SUBJECT: MATHEMATICS
CLASS : X
MAX. MARKS : 80
DURATION : 3 HRS

General Instruction:
(i) All the questions are compulsory.
(ii) The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION – A
Questions 1 to 20 carry 1 mark each.

1. The HCF of 90 and 144 is :
   (a) 9
   (b) 18
   (c) 3
   (d) None of these
   Ans: (b) 18

2. For some integer q, every odd integer is of the form :
   (a) q
   (b) q + 1
   (c) 2q
   (d) 2q + 1
   Ans: (d) 2q + 1

3. After how many decimal places will the decimal expansion of \( \frac{23}{2^2 \times 5^2} \) terminate?
   (a) 1 decimal place
   (b) 2 decimal places
   (c) 3 decimal places
   (d) 4 decimal places
   Ans: (b) 2 decimal places

4. If p and q are the zeroes of the quadratic polynomial \( f(x) = 2x^2 - 7x + 3 \), find the value of \( p + q - pq \) is
   (a) 1
   (b) 2
   (c) 3
   (d) None of these
   Ans: (b) 2

5. The number of zeroes of the given polynomial \( y = f(x) \) from the graph is
   (a) 2
   (b) 3
   (c) 4
   (d) 5
   Ans: (d) 5

6. The distance of the point \((-4, -7)\) from the y-axis is
   (a) 4 units
   (a) 7 units
   (c) 11 units
   (d) none of these
   Ans: (a) 4 units
7. The distance between the points (0, 5) and (–5, 0) is
   (a) 5  (b) \(5\sqrt{2}\)  (c) \(2\sqrt{5}\)  (d) 10
   Ans: (b) \(5\sqrt{2}\)

8. If PQ and PR are two tangents to a circle with center O. If \(\angle QPR = 46^\circ\), then \(\angle QOR\) equals:
   (a) 67°  (b) 134°  (c) 44°  (d) 46°
   Ans: (b) 134°.

9. The abscissa of the point of intersection of the less than type and of the more than type ogives gives its
   (a) mean  (b) median  (c) mode  (d) all the three
   Ans: (b) median

10. When a die is thrown, the probability of getting an odd number less than 3 is
    (a) \(\frac{1}{6}\)  (b) \(\frac{1}{3}\)  (c) \(\frac{1}{2}\)  (d) 0
    Ans: (a) \(\frac{1}{6}\)

11. If \(x = \frac{1}{2}\) is a solution of the quadratic equation \(x^2 + kx - \frac{5}{4} = 0\) then the value of \(k\) is ______
    Ans: \(k = 2\)

OR

The value of \(k\) for which the equations 3x – 2y + 8 = 0 and 6x – ky + 16 = 0 represent coincident lines is ______
Ans: \(k = 4\)

12. Sides of two similar triangles are in the ratio 4 : 7, then the ratio of the areas of these triangles is ______.
    Ans: 16 : 49

13. The ordinate of a point A on y-axis is 5 and B has coordinates (-3, 1) then the length AB is ______
    Ans: 5 units

14. If \(\cos A = \frac{4}{5}\), then the value of \(\tan A\) is ______
    Ans: \(\tan A = \frac{3}{4}\)

15. In \(\triangle ABC\) is right angled at C, then the value of \(\cos(A + B)\) is ______
    Ans: 0

16. In the below \(\triangle ABC\), DE \parallel BC, then find the value of x.
    Ans: \(x = 2\)
17. If $\sin A = \frac{1}{2}$ and $\cos B = \frac{1}{2}$, then find value of $A + B$.  
   Ans: $90^0$.  

OR  

Find the value of $\frac{\sin 25^0}{\cos 65^0} + \frac{\tan 23^0}{\cot 67^0}$  
   Ans: 2  

18. Find the $30^{th}$ term of the AP: 10, 7, 4, 1, ............  
   Ans: $-77$  

19. A bag contains 6 red and 5 blue balls. Find the probability that the ball drawn is not red.  
   Ans: $\frac{5}{11}$  

20. Find the radius of a circle whose circumference is equal to the sum of the circumference of the two circles of diameters 36cm and 20cm.  
   Ans: $2\pi r = 2\pi r_1 + 2\pi r_2 \Rightarrow r = r_1 + r_2 \Rightarrow r = 18 \text{ cm} + 10 \text{ cm} = 28 \text{ cm}$  

SECTION – B  
Questions 21 to 26 carry 2 marks each.  

21. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.  
   Ans: Number of total possible outcomes = 8  
   The possible outcomes are {HHH, TTT, HHT, HTH, THH, TTH, THT, HTT}  
   Number of favourable outcomes = 6 {i.e. HHT, HTH, THH, TTH, THT, HTT}  
   $P (\text{Hanif will lose the game}) = \frac{6}{8} = \frac{3}{4}$  

   OR  

One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a red face card (ii) the queen of spade.  
   Ans: Total number of cards in a well-shuffled deck = 52  
   (i) Total number of red face cards = 6  
   $P (\text{getting a red face card}) = \frac{6}{52} = \frac{3}{26}$  
   (ii) Total number of queen of spade = 1  
   $P (\text{getting a queen of spade}) = \frac{1}{52}$  

22. A die is thrown twice. What is the probability that (i) 5 will not come up either time? (ii) 5 will come up at least once?  
   Ans: Total number of outcomes = $6 \times 6 = 36$  
   (i)Total number of outcomes when 5 comes up on either time are (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (1, 5), (2, 5), (3, 5), (4, 5), (6, 5)  
   Hence, total number of favourable cases = 11  
   $P (\text{5 will come up either time}) = \frac{11}{36}$  
   $P (\text{5 will not come up either time}) = 1 - \frac{11}{36} = \frac{25}{36}$
(ii) Total number of cases, when 5 can come at least once = 11

\[ P(5 \text{ will come at least once}) = \frac{11}{36} \]

23. If \( \sec 4A = \csc (A - 20^\circ) \), where \( 4A \) is an acute angle, find the value of \( A \).
   Ans: Given that, \( \sec 4A = \csc (A - 20^\circ) \)
   \[ \Rightarrow \csc (90^\circ - 4A) = \csc (A - 20^\circ) \]
   \[ \Rightarrow 90^\circ - 4A = A - 20^\circ \Rightarrow 110^\circ = 5A \Rightarrow A = 22^\circ \]

   OR
   In \( \Delta ABC \), right-angled at \( B \), \( AB = 5 \text{ cm} \) and \( \angle ACB = 30^\circ \). Determine the lengths of the sides \( BC \) and \( AC \).
   Ans: In \( \Delta ABC \), \( \tan C = \frac{AB}{BC} \Rightarrow \tan 30^\circ = \frac{5}{BC} = \frac{1}{\sqrt{3}} \Rightarrow BC = 5\sqrt{3} \text{ cm} \)
   Now, \( \sin C = \frac{AB}{AC} \Rightarrow \sin 30^\circ = \frac{5}{AC} = \frac{1}{2} \Rightarrow AC = 10 \text{ cm} \)

24. Divide \( x^3 - 3x^2 + 5x - 3 \) by \( x^2 - 2 \) and find the quotient and remainder.
   Ans: Quotient, \( q(x) = x - 3 \) and Remainder, \( r(x) = 7x - 9 \)

25. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding minor segment (Use \( \pi = 3.14 \))
   Ans: Area of minor sector OACB = \( \frac{90^\circ}{360^\circ} \times \pi r^2 = \frac{1}{4} \times 3.14 \times 10 \times 10 = 78.5 \text{ cm}^2 \)
   Area of \( \Delta OAB = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2 \)
   Area of minor segment ACB = Area of minor sector OACB – Area of \( \Delta OAB \)
   = 78.5 - 50 = 28.5 \text{ cm}^2

26. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that \( AB + CD = AD + BC \)
   Ans: Using theorem, the lengths of tangents drawn from an external point to a circle are equal. \( AP = AS \ldots(i) \)
   \( BP = BQ \ldots(ii) \)
   \( CQ = RC \ldots(iii) \)
   \( SD = RD \ldots(iv) \)
   On adding Eqs.(i), (ii), (iii) and (iv), we get
   \( (AP + BP) + (RC + RD) = (AS + BQ) + (CQ + SD) \)
   \[ \Rightarrow AB + CD = (AS + SD) + (BQ + CQ) \]
   \[ \Rightarrow AB + CD = AD + BC \]

SECTION – C
Questions 27 to 34 carry 3 marks each.

27. Prove that \( \sqrt{5} \) is an irrational number.
   Ans: Let \( \sqrt{5} \) be a rational number
   Therefore, \( \sqrt{5} = \frac{p}{q} \) where \( p, q \) are co-primes and \( q \neq 0 \)
   On squaring both sides, we get \( p^2 = 5q^2 \ldots(1) \)
   \( \Rightarrow 5 \) is a factor of \( p^2 \) [since, \( 5q^2 = p^2 \) \( \Rightarrow 5 \) is a factor of \( p \)
   Let \( p = 5m \) for all \( m \) (where \( m \) is a positive integer)
   Squaring both sides, we get \( p^2 = 25m^2 \ldots(2) \)
   From (1) and (2), we get \( 5q^2 = 25m^2 \Rightarrow q^2 = 5m^2 \)
   \( \Rightarrow 5 \) is a factor of \( q^2 \) [since, \( q^2 = 5m^2 \) \( \Rightarrow 5 \) is a factor of \( q \)
   Thus, we see that both \( p \) and \( q \) have common factor 5 which is a contradiction that \( p, q \) are co-primes.
Therefore, Our assumption is wrong
Hence \( \sqrt{5} \) is not a rational number i.e., irrational number.

OR

Show that any positive odd integer is of the form \( 6q + 1 \), or \( 6q + 3 \), or \( 6q + 5 \), where \( q \) is some integer.

Ans: Let \( x \) be the positive odd integer which when divided by 6 gives \( q \) as quotient and \( r \) as remainder. According to Euclid’s division lemma, we have \( x = bq + r \)
\[ \Rightarrow x = 6q + r \text{ where, } r = 0,1,2,3,4,5 \]
then, \( x = 6q + 1 \) or \( 6q + 1 \) or \( 6q + 2 \) or \( 6q + 3 \) or \( 6q + 4 \) or \( 6q + 5 \)

Now, \( 6q = 2 \times 3q \)
\( 6q \) is an even integer

We know that the sum of two even integers is always an even integer
Therefore, \( 6q + 2 \) and \( 6q + 4 \) are even integers

We know that the sum of even and odd integer is always an odd integer.
Therefore, \( 6q + 1 \), \( 6q + 3 \), \( 6q + 5 \) are odd integers

Hence, any positive odd integer is of the form \( 6q + 1 \), or \( 6q + 3 \), or \( 6q + 5 \), where \( q \) is some integer

28. Prove that "The lengths of the two tangents from an external point to a circle are equal."

Given, To prove, Diagram, Construction and Proof

29. In the below figure, \( AB \) and \( CD \) are two diameters of a circle (with centre \( O \)) perpendicular to each other and \( OD \) is the diameter of the smaller circle. If \( OA = 7 \) cm, find the area of the shaded region.

Ans:

Given, \( OA = 7 \) cm \quad \( OD = 7 \) cm

Now, area of smaller circle whose diameter \( (OD = 7 \) cm) is
\[ = \pi r^2 = \pi \left( \frac{7}{2} \right)^2 = \frac{22}{7} \times \frac{49}{4} = \frac{77}{2} \text{ cm}^2 \]

Now, \( \triangle ABC = \frac{1}{2} \times AB \times OC = \frac{1}{2} \times 2 \times OA \times OC \)
\[ = \frac{1}{2} \times 14 \times 7 \quad (\because OA = OC) = 49 \text{ cm}^2 \]

and \( \text{area of semi-circle } ABCA = \frac{\pi r^2}{2} = \frac{22}{7} \times \frac{7}{2} \)
\[ = 77 \text{ cm}^2 \]

\[ \therefore \text{Area of segment } BC \text{ and } AC = \text{Area of semi-circle} - \text{Area of } \triangle ABC \]
\[ = 77 - 49 = 28 \text{ cm}^2 \]

\[ \therefore \text{Area of total shaded region} = \text{Area of small circle} \]
\[ + \text{Area of segment } BC \text{ and } AC \]
\[ = \frac{77}{2} + 28 = 38.5 + 28 = 66.5 \text{ cm}^2 \]

30. Find the zeroes of the quadratic polynomial \( x^2 - 2x - 8 \) and verify the relationship between the zeroes and the coefficients.

Ans: \( x^2 - 2x - 8 = 0 \)
\[ \Rightarrow x^2 - 4x + 2x - 8 = 0 \]
\[ \Rightarrow x(x - 4) + 2(x - 4) = 0 \]
\[ \Rightarrow (x + 2) (x - 4) = 0 \]
\[ \Rightarrow x = -2, 4 \]
Now, $\alpha + \beta = -2 + 4 = 2$ and $\frac{-b}{a} = \frac{-(-2)}{1} = 2 \Rightarrow \alpha + \beta = \frac{-b}{a}$

$\alpha \beta = -2 \times 4 = -8$ and $\frac{c}{a} = \frac{-8}{1} = -8 \Rightarrow \alpha \beta = \frac{c}{a}$

31. Prove that: \((\sin A + \cosec A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A\)

Ans: LHS = \((\sin A + \cosec A)^2 + (\cos A + \sec A)^2\)

\[
= \sin^2 A + \cosec^2 A + 2\sin A\cosec A + \cos^2 A + \sec^2 A + 2\cos A\sec A
\]
\[
\therefore 1 + \tan^2 A = \sec^2 A, 1 + \cot^2 A = \cosec^2 A
\]
\[
= \sin^2 A + \cos^2 A + 1 + 2 + 1 + 2 + \tan^2 A + \cot^2 A
\]
\[
= 7 + \tan^2 A + \cot^2 A \quad (\therefore \sin^2 A + \cos^2 A = 1) = \text{RHS}
\]

OR

Prove that: \(\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \cos ec A + \cot A\)

Ans:

LHS = \(\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}\)

On dividing numerator and denominator by \(\sin A\), we get

\[
\frac{\cot A - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\cot A + \frac{1}{\sin A} - \cot A}
\]
\[
= \frac{\cot A + \sec A - 1}{\cot A + 1 - \sec A}
\]
\[
\therefore 1 = \cosec^2 A - \cot^2 A
\]
\[
= \frac{\cot A + \sec A}{\cot A + 1 - \cosec A}
\]
\[
\therefore (a^2 - b^2) = (a + b)(a - b)
\]
\[
= \frac{(\cot A + \cosec A)(1 - (\cosec A - \cot A))}{\cot A + 1 - \cosec A}
\]
\[
= \frac{\cot A + \cosec A}{\cot A + 1 - \cosec A} = \cosec A + \cot A = \text{RHS}
\]

32. Draw the graphs of the equations \(5x - y = 5\) and \(3x - y = 3\). Determine the co-ordinates of the vertices of the triangle formed by these lines and the y axis.

Ans: It can be observed that the required triangle is \(\Delta ABC\) formed by these lines and y-axis. The coordinates of vertices are A (1, 0), B (0, -3), C (0, -5).
33. Draw a triangle $ABC$ with side $BC = 7$ cm, $\angle B = 45^\circ$, $\angle A = 105^\circ$. Then, construct a triangle whose sides are $\frac{3}{5}$ times the corresponding sides of $\triangle ABC$.

**Ans:** Correct Construction of a triangle $ABC$ with $BC = 7$ cm, $\angle B = 45^\circ$ & $\angle A = 105^\circ$  
Correct Construction of another triangle whose sides are $\frac{3}{5}$ times the corresponding sides of $\triangle ABC$.

**OR**

Draw a line segment of length 9.6 cm and divide it in the ratio 3 : 4.

**Ans:** Correct line segment of length 9.6 cm  
Correct division of line segment in the ratio 3 : 4

34. Four friends Aditya(A), Bunny(B), Chotu(C) and Dhanush(D) are sitting in a park and they are talking to each other using walkie-talkie. Aditya told his friends that their positions will form a quadrilateral in a park. All friends also agree with Aditya. They got the coordinates of their positions as $A(-5, 7)$, $B(-4, -5)$, $C(-1, -6)$ and $D(4, 5)$ by taking origin in the centre of the park. After obtaining the coordinates they have calculate the area of the quadrilateral formed. How much area they calculated?

**Ans:**

By joining B to D, you will get two triangles $ABD$ and $BCD$.

Now the area of $\triangle ABD = \frac{1}{2} \left[ -5(-5 - 5) + (-4)(5 - 7) + 4(7 + 5) \right]$
\[= \frac{1}{2} (50 + 8 + 48) = \frac{106}{2} = 53 \text{ square units} \]

Also, the area of $\triangle BCD = \frac{1}{2} \left[ -4(-5 - 5) - 1(5 + 5) + 4(-5 + 6) \right]$
\[= \frac{1}{2} (44 - 10 + 4) = 19 \text{ square units} \]

So, the area of quadrilateral $ABCD = 53 + 19 = 72 \text{ square units}$.  

**SECTION – D**

Questions 35 to 40 carry 4 marks each.

35. The angle of elevation of the top of a building from the foot of the tower is $30^\circ$ and the angle of elevation of the top of the tower from the foot of the building is $60^\circ$. If the tower is 50 m high, find the height of the building.

**Ans:**

36. In a class test, the sum of Shefali’s marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.
37. Prove that “The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.”

Given, To prove, Diagram, Construction and Proof

OR

State and prove Pythagoras theorem.

Statement, Given, To prove, Diagram, Construction and Proof

38. Find the sum of first 24 terms of the list of numbers whose nth term is given by \( a_n = 3 + 2n \)

Ans: As \( a_n = 3 + 2n \), so, \( a_1 = 3 + 2 = 5 \), \( a_2 = 3 + 2 \times 2 = 7 \), \( a_3 = 3 + 2 \times 3 = 9 \) ....

List of numbers becomes 5, 7, 9, 11, . . .

Here, 7 – 5 = 9 – 7 = 11 – 9 = 2 and so on. So, it forms an AP with common difference \( d = 2 \).

To find \( S_{24} \), we have \( n = 24, a = 5, d = 2 \).

Therefore, \( S_{24} = \frac{24}{2}[2 \times 5 + (24 - 1)\times 2] = 12[10 + 46] = 672 \)

So, sum of first 24 terms of the list of numbers is 672.

OR

How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

Ans: Here, \( a = 24, d = 21 - 24 = -3, S_n = 78 \). We need to find \( n \).

We know that \( S_n = \frac{n}{2}[2a + (n - 1)d] \). So, \( 78 = \frac{n}{2}[48 + (n - 1)(-3)] = \frac{n}{2}[51 - 3n] \)

\( \Rightarrow 3n^2 - 51n + 156 = 0 \) \( \Rightarrow n^2 - 17n + 52 = 0 \)

\( \Rightarrow (n - 4)(n - 13) = 0 \) \( \Rightarrow n = 4 \) or 13

Both values of \( n \) are admissible. So, the number of terms is either 4 or 13.

39. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm \( \times \) 10 cm \( \times \) 3.5 cm?

Ans: Coins are cylindrical in shape.

Height \( (h_1) \) of cylindrical coins = 2 mm = 0.2 cm

Radius \( (r) \) of circular end of coins = 1.75/2 = 0.875 cm

Let \( n \) coins be melted to form the required cuboids.

Volume of \( n \) coins = Volume of cuboids

\( n \pi r^2 h_1 = l \times b \times h \)

\( n \pi \times (0.875)^2 \times 0.2 = 5.5 \times 10 \times 3.5 \)

\( n = \frac{5.5 \times 10 \times 3.5 \times 7}{(0.875)^2 \times 0.2 \times 22} = 400 \)

Therefore, the number of coins melted to form such a cuboid is 400.
A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

**Ans:** The radius of the conical part and the hemispherical part is same (i.e., 3.5 cm).

Height of hemispherical part = Radius \( (r) = 3.5 = \frac{7}{2} \) cm

Height of conical part (h) = 15.5 – 3.5 = 12 cm

Slant height \( (l) \) of conical part = \( \sqrt{r^2 + h^2} \)

\[
= \sqrt{\left(\frac{7}{2}\right)^2 + (12)^2} = \sqrt{\frac{49}{4} + 144} = \sqrt{\frac{49 + 576}{4}} = \sqrt{\frac{625}{4}} = \frac{25}{2}
\]

Total surface area of toy = CSA of conical part + CSA of hemispherical part

\[
= \pi rl + 2\pi r^2
\]

\[
= \frac{22}{7} \times \frac{7}{2} \times \frac{25}{2} + 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 137.5 + 77 = 214.5 \text{ cm}^2
\]

40. Draw less than type ogive of the following table and hence find median from the graph

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**Ans:**

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Median = 43