roots

8. Which of the following equations has two distinct real roots?

(a) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ (b) $x^2 + x - 5 = 0$

(c) $x^2 + 3x + 2\sqrt{2} = 0$ (d) $5x^2 - 3x + 1 = 0$

9. Which of the following equations has no real roots?

(a) $x^2 - 4x + 3\sqrt{2} = 0$ (b) $x^2 + 4x - 3\sqrt{2} = 0$

(c) $x^2 - 4x - 3\sqrt{2} = 0$ (d) $3x^2 + 4\sqrt{3}x + 4 = 0$

10. $(x^2 + 1)^2 - x^2 = 0$ has

(a) four real roots (b) two real roots

(c) no real roots (d) one real root.

11. If 2 is the root of the equation $x^2 + bx + 12 = 0$ and the equation $x^2 + bx + q = 0$ has equal roots then q =

(a) 8 (b) 16 (c) –8 (d) –16

12. If the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + c^2 + d^2 = 0$ has equal roots then

(a) ab = cd (b) ad = bc (c) ad = $\sqrt{bc}$ (d) ab = $\sqrt{cd}$

13. If a and b can take values 1, 2, 3, 4. Then the number of the equations of the form $ax^2 + bx + c = 0$ having real roots is

(a) 6 (b) 7 (c) 10 (d) 12

14. The number of quadratic equations having real roots and which do not change by squaring their roots is

(a) 4 (b) 3 (c) 2 (d) 1

15. If one of the roots of the quadratic equation $(k^2 + 4)x^2 + 13x + 4k$ is reciprocal of the other then k =

(a) 2 (b) 1 (c) –1 (d) – 2

16. If α, β are the roots of the quadratic equation $4x^2 + 3x + 7 = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta}$

(a) $\frac{7}{3}$ (b) $\frac{-7}{3}$ (c) $\frac{3}{7}$ (d) $\frac{-3}{7}$

17. If α, β are the roots of the quadratic equation $x^2 - p(x + 1) - c = 0$, then $(\alpha + 1)(\beta + 1)$ =

(a) c – 1 (b) 1 – c (c) c (d) 1 + c

18. Find the values of k for which the quadratic equation $2x^2 + kx + 3 = 0$ has real equal roots.

(a) $\pm 2\sqrt{6}$ (b) $2\sqrt{6}$ (c) 0 (d) ± 2

19. Find the values of k for which the quadratic equation $kx(x - 3) + 9 = 0$ has real equal roots.

(a) k = 0 or k = 4 (b) k = 1 or k = 4 (c) k = –3 or k = 3 (d) k = –4 or k = 4

20. Find the values of k for which the quadratic equation $4x^2 - 3kx + 1 = 0$ has real and equal roots.

(a) $\pm\frac{4}{3}$ (b) $\pm\frac{2}{3}$ (c) ± 2 (d) none of these

21. The value of k for which equation $9x^2 + 8xk + 8 = 0$ has equal roots is:

(a) only 3 (b) only –3 (c) ±3 (d) 9

22. Which of the following is not a quadratic equation?

(a) $x - \frac{3}{x} = 4$ (b) $3x - \frac{5}{x} = x^2$ (c) $x + \frac{1}{x} = 3$ (d) $x^2 - 3 = 4x^2 - 4x$

23. Which of the following is a solution of the quadratic equation $2x^2 + x - 6 = 0$?

(a) x = 2 (b) x = –12 (c) x = $\frac{3}{2}$ (d) x = –3

24. The value of k for which x = –2 is a root of the quadratic equation $kx^2 + x - 6 = 0$

(a) –1 (b) –2 (c) 2 (d) $-\frac{3}{2}$

25. The value of p so that the quadratics equation $x^2 + 5px + 16 = 0$ has no real root, is

(a) p>8 (b) p<5 (c) $\frac{-8}{5} < x < \frac{8}{5}$ (d) $\frac{-8}{5} \le x < 0$

26. If $px^2 + 3w + q = 0$ has two roots x = –1 and x = –2, the value of q – p is

(a) –1 (b) –2 (c) 1 (d) 2

27. The common root of the quadratic equation $x^2 - 3x + 2 = 0$ and $2x^2 - 5x + 2 = 0$ is:

(a) x = 2 (b) x = –2 (c) x = $\frac{1}{2}$ (d) x = 1

28. If $x^2 - 5x + 1 = 0$, the value of $\left(x + \frac{1}{x}\right)$ is:

(a) –5 (b) –2 (c) 5 (d) 3

29. If $a - 3 = \frac{10}{a}$, the value of a are

(a) –5, 2 (b) 5, –2 (c) 5, 2 (d) 5, 0

30. If the roots of the quadratic equation $kx^2 + (a + b)x + ab = 0$ are (–1, –b), the value of k is:

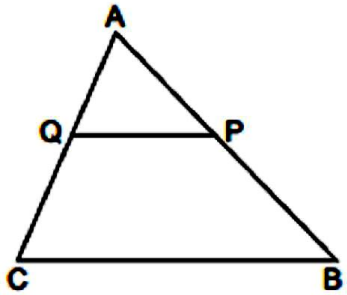
(a) –1 (b) –2 (c) 1 (d) 2

CHAPTER – 6
TRIANGLES

IMPORTANT 1 MARK QUESTIONS

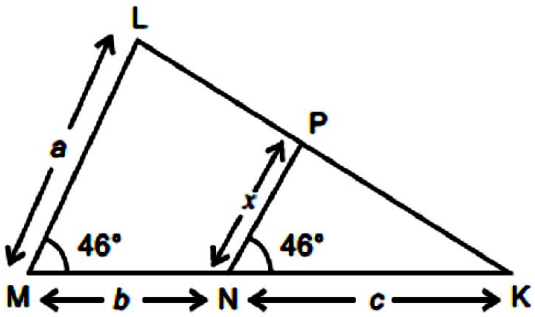
1. In ΔABC, D and E are points on sides AB and AC respectively such that DE || BC and AD : DB = 3 : 1. If EA = 6.6 cm then find AC.

2. In the fig., P and Q are points on the sides AB and AC respectively of ΔABC such that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm and QC = 6 cm. I f PQ = 4.5 cm, find BC.

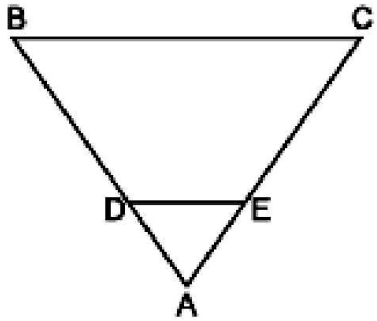


3. The perimeter of two similar triangles ABC and LMN are 60 cm and 48 cm respectively. If LM = 8 cm, then what is the length of AB ?

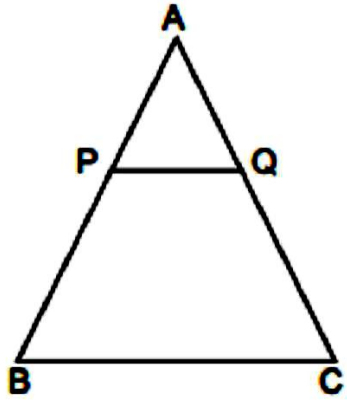
4. In fig. ∠M = ∠N = 46°, express x in terms of a, b and c, where a, b and c are lengths of LM, MN and NK respectively.



5. In figure, DE || BC in ΔABC such that BC = 8 cm, AB = 6 cm and DA = 1.5 cm. Find DE.



6. In the fig., PQ || BC and AP : PB = 1 : 2. Find $\frac{ar(\Delta APQ)}{ar(\Delta ABC)}$

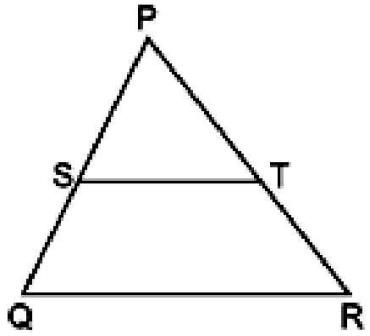


7. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time a tower casts the shadow 40 m long on the ground. Determine the height of the tower.

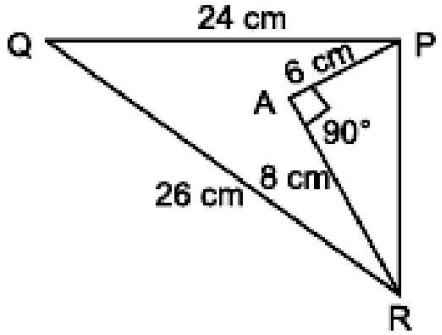
8. If ΔABC and ΔDEF are similar triangles such that ∠A = 57° and ∠E = 83°. Find C.

9. If the areas of two similar triangles are in ratio 25 : 64, write the ratio of their corresponding sides.

10. In figure, S and T are points on the sides PQ and PR, respectively of ΔPQR, such that PT = 2 cm, TR = 4 cm and ST is parallel to QR. Find the ratio of the areas of ΔPST and ΔPQR.

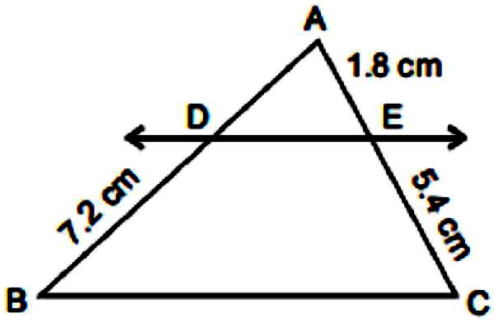


11. In the fig., PQ = 24 cm, QR = 26 cm, ∠PAR = 90°, PA = 6 cm and AR = 8 cm. Find ∠QPR.



12. The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus.

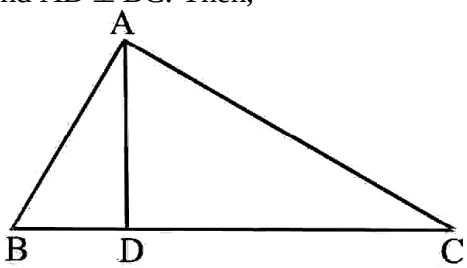
13. In the given figure, DE || BC. Find AD.



14. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm., what is the corresponding side of the other triangle ?

MCQ QUESTIONS (1 mark)

1. In the below, ∠BAC = 90° and AD ⊥ BC. Then,



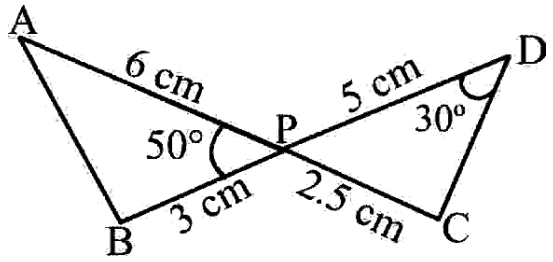
(a) BD . CD = BC^2 (b) AB . AC = BC^2 (c) BD . CD = AD^2 (d) AB . AC = AD^2

2. The lengths of the diagonals of a rhombus are 16 cm and 12 cm. Then, the length of the side of the rhombus is
(a) 9 cm (b) 10 cm (c) 8 cm (d) 20 cm

3. If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true?
(a) BC . EF = AC. FD (b) AB . EF = AC . DE (c) BC . DE = AB . EF (d) BC . DE = AB . FD

4. If in two triangles ABC and PQR, AB = BC = CA, then

5. In the below figure, two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, $\angle APB = 50°$ and $\angle CDP = 30°$. Then, $\angle PBA$ is equal to



(a) 50° (b) 30° (c) 60° (d) 100°

6. If in two triangles DEF and PQR, $\angle D = \angle Q$ and $\angle R = \angle E$, then which of the following is not true?
(a) $\frac{EF}{PR} = \frac{DF}{PQ}$ (b) $\frac{DE}{PQ} = \frac{EF}{RP}$ (c) $\frac{DE}{QR} = \frac{DF}{PQ}$ (d) $\frac{EF}{RP} = \frac{DE}{QR}$

7. In triangles ABC and DEF, $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3 DE. Then, the two triangles are
(a) congruent but not similar (b) similar but not congruent
(c) neither congruent nor similar (d) congruent as well as similar

8. It is given that $\Delta ABC \sim \Delta PQR$, with $\frac{BC}{QR} = \frac{1}{3}$. Then $\frac{ar(PRQ)}{ar(BCA)}$ is equal to
(a) 9 (b) 3 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$

9. It is given that $\Delta ABC \sim \Delta DFE$, $\angle A = 30°$, $\angle C = 50°$, AB = 5 cm, AC = 8 cm and DF= 7.5 cm. Then, the following is true:
(a) DE = 12 cm, $\angle F = 50°$ (b) DE = 12 cm, $\angle F = 100°$
(c) EF = 12 cm, $\angle D = 100°$ (d) EF = 12 cm, $\angle D = 30°$

10. If in triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when
(a) $\angle B = \angle E$ (b) $\angle A = \angle D$
(c) $\angle B = \angle D$ (d) $\angle A = \angle F$

11. If $\Delta ABC \sim \Delta QRP$, $\frac{ar(ABC)}{ar(PQR)} = \frac{9}{4}$, AB = 18 cm and BC = 15 cm, then PR is equal to
(a) 10 cm (b) 12 cm (c) 20/3 cm (d) 8 cm

12. If S is a point on side PQ of a ΔPQR such that PS = QS = RS, then

(a) PR . QR = RS^2 (b) $QS^2 + RS^2 = QR^2$
(c) $PR^2 + QR^2 = PQ^2$ (d) $PS^2 + RS^2 = PR^2$

13. A vertical pole of length 20 m casts a shadow 10 m long on the ground and at the same time a tower casts a shadow 50 m long, then the height of the tower.
(a) 100 m (b) 120 m (c) 25 m (d) none of these

14. The areas of two similar triangles are in the ratio 4 : 9. The corresponding sides of these triangles are in the ratio
(a) 2 : 3 (b) 4 : 9 (c) 81 : 16 (d) 16 : 81

15. The areas of two similar triangles ΔABC and ΔDEF are 144 cm^2 and 81 cm^2, respectively. If the longest side of larger ΔABC be 36 cm, then the longest side of the similar triangle ΔDEF is
(a) 20 cm (b) 26 cm (c) 27 cm (d) 30 cm

16. The areas of two similar triangles are in respectively 9 cm^2 and 16 cm^2. The ratio of their corresponding sides is
(a) 2 : 3 (b) 3 : 4 (c) 4 : 3 (d) 4 : 5

17. Two isosceles triangles have equal angles and their areas are in the ratio 16 : 25. The ratio of their corresponding heights is
(a) 3 : 2 (b) 5 : 4 (c) 5 : 7 (d) 4 : 5

18. If ΔABC and ΔDEF are similar such that 2AB = DE and BC = 8 cm, then EF =
(a) 16 cm (b) 112 cm (c) 8 cm (d) 4 cm

19. XY is drawn parallel to the base BC of a ΔABC cutting AB at X and AC at Y. If AB = 4BX and YC = 2 cm, then AY =
(a) 2 cm (b) 6 cm (c) 8 cm (d) 4 cm

20. Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, the distance between their tops is
(a) 14 cm (b) 12 cm (c) 13 cm (d) 11 cm

21. If D, E, F are midpoints of sides BC, CA and AB respectively of ΔABC, then the ratio of the areas of triangles DEF and ABC is
(a) 2 : 3 (b) 1 : 4 (c) 1 : 2 (d) 4 : 5

22. If ΔABC and ΔDEF are two triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{2}{5}$, then $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} =$
(a) 2 : 5 (b) 4 : 25 (c) 4 : 15 (d) 8 : 125

23. In triangles ABC and DEF, $\angle A = \angle E = 40^0$, AB : ED = AC : EF and $\angle F = 65^0$, then $\angle B$ =
(a) 35^0 (b) 65^0 (c) 75^0 (d) 85^0

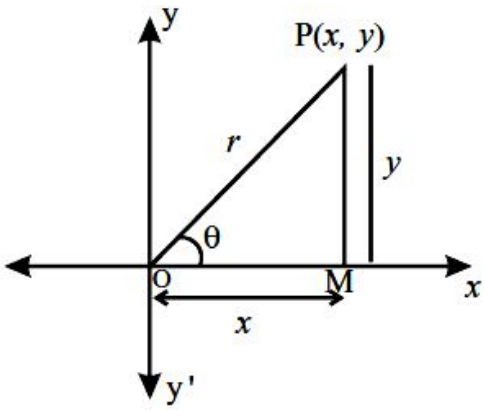
24. If ABC and DEF are similar triangles such that $\angle A = 47^0$ and $\angle E = 83^0$, then $\angle C$=
(a) 50^0 (b) 60^0 (c) 70^0 (d) 80^0

CHAPTER – 8 & 9
TRIGONOMETRY

Trigonometric Ratios (T - Ratios) of an acute angle of a right triangle

In XOY-plane, let a revolving line OP starting from OX, trace out ∠XOP=θ. From P (x, y)draw PM ⊥ to OX.

In right angled triangle OMP. OM = x (Adjacent side); PM = y (opposite side); OP = r (hypotenuse).



$$\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{y}{r}, \quad \cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{x}{r}, \quad \tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{y}{x}$$

$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{r}{y}, \quad \sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{r}{x}, \quad \cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{x}{y}$$

Reciprocal Relations

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$ **and** $\cot\theta = \dfrac{1}{\tan\theta}$

Quotient Relations

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

IMPORTANT QUESTIONS

If $\tan A = \dfrac{4}{3}$ **, find the value of all T– ratios of** θ**.**

Solution: Given that, In right Δ ABC, $\tan A = \dfrac{BC}{AB} = \dfrac{4}{3}$

Therefore, if BC = 4k, then AB = 3k, where k is a positive number.

Now, by using the Pythagoras Theorem, we have

$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2 = 25k^2$

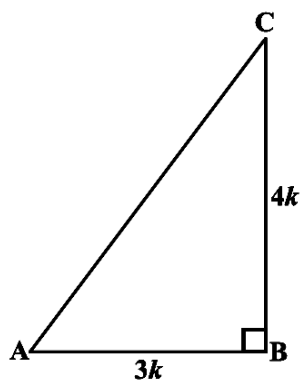
So, AC = 5k

Now, we can write all the trigonometric ratios using their definitions

$\sin A = \dfrac{BC}{AC} = \dfrac{4k}{5k} = \dfrac{4}{5}$, $\cos A = \dfrac{AB}{AC} = \dfrac{3k}{5k} = \dfrac{3}{5}$

and $\cot A = \dfrac{1}{\tan A} = \dfrac{3}{4}$,

$\operatorname{cosec} A = \dfrac{1}{\sin A} = \dfrac{5}{4}$,



$\sec A = \frac{1}{\cos A} = \frac{5}{3}$

Questions for Practice

1. If $\sin\theta = \frac{5}{13}$, find the value of all T– ratios of θ.
2. If $\cos\theta = \frac{7}{25}$, find the value of all T– ratios of θ.
3. If $\tan\theta = \frac{15}{8}$, find the value of all T– ratios of θ.
4. If $\cot\theta = 2$, find the value of all T– ratios of θ.
5. If $\text{cosec}\,\theta = \sqrt{10}$, find the value of all T– ratios of θ.
6. In Δ OPQ, right-angled at P, OP = 7 cm and OQ – PQ = 1 cm. Determine the values of sin Q and cos Q.
7. In Δ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Trigonometric ratios for angle of measure.

0^0, 30^0,45^0, 60^0 and 90^0 in tabular form.

$\angle$**A**	0^0	30^0	45^0	60^0	90^0
sinA	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosA	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanA	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosecA	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
secA	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cotA	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

IMPORTANT QUESTIONS

If cos (A – B) = $\frac{\sqrt{3}}{2}$ and sin (A + B) = 1, then find the value of A and B.

Solution: Given that $\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30^0$

$\Rightarrow A - B = 30^0$ ……. (1)

and $\sin(A+B) = 1 = \sin 90^0$

$\Rightarrow A + B = 90^0$ …… (2)

Solving equations (1) and (2), we get A = 60^0 and B = 30^0.

Questions for Practice
Evaluate each of the following:

1. $\sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0$
2. $\cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0$
3. $\cos 45^0 \cos 30^0 + \sin 45^0 \sin 30^0$
4. $\sin 60^0 \sin 45^0 - \cos 60^0 \cos 45^0$
5. $(\sin^2 30^0 + 4\cot^2 45^0 - \sec^2 60^0)(\cos ec^2 45^0 \sec^2 30^0)$
6. If sin (A – B) = $\frac{1}{2}$ and cos(A + B) = $\frac{1}{2}$, then find the value of A and B.
7. If tan (A – B) = $\frac{1}{\sqrt{3}}$ and tan (A + B) = $\sqrt{3}$, then find the value of A and B.

Trigonometric ratios of Complementary angles.

sin (90 – θ) = cos θ	cos (90 – θ) = sin θ
tan (90 – θ) = cot θ	cot (90 – θ) = tan θ
sec (90 – θ) = cosec θ	cosec (90 – θ) = sec θ.

IMPORTANT QUESTIONS

If sin 3A = cos (A – 26°), where 3A is an acute angle, find the value of A.
Solution: Given that sin 3A = cos (A – 26°). (1)
Since sin 3A = cos (90° – 3A), we can write (1) as
cos (90° – 3A) = cos (A – 26°)
Since 90° – 3A and A – 26° are both acute angles, therefore comparing both sides we get,
90° – 3A = A – 26° which gives A = 29°

Questions for Practice

1. Express cot 85° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.
2. Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.
3. If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.
4. If tan A = cot B, prove that A + B = 90°.
5. If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A.
6. If A, B and C are interior angles of a triangle ABC, then show that

TRIGONOMETRIC IDENTITIES

An equation involving trigonometric ratios of an angle is said to be a trigonometric identity if it is satisfied for all values of θ for which the given trigonometric ratios are defined.

Identity (1) : $\sin^2\theta + \cos^2\theta = 1$
$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$.
Identity (2) : $\sec^2\theta = 1 + \tan^2\theta$
$\Rightarrow \sec^2\theta - \tan^2\theta = 1$ and $\tan^2\theta = \sec^2\theta - 1$.
Identity (3) : $\text{cosec}^2\theta = 1 + \cot^2\theta$
$\Rightarrow \text{cosec}^2\theta - \cot^2\theta = 1$ and $\cot^2\theta = \text{cosec}^2\theta - 1$.

IMPORTANT QUESTIONS

Prove that: $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \cos ecA + \cot A$

Solution: LHS = $\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}$

(Dividing Numerator and Denominator by sinA, we get)

$$=\frac{\dfrac{\cos A}{\sin A}-\dfrac{\sin A}{\sin A}+\dfrac{1}{\sin A}}{\dfrac{\cos A}{\sin A}+\dfrac{\sin A}{\sin A}-\dfrac{1}{\sin A}}=\frac{\cot A-1+\cos ecA}{\cot A+1-\cos ecA} \qquad \left[\because \cot A=\frac{\cos A}{\sin A}, \cos ecA=\frac{1}{\sin A}\right]$$

$$=\frac{\cot A+\cos ecA-1}{\cot A+1-\cos ecA}=\frac{\cot A+\cos ecA-(\cos ec^2 A-\cot^2 A)}{\cot A+1-\cos ecA} \quad [\because \cos ec^2 A-\cot^2 A=1]$$

$$=\frac{\cot A+\cos ecA-(\cos ecA+\cot A)(\cos ecA-\cot A)}{\cot A+1-\cos ecA}$$

$$=\frac{(\cos ecA+\cot A)(1-\cos ecA+\cot A)}{\cot A+1-\cos ecA}=\cos ecA+\cot A=RHS$$

Questions for Practice

Prove the following identities:

1. sec A (1 – sin A)(sec A + tan A) = 1.

2. $\dfrac{\cot A-\cos A}{\cot A+\cos A}=\dfrac{\cos ecA-1}{\cos ecA+1}$

3. $\dfrac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\dfrac{1}{\sec\theta-\tan\theta}$

4. $(\cos ec\theta-\cot\theta)^2=\dfrac{1-\cos\theta}{1+\cos\theta}$

5. $\dfrac{\cos A}{1+\sin A}+\dfrac{1+\sin A}{\cos A}=2\sec A$

6. $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta\cos ec\theta$

7. $\dfrac{1+\sec A}{\sec A}=\dfrac{\sin^2 A}{1-\cos A}$

8. $\sqrt{\dfrac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

9. $\dfrac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

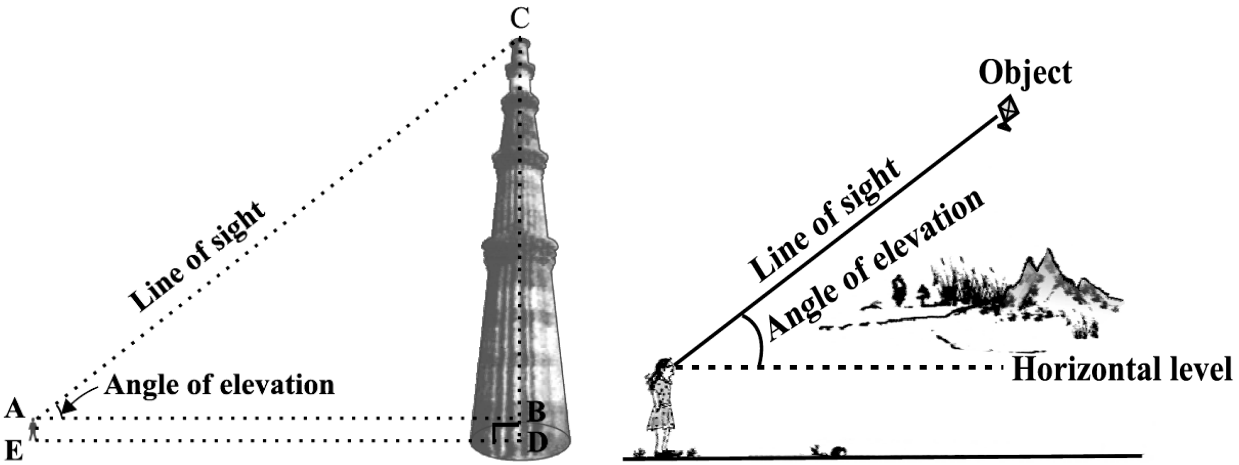
10. $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

11. $(\cos ecA-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$

12. $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

ANGLE OF ELEVATION

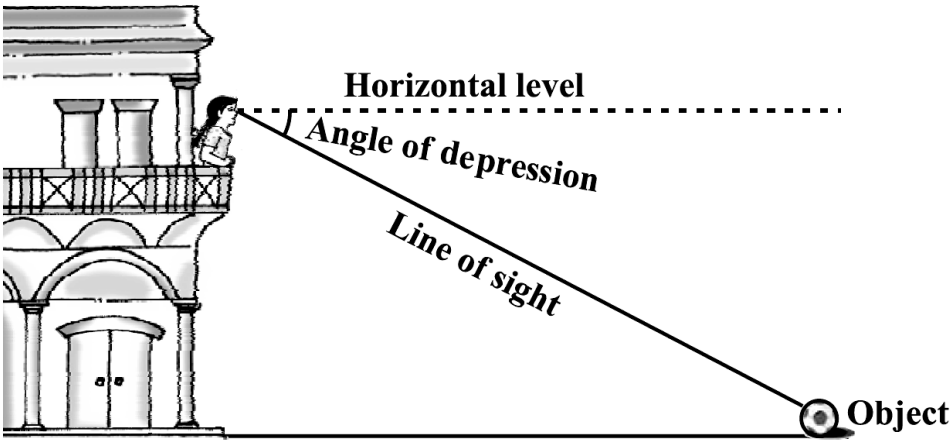
In the below figure, the line AC drawn from the eye of the student to the top of the minar is called the *line of sight*. The student is looking at the top of the minar. The angle BAC, so formed by the line of sight with the horizontal, is called the *angle of elevation* of the top of the minar from the eye of the student. Thus, the **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.



The **angle of elevation** of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object

ANGLE OF DEPRESSION

In the below figure, the girl sitting on the balcony is *looking down* at a flower pot placed on a stair of the temple. In this case, the line of sight is *below* the horizontal level. The angle so formed by the line of sight with the horizontal is called the *angle of depression*. Thus, the **angle of depression** of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed



IMPORTANT QUESTIONS

The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45°, respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

Solution : Let PC = h m be the height of multistoryed building and AB denotes the 8 m tall building.

BD = AC = x m, PC = h = PD + DC = PD + AB = PD + 8 m

So, PD = h – 8 m

Now, $\angle$QPB = $\angle$PBD = 30°

Similarly, $\angle$QPA = $\angle$PAC = 45°.

In right Δ PBD, $\tan 30^0 = \frac{PD}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h-8}{x}$

$\Rightarrow x = (h-8)\sqrt{3}\ m$ ……. (1)

Also, In right Δ PAC, $\tan 45^0 = \frac{PC}{AC} \Rightarrow 1 = \frac{h}{x}$

$\Rightarrow x = h \ \ m$ …….. (2)

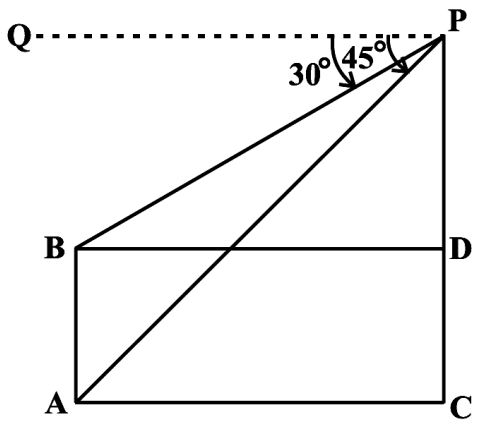
From equations (1) and (2), we get $h = (h-8)\sqrt{3}$

$\Rightarrow h = h\sqrt{3} - 8\sqrt{3} \Rightarrow h\sqrt{3} - h = 8\sqrt{3}$

$\Rightarrow h(\sqrt{3}-1) = 8\sqrt{3} \Rightarrow h = \frac{8\sqrt{3}}{\sqrt{3}-1}$

$\Rightarrow h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{8\sqrt{3}(\sqrt{3}+1)}{3-1}$

$\Rightarrow h = \frac{8(3+\sqrt{3})}{2} = 4(3+\sqrt{3})m$



Hence, the height of the multi-storeyed building is $4(3+\sqrt{3})m$ and the distance between the two buildings is also $4(3+\sqrt{3})m$.

From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 3 m from the banks, find the width of the river.

Solution: Let A and B represent points on the bank on opposite sides of the river, so that AB is the width of the river. P is a point on the bridge at a height of 3 m, i.e., DP = 3 m.

Now, AB = AD + DB

In right Δ APD, $\tan 30^0 = \frac{PD}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AD}$

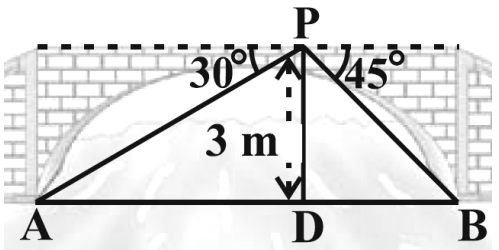
$\Rightarrow AD = 3\sqrt{3} \ \ m$

Also, in right Δ PBD, $\tan 45^0 = \frac{PD}{BD} \Rightarrow 1 = \frac{3}{BD}$

$\Rightarrow BD = 3 \ \ m$

Now, AB = BD + AD = $3 + 3\sqrt{3} = 3(1+\sqrt{3}) \ \ m$

Therefore, the width of the river is $3(1+\sqrt{3}) \ \ m$



Questions for Practice

1. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.
2. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.
3. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.
4. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.
5. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.
6. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

7. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

8. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joing this point to the foot of the tower, the angle of elevation of the top of the tower is 30° . Find the height of the tower and the width of the canal.

9. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

10. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

11. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° . Find the distance travelled by the balloon during the interval.

12. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

13. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

MCQ QUESTIONS (1 mark)

1. The value of (sin30° + cos30°) – (sin60° + cos60°) is
(a) – 1 (b) 0 (c) 1 (d) 2

2. The value of $\frac{\tan 30^0}{\cot 60^0}$ is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 1

3. The value of (sin 45° + cos 45°) is
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

4. If cos A = $\frac{4}{5}$, then the value of tan A is
(a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

5. If sin A = $\frac{1}{2}$, then the value of cot A is
(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

6. The value of the expression [cosec (75° + θ) – sec (15° – θ) – tan (55° + θ) + cot (35° – θ)] is

(a) – 1 (b) 0 (c) 1 (d) $\frac{3}{2}$

7. If cos (α + β) = 0, then sin (α – β) can be reduced to
(a) cos β (b) cos 2β (c) sin α (d) sin 2α

8. The value of (tan1° tan2° tan3° ... tan89°) is
(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

9. If cos 9α = sinα and 9α < 90° , then the value of tan5α is
(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c)1 (d) 0

10. If ΔABC is right angled at C, then the value of cos (A+B) is
(a) 0 (b)1 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

11. If sinA + $\sin^2$A = 1, then the value of the expression ($\cos^2$A + $\cos^4$A) is
(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

12. Given that sinθ = $\frac{a}{b}$, then cosθ is equal to
(a) $\frac{b}{\sqrt{b^2-a^2}}$ (b) $\frac{b}{a}$ (c) $\frac{\sqrt{b^2-a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2-a^2}}$

13. Given that sinα = $\frac{1}{2}$ and cosβ = $\frac{1}{2}$, then the value of (α + β) is
(a) 0° (b) 30° (c) 60° (d) 90°

14. The value of the expression $\left[\frac{\sin^2 22^0+\sin^2 68^0}{\cos^2 22^0+\cos^2 68^0}+\sin^2 63^0+\cos 63^0 \sin 27^0\right]$ is
(a) 3 (b) 2 (c) 1 (d) 0

15. If sinθ – cosθ = 0, then the value of ($\sin^4$θ + $\cos^4$θ) is
(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

16. sin (45° + θ) – cos (45° – θ) is equal to
(a) 2cosθ (b) 0 (c) 2sinθ (d) 1

17. If $4\tan\theta = 3$, then $\left(\frac{4\sin\theta-\cos\theta}{4\sin\theta+\cos\theta}\right)$ is equal to
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

CHAPTER – 12

AREAS RELATED TO CIRCLES

AREA AND PERIMETER OF CIRCLE, QUADRANT, SEMICIRCLE

Area of Circle = πr^2, Perimeter of Circle = Circumference = $2\pi r$

Area of Semicircle = $\frac{1}{2}\pi r^2$, Perimeter of Semicircle = $\pi r + 2r$

Area of Quadrant = $\frac{1}{4}\pi r^2$, Perimeter of Quadrant = $\frac{1}{2}\pi r + 2r$

IMPORTANT QUESTONS

Find the diameter of the circle whose area is equal to the sum of the areas of the two circles of diameters 20 cm and 48 cm.

Solution: Here, radius r_1 of first circle = 20/2 cm = 10 cm
and radius r_2 of the second circle = 48/2 cm = 24 cm
Therefore, sum of their areas = $\pi r_1^2 + \pi r_2^2 = \pi (10)^2 + \pi (24)^2 = \pi \times 676$
Let the radius of the new circle be r cm. Its area = πr^2
Therefore, $\pi r^2 = \pi \times 676 \quad \Rightarrow r^2 = 676 \quad \Rightarrow r = 26$
Thus, radius of the new circle = 26 cm
Hence, diameter of the new circle = 2×26 cm = 52 cm

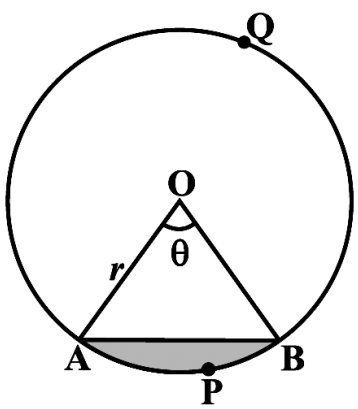
Questions for Practice

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.
2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.
3. Find the diameter of the circle whose area is equal to the sum of the areas of the two circles of diameters 20 cm and 48 cm.
4. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?
5. Find the area of a quadrant of a circle whose circumference is 22 cm.

AREAS OF SECTOR AND SEGMENT OF A CIRCLE

Area of the sector of angle θ $= \frac{\theta}{360^0} \times \pi r^2$, where r is the radius of the circle and θ the angle of the sector in degrees

length of an arc of a sector of angle θ $= \frac{\theta}{360^0} \times 2\pi r$, where r is the radius of the circle and θ the angle of the sector in degrees



Area of the segment APB = Area of the sector OAPB – Area of Δ OAB

$$= \frac{\theta}{360^0} \times \pi r^2 - \text{area of } \Delta \text{ OAB}$$

- Area of the major sector OAQB = πr^2 – Area of the minor sector OAPB
- Area of major segment AQB = πr^2 – Area of the minor segment APB
- Area of segment of a circle = Area of the corresponding sector – Area of the corresponding triangle

IMPORTANT QUESTIONS

Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector (Use π = 3.14).

Solution : Here, radius, r = 4 cm, $\theta = 30^0$,

We know that Area of sector = $\frac{\theta}{360^0} \times \pi r^2 = \frac{30^0}{360^0} \times 3.14 \times 4 \times 4 = \frac{1}{12} \times 3.14 \times 4 \times 4$

$= \frac{12.56}{3} = 4.19 cm^2$ (approx.)

Area of the corresponding major sector

= πr^2 – area of sector OAPB

= (3.14 × 16 – 4.19) cm^2

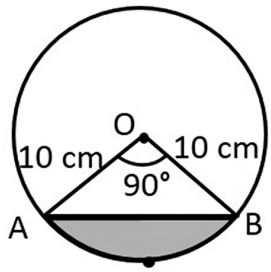
= 46.05 cm2 = 46.1 cm^2(approx.)

A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding : (i) minor segment (ii) major sector. (Use π = 3.14)

Solutions: Here, radius, r = 10 cm, $\theta = 90^0$,

We know that Area of minor sector = $\frac{\theta}{360^0} \times \pi r^2 = \frac{90^0}{360^0} \times 3.14 \times 10 \times 10 = \frac{1}{4} \times 314 = 78.5 cm^2$

and Area of triangle AOB = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 10 \times 10 = 50 cm^2$



Area of minor segment = Area of minor sector –

Area of triangle AOB = 78.5 – 50 = 28.5 cm^2.

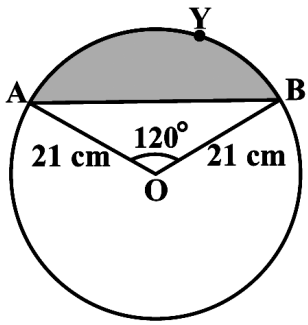
Area of circle = $\pi r^2 = 3.14 \times 10 \times 10 = 314 cm^2$

Area of major sector = Area of circle – Area of minor sector

= 314 – 78.5 = 235.5 cm^2

Questions for Practice

1. Find the area of the segment AYB shown in below figure, if radius of the circle is 21 cm and ∠AOB = 120°.



2. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

4. Find the area of a sector of circle of radius 21 cm and central angle 120°.

5. Area of a sector of a circle of radius 36 cm is 54 π cm2. Find the length of the corresponding arc of the sector.

6. The wheel of a motor cycle is of radius 35 cm. How many revolutions per minute must the wheel make so as to keep a speed of 66 km/h?

7. Find the area of the minor segment of a circle of radius 14 cm, when the angle of the corresponding sector is 60°.

8. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 20m × 16m. Find the area of the field in which the cow can graze.

9. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find (i) the area of that part of the field in which the horse can graze. (ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use $\pi = 3.14$)

10. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors. Find : (i) the total length of the silver wire required. (ii) the area of each sector of the brooch.

11. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find: (i) the length of the arc (ii) area of the sector formed by the arc (iii) area of the segment formed by the corresponding chord

12. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

13. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

14. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

15. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)

16. Find the number of revolutions made by a circular wheel of area 1.54 m^2 in rolling a distance of 176 m.

17. The area of a circular playground is 22176 m^2. Find the cost of fencing this ground at the rate of Rs 50 per metre.

18. Find the area of the sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm.

19. Area of a sector of central angle 200° of a circle is 770 cm2. Find the length of the corresponding arc of this sector.

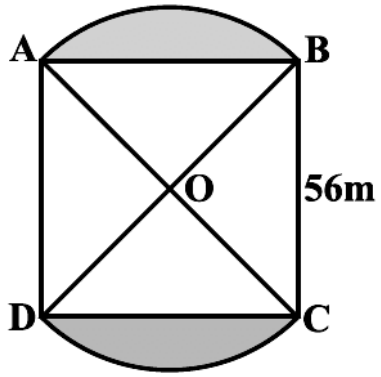
20. Find the area of the segment of a circle of radius 12 cm whose corresponding sector has a central angle of 60° (Use $\pi = 3.14$).

AREA OF SHADED REGION BASED QUESTIONS

IMPORTANT QUESTIONS

In the adjoining figure, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and the flower beds.

Solution: Here, side of square ABCD, a = 56 m



diagonal of square = $a\sqrt{2} = 56\sqrt{2}$

radius, r = OA = OB = OC = OD = $\frac{56\sqrt{2}}{2} = 28\sqrt{2}cm$

Now, Area of sector OAB = Area of sector ODC

$= \frac{\theta}{360^0} \times \pi r^2 = \frac{90^0}{360^0} \times \frac{22}{7} \times r^2 = \frac{1}{4} \times \frac{22}{7} \times r^2$

and Area of Δ OAD = Area of Δ OBC = $\frac{1}{2} \times r \times r = \frac{1}{2} \times r^2$

Total area = Area of sector OAB + Area of sector ODC
+ Area of Δ OAD + Area of Δ OBC

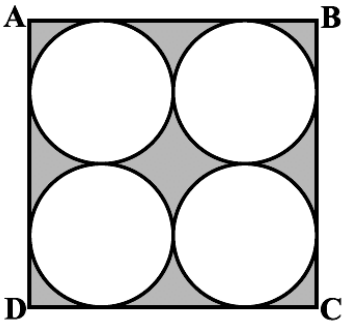
$= \frac{1}{4} \times \frac{22}{7} \times r^2 + \frac{1}{4} \times \frac{22}{7} \times r^2 + \frac{1}{2} \times r^2 + \frac{1}{2} \times r^2$

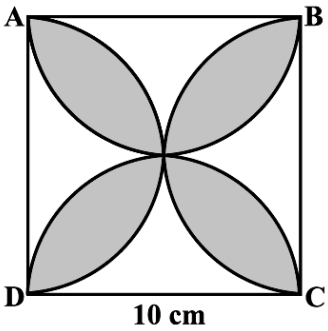
$= 2 \times \frac{1}{4} \times \frac{22}{7} \times r^2 + 2 \times \frac{1}{2} \times r^2 = \frac{11}{7} \times r^2 + r^2 = \left(\frac{11}{7} + 1\right) r^2$

$= \frac{18}{7} \times 28 \times 28 \times 2 = 4032 cm^2$

Questions for Practice

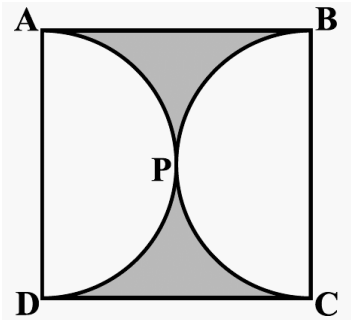
1. Find the area of the shaded region in below left figure, where ABCD is a square of side 14 cm.

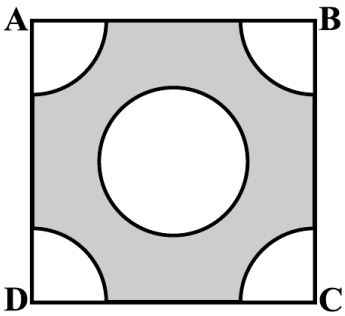




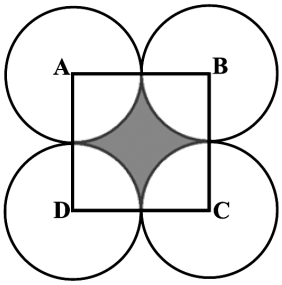
2. Find the area of the shaded design in above right figure, where ABCD is a square of side 10 cm and semicircles are drawn with each side of the square as diameter. (Use $\pi = 3.14$)

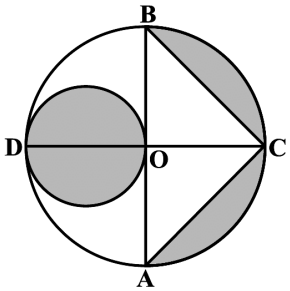
3. Find the area of the shaded region in below left figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.





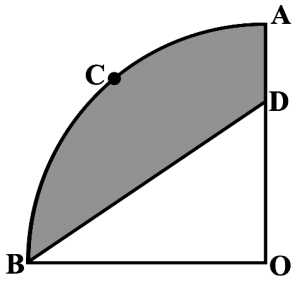
4. In the below left figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.

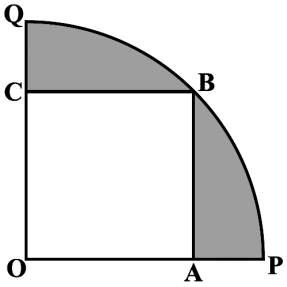




5. In the above right sided figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

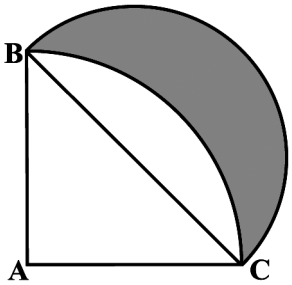
6. In the below left figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.

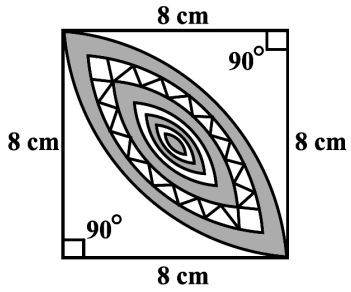




7. In the above right sided figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the (i) quadrant OACB, (ii) shaded region.

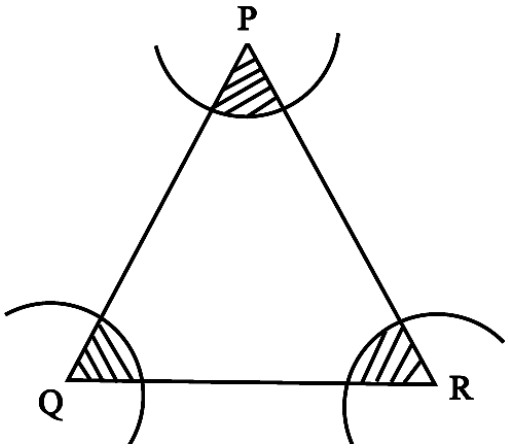
8. In the below figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use $\pi = 3.14$)

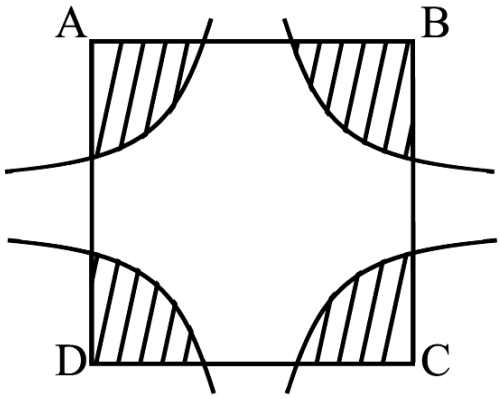




9. Calculate the area of the designed region in above right sided figure, common between the two quadrants of circles of radius 8 cm each.

10. In the below figure, arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.



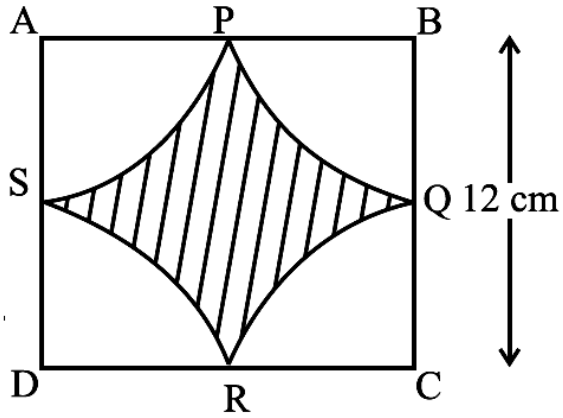


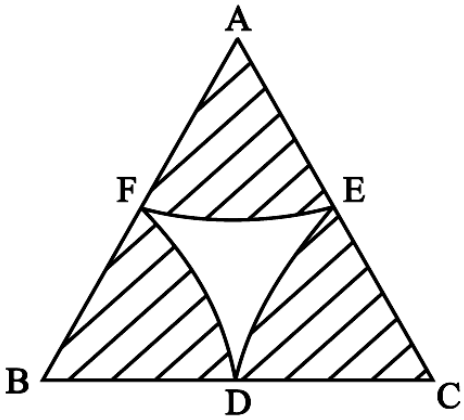
11. In the above right sided figure, arcs have been drawn of radius 21 cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of the shaded region.

12. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in above right sided figure. Find the area of the remaining portion of the square.

13. A circular park is surrounded by a road 21 m wide. If the radius of the park is 105 m, find the area of the road.

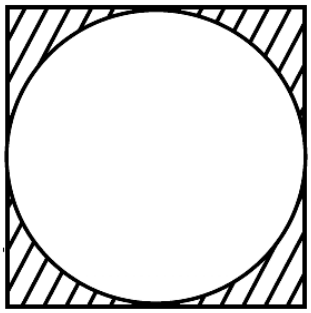
14. Find the area of the shaded region in the below figure, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA, respectively of a square ABCD (Use $\pi = 3.14$).

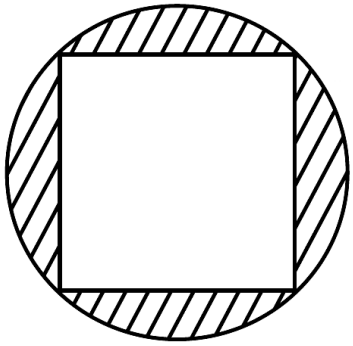




15. In the above right sided figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm. to intersect the sides BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region (Use $\pi = 3.14$).

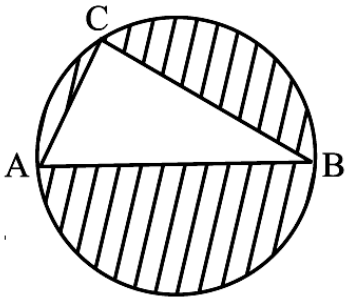
16. In below figure, a circle of radius 7.5 cm is inscribed in a square. Find the area of the shaded region (Use $\pi = 3.14$)

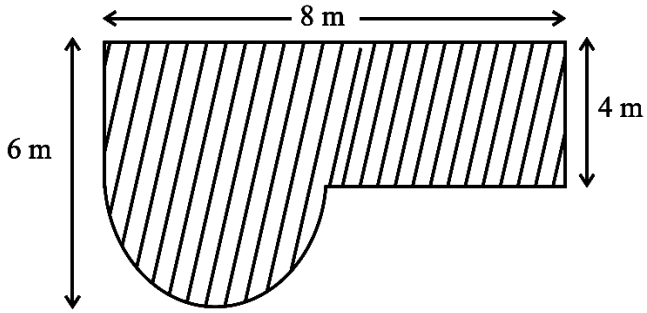




17. In above right sided figure, a square of diagonal 8 cm is inscribed in a circle. Find the area of the shaded region.

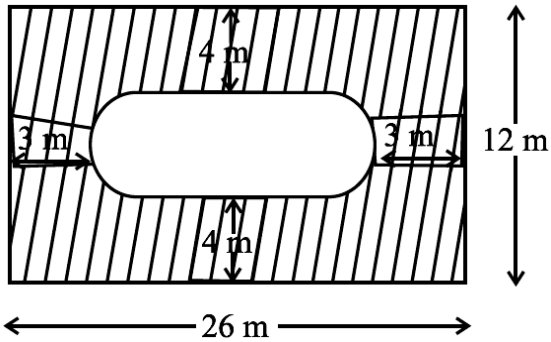
18. In below figure, AB is a diameter of the circle, AC = 6 cm and BC = 8 cm. Find the area of the shaded region (Use $\pi = 3.14$).

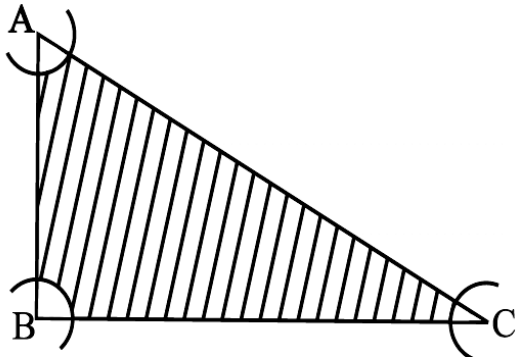




19. Find the area of the shaded field shown in above right sided figure.

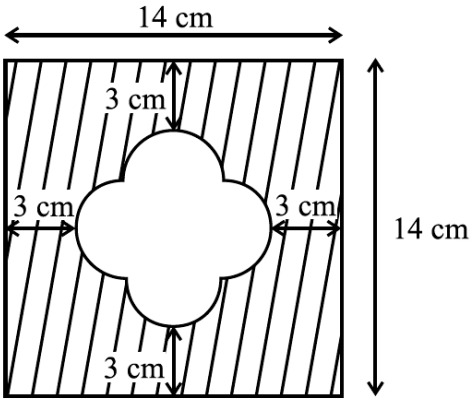
20. Find the area of the shaded field shown in below figure.

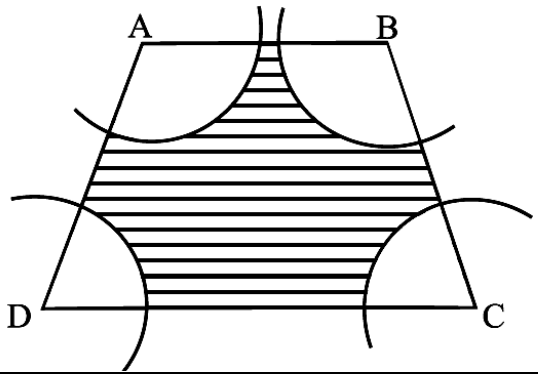




21. With the vertices A, B and C of a triangle ABC as centres, arcs are drawn with radii 5 cm each as shown in above right sided figure. If AB = 14 cm, BC = 48 cm and CA = 50 cm, then find the area of the shaded region. (Use $\pi = 3.14$).

22. Find the area of the shaded field shown in below figure.

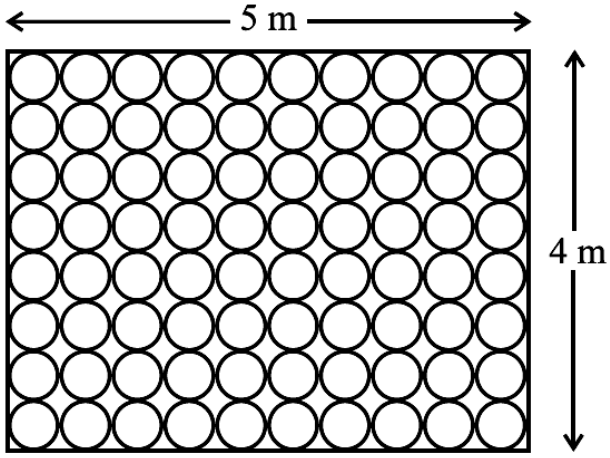




23. In above right sided figure, ABCD is a trapezium with AB || DC, AB = 18 cm, DC = 32 cm and distance between AB and DC = 14 cm. If arcs of equal radii 7 cm with centres A, B, C and D have been drawn, then find the area of the shaded region of the figure.

24. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if area of the circle is 1256 cm^2 . (Use $\pi = 3.14$).

25. Floor of a room is of dimensions 5 m × 4 m and it is covered with circular tiles of diameters 50 cm each as shown in below figure. Find the area of floor that remains uncovered with tiles. (Use $\pi = 3.14$)



MCQ QUESTIONS (1 mark)

1. If the area of a circle is 154 cm^2, then its perimeter is
(a) 11 cm (b) 22 cm (c) 44 cm (d) 55 cm

2. If θ is the angle (in degrees) of a sector of a circle of radius r, then area of the sector is
(a) $\frac{\pi r^2\theta}{360^0}$ (b) $\frac{\pi r^2\theta}{180^0}$ (c) $\frac{2\pi r\theta}{360^0}$ (d) $\frac{2\pi r\theta}{180^0}$

3. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then
(a) $R_1 + R_2 = R$ (b) $R_1^2 + R_2^2 = R^2$ (c) $R_1 + R_2 < R$ (d) $R_1^2 + R_2^2 < R^2$

4. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then
(a) $R_1 + R_2 = R$ (b) $R_1 + R_2 > R$
(c) $R_1 + R_2 < R$ (d) Nothing definite can be said about the relation among R_1, R_2 and R.

5. If the circumference of a circle and the perimeter of a square are equal, then
(a) Area of the circle = Area of the square
(b) Area of the circle > Area of the square
(c) Area of the circle < Area of the square
(d) Nothing definite can be said about the relation between the areas of the circle and square.

6. Area of the largest triangle that can be inscribed in a semi-circle of radius r units is
(a) r^2 sq. units (b) $\frac{1}{2}$ r^2 sq. units (c) 2 r^2 sq. units (d) $\sqrt{2}$ r^2 sq. units

7. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is
(a) 22 : 7 (b) 14 : 11 (c) 7 : 22 (d) 11: 14

8. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be
(a) 10 m (b) 15 m (c) 20 m (d) 24 m

9. The area of the circle that can be inscribed in a square of side 6 cm is
(a) 36π cm^2 (b) 18π cm^2 (c) 12π cm^2 (d) 9π cm^2

10. The area of the square that can be inscribed in a circle of radius 8 cm is
(a) 256 cm^2 (b) 128 cm^2 (c) $64\sqrt{2}$ cm^2 (d) 64 cm^2

11. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36cm and 20 cm is
(a) 56 cm (b) 42 cm (c) 28 cm (d) 16 cm

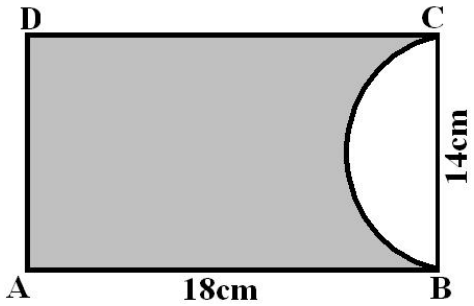
12. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is
(a) 31 cm (b) 25 cm (c) 62 cm (d) 50 cm

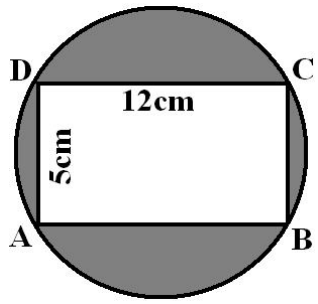
13. A wire is looped in the form of a circle of radius 28 cm. It is rebent into a square form. Determine the length of the side of the square.
(a) 42 cm (b) 44 cm (c) 46 cm (d) 48 cm

14. A circular part, 42 m in diameter has a path 3.5 m wide running round it on the outside. Find the cost of gravelling the path at Rs. 4 per m^2.
(a) Rs. 2800 (b) Rs. 2020 (c) Rs. 2002 (d) none of these

15. The diameter of the wheels of a bus is 140 cm. How many revolutions per minute must a wheel make in order to move a t a speed of 66km/hr?
(a) 240 (b) 250 (c) 260 (d) 270

16. A paper is in the form of a rectangle ABCD in which AB = 18cm and BC = 14cm. A semicircular portion with BC as diameter is cut off. Find the area of the remaining paper (see in below figure).
(a) 175 cm^2 (b) 165 cm^2 (c) 145 cm^2 (d) none of these





17. Find the area of the shaded region in the above sided figure. Take $\pi = 3.14$
(a) 75 cm^2 (b) 72 cm^2 (c) 70 cm^2 (d) none of these

18. A square ABCD is inscribed in a circle of radius 'r'. Find the area of the square in sq. units.
(a) $3r^2$ (b) $2r^2$ (c) $4r^2$ (d) none of these

CHAPTER – 13
SURFACE AREAS AND VOLUMES

IMPORTANT FORMULAE

Name of the Solid	Curved Surface Area	Total Surface Area	Volume
Cuboid	$2h(l+b)$	$2(lb+bh+hl)$	lbh
Cube	$4a^2$	$6a^2$	a^3
Right Circular Cylinder	$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$
Right Circular Cone	πrl	$2\pi r(r+l)$	$\frac{1}{3}\pi r^2 h$
Sphere	–	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Hemisphere	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
Frustum of a Cone	$\pi(r_1+r_2)l$ where $l=\sqrt{h^2+(r_1-r_2)^2}$	$\pi(r_1+r_2)l$ + $\pi r_1^2+\pi r_2^2$	$\frac{1}{3}\pi h(r_1^2+r_2^2+r_1r_2)$

COMBINATIONAL FIGURE BASED QUESTIONS

IMPORTANT QUESTIONS

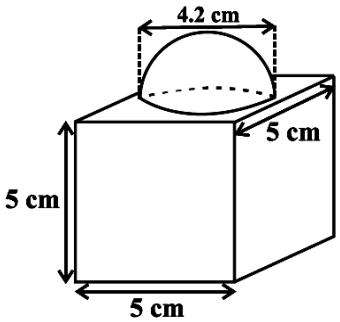
The decorative block is shown in below left figure made of two solids — a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block.

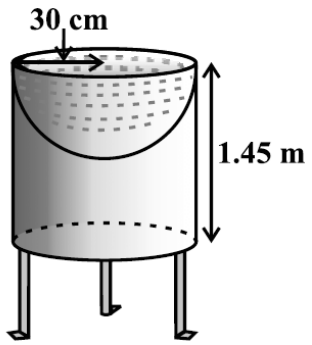
Solution: The total surface area of the cube = 6 × (edge)^2 = 6 × 5 × 5 cm^2 = 150 cm^2.

So, the surface area of the block = TSA of cube – base area of hemisphere + CSA of hemisphere

= 150 – πr^2 + $2\pi r^2$ = (150 + πr^2) cm2

$$=150+\left(\frac{22}{7}\times\frac{4.2}{2}\times\frac{4.2}{2}\right)cm^2=150+13.86cm^2=163.86cm^2$$





Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath.

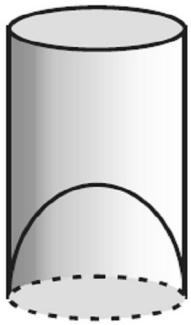
Solution : Let h be height of the cylinder, and r the common radius of the cylinder and hemisphere. (See above right sided figure)

Total surface area of the bird-bath = CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r2 = 2\pi\ r\ (h + r)\ = 2\times\frac{22}{7}\times 30(145+30) = 2\times\frac{22}{7}\times 30\times 175\ = 33000cm^2 = 3.3m^2$

A juice seller was serving his customers using glasses as shown in below figure. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14.)

Solution: Here, inner diameter = 5 cm. height, h = 10 cm



So, radius, r = $\frac{5}{2}$ cm

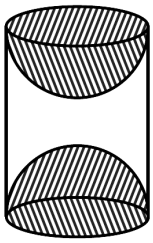
Apparent capacity of the glass = Volume of cylinder – Volume of hemisphere

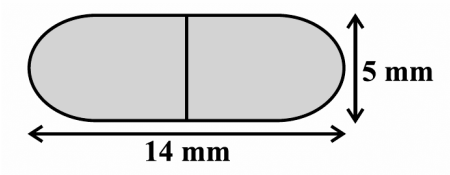
$=\pi r^2 h-\frac{2}{3}\pi r^3 = \pi r^2\left(h-\frac{2}{3}r\right) = 3.14\times\frac{5}{2}\times\frac{5}{2}\times\left(10-\frac{2}{3}\times\frac{5}{2}\right)$

$=3.14\times\frac{25}{4}\times\frac{25}{3}=\frac{19625}{12}=163.54cm^3$

Questions for Practice

1. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder (see below left figure). If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.





2. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see above right sided figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

3. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2.

4. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

5. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

6. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take π = 3.14)

7. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm

8. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm3 of iron has approximately 8g mass. (Use π = 3.14)

9. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it

touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

CONVERSION BASED QUESTIONS

IMPORTANT QUESTIONS

A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.

Solution: Here, radius of cone, r = 6 cm, height of cone, h = 24 cm

Let the radius of the sphere be R cm, then we have

Volume of Sphere = Volume of cone

$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h \;\Rightarrow 4R^3 = r^2 h \Rightarrow R^3 = \frac{r^2 h}{4} = \frac{6\times 6\times 24}{4} = 6\times 6\times 6 \;\Rightarrow R = 6cm$$

Therefore, the radius of the sphere is 6 cm.

Questions for Practice

1. A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire.
2. A hemispherical tank full of water is emptied by a pipe at the rate of litres per second. How much time will it take to empty half the tank, if it is 3m in diameter?
3. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.
4. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.
5. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?
6. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

FRUSTUM OF A CONE BASED QUESTIONS

IMPORTANT QUESTIONS

A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Solution: Here, height of frustum of cone, h = 14 cm, diameters of its two circular ends are 4 cm and 2 cm

So, radii of its two circular ends are R = 2 cm and r = 1 cm

Now, Capacity of the glass = Volume of a frustum of a cone

$$= \frac{\pi h}{3}(R^2 + r^2 + Rr) = \frac{22}{7}\times\frac{14}{3}(2^2 + 1^2 + 2\times 1) = \frac{44}{3}(4+1+2) \;= \frac{44}{3}\times 7 = \frac{308}{3} = 102.67cm^3$$

The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Solution: Here, slant height of a frustum of a cone, l = 4 cm,

Circumference of upper end = 2πr = 6 cm

So, πr = 3 cm

and Circumference of upper end = 2πR = 18 cm

So, πR = 9 cm

Now, curved surface area of the frustum = πl(R + r) = l x (πR + πr)

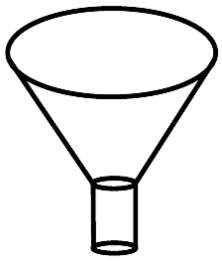
= 4 x (9 + 3) = 4 x 12 = 48cm^2

Questions for Practice

1. The radii of the ends of a frustum of a cone 45 cm high are 28 cm and 7 cm. Find its volume, the curved surface area and the total surface area
2. An open metal bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metallic sheet. The diameters of the two circular ends of the bucket are 45 cm and 25 cm, the total vertical height of the bucket is 40 cm and that of the cylindrical base is 6 cm. Find the area of the metallic sheet used to make the bucket, where we do not take into account the handle of the bucket. Also, find the volume of water the bucket can hold.
3. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm2. (Take π = 3.14)
4. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter cm, find the length of the wire.

MCQ QUESTIONS (1 mark)

1. A funnel (see below figure) is the combination of
   (a) a cone and a cylinder (b) frustum of a cone and a cylinder
   (c) a hemisphere and a cylinder (d) a hemisphere and a cone



2. If a marble of radius 2.1 cm is put into a cylindrical cup full of water of radius 5cm and height 6 cm, then how much water flows out of the cylindrical cup?
   (a) 38.8 cm^3 (b) 55.4 cm^3 (c) 19.4 cm^3 (d) 471.4 cm^3

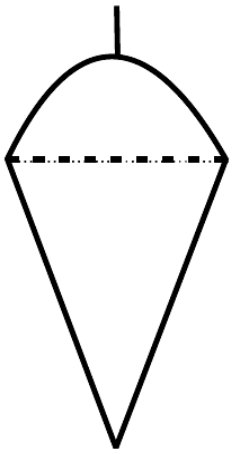
3. A cubical ice cream brick of edge 22 cm is to be distributed among some children by filling ice cream cones of radius 2 cm and height 7 cm upto its brim. How many children will get the ice cream cones?
   (a) 163 (b) 263 (c) 363 (d) 463

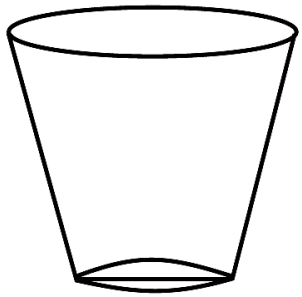
4. The volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is
   (a) 9.7 cm^3 (b) 77.6 cm^3 (c) 58.2 cm^3 (d) 19.4 cm^3

5. A cylindrical pencil sharpened at one edge is the combination of
   (a) a cone and a cylinder (b) frustum of a cone and a cylinder
   (c) a hemisphere and a cylinder (d) two cylinders.

6. A surahi is the combination of
   (a) a sphere and a cylinder (b) a hemisphere and a cylinder
   (c) two hemispheres (d) a cylinder and a cone.

7. A plumbline (sahul) is the combination of (see below figure)
(a) a cone and a cylinder (b) a hemisphere and a cone
(c) frustum of a cone and a cylinder (d) sphere and cylinder





8. The shape of a glass (tumbler) (see above right figure) is usually in the form of
(a) a cone (b) frustum of a cone (c) a cylinder (d) a sphere

9. The shape of a gilli, in the gilli-danda game (see below figure), is a combination of
(a) two cylinders (b) a cone and a cylinder
(c) two cones and a cylinder (d) two cylinders and a cone

10. A shuttle cock used for playing badminton has the shape of the combination of
(a) a cylinder and a sphere (b) a cylinder and a hemisphere
(c) a sphere and a cone (d) frustum of a cone and a hemisphere

11. A cone is cut through a plane parallel to its base and then the cone that is formed on one side of that plane is removed. The new part that is left over on the other side of the plane is called
(a) a frustum of a cone (b) cone (c) cylinder (d) sphere

12. A hollow cube of internal edge 22cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that 1/8 space of the cube remains unfilled. Then the number of marbles that the cube can accommodate is
(a) 142296 (b) 142396 (c) 142496 (d) 142596

13. A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively is melted and recast into the form a cone of base diameter 8cm. The height of the cone is
(a) 12cm (b) 14cm (c) 15cm (d) 18cm

14. A solid piece of iron in the form of a cuboid of dimensions 49cm × 33cm × 24cm, is moulded to form a solid sphere. The radius of the sphere is
(a) 21cm (b) 23cm (c) 25cm (d) 19cm

15. A mason constructs a wall of dimensions 270cm× 300cm × 350cm with the bricks each of size 22.5cm × 11.25cm × 8.75cm and it is assumed that 1/8 space is covered by the mortar. Then the number of bricks used to construct the wall is
(a) 11100 (b) 11200 (c) 11000 (d) 11300

16. Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is
(a) 4 cm (b) 3 cm (c) 2 cm (d) 6 cm

17. The radii of the top and bottom of a bucket of slant height 45 cm are 28 cm and 7 cm, respectively. The curved surface area of the bucket is
(a) 4950 cm^2 (b) 4951 cm^2 (c) 4952 cm^2 (d) 4953 cm^2

18. A medicine-capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of entire capsule is 2 cm. The capacity of the capsule is
(a) 0.36 cm^3 (b) 0.35 cm^3 (c) 0.34 cm^3 (d) 0.33 cm^3

19. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is
(a) $4\pi r^2$ (b) $6\pi r^2$ (c) $3\pi r^2$ (d) $8\pi r^2$

20. A right circular cylinder of radius r cm and height h cm (h>2r) just encloses a sphere of diameter
(a) r cm (b) 2r cm (c) h cm (d) 2h cm

21. During conversion of a solid from one shape to another, the volume of the new shape will
(a) increase (b) decrease (c) remain unaltered (d) be doubled

22. The diameters of the two circular ends of the bucket are 44 cm and 24 cm. The height of the bucket is 35 cm. The capacity of the bucket is
(a) 32.7 litres (b) 33.7 litres (c) 34.7 litres (d) 31.7 litres

23. In a right circular cone, the cross-section made by a plane parallel to the base is a
(a) circle (b) frustum of a cone (c) sphere (d) hemisphere

24. Volumes of two spheres are in the ratio 64:27. The ratio of their surface areas is
(a) 3 : 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9