SECTION - A

Questions 1 to 20 carry 1 mark each.

1. The decimal expansion of the rational number \( \frac{11}{2^4 \times 5^3} \) will terminate after:
   (a) one decimal place    (b) two decimal places    (c) three decimal places    (d) more than three decimal places.
   \[ \text{Ans: (d) more than three decimal places.} \]

2. If \( n \) is an even natural number, then the largest natural number by which \( n(n + 1) (n + 2) \) is divisible is
   (a) 6    (b) 8    (c) 12    (d) 24
   \[ \text{Ans: (d) 24} \]

3. For what value of \( k \), does the pair of linear equations given below has a unique solution?
   \[ 2x + ky = 6 \quad \text{and} \quad 4x + 6y = 0 \]
   (a) \( k = 3 \)    (b) \( k \neq 3 \)    (c) \( k \neq -3 \)    (d) none of these
   \[ \text{Ans: (b) } k \neq 3 \]

4. The point which divides the line segment joining the points \( (7, -6) \) and \( (3, 4) \) in ratio 1 : 2 internally lies in the
   (a) I quadrant    (b) II quadrant    (c) III quadrant    (d) IV quadrant
   \[ \text{Ans: (d) IV quadrant} \]

5. If the points \( (7, -2), (5, 1) \) and \( (3, k) \) are collinear then the value of \( k \) is
   (a) 4    (b) 10    (c) -4    (d) 0
   \[ \text{Ans: (a) 4} \]

6. The point on x-axis which is equidistant from points \( (-1, 0) \) and \( (5, 0) \) is
   (a) \( (0, 2) \)    (b) \( (2, 0) \)    (c) \( (3, 0) \)    (d) \( (0, 3) \)
   \[ \text{Ans: (b) (2, 0)} \]

7. The value of the expression \( \left[ \frac{\sin^2 22^0 + \sin^2 68^0 + \sin^2 63^0 + \cos 63^0 \sin 27^0 + \tan^2 45^0}{\cos^2 22^0 + \cos^2 68^0} \right] \) is
   (a) 3    (b) 0    (c) 1    (d) 2
   \[ \text{Ans: (a) 3} \]
8. If \( \sin 3A = \cos (A - 26^\circ) \), where 3A is an acute angle, find the value of A.
   (a) \( 29^\circ \)  (b) \( 30^\circ \)  (c) \( 26^\circ \)  (d) \( 36^\circ \)
   Ans: (a) \( 29^\circ \)

9. In a right triangle ABC, right-angled at B, if \( \tan A = 1 \), then the value of \( 2 \sin A \cos A = \)
   (a) 0  (b) 1  (c) \( \frac{1}{2} \)  (d) not defined
   Ans: (b) 1

10. For the following distribution:

<table>
<thead>
<tr>
<th>Class</th>
<th>0 – 5</th>
<th>5 – 10</th>
<th>10 – 15</th>
<th>15 – 20</th>
<th>20 – 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>10</td>
<td>15</td>
<td>12</td>
<td>20</td>
<td>9</td>
</tr>
</tbody>
</table>

The sum of lower limits of the median class and the modal class is
   (a) 15  (b) 25  (c) 30  (d) 35
   Ans: (b) 25

11. In an AP, if \( a = 3 \), \( n = 8 \), \( S_n = 192 \), then the value of d is _____
   Ans: \( d = 6 \)

12. If the probability of an event is \( p \), the probability of its complementary event will be ______
   Ans: \( 1 - p \)

13. A shuttle cock used for playing badminton has the shape of the combination of ___________
   Ans: frustum of cone and hemisphere

14. P and Q are points on the sides AB and AC respectively of a triangle ABC. PQ is parallel to BC and divides the triangle ABC into 2 parts, equal in area. The ratio of PA:AB =______
   Ans: \( 1 : \sqrt{2} \)

15. The value(s) of k for which the equation \( x^2 + 5kx + 16 = 0 \) has real and equal roots is _____
   Ans: Here, \( a = 1 \), \( b = 5k \), \( c = 16 \)
   Now, \( b^2 - 4ac = 0 \Rightarrow 25k^2 - 4(1)(16) = 0 \Rightarrow 25k^2 = 64 \Rightarrow k^2 = 64/25 \Rightarrow k = \pm\frac{8}{5} \)

   OR

   If one of the zeroes of the cubic polynomial \( x^3 + ax^2 + bx + c \) is \( -1 \), then the product of the other two zeroes is ________
   Ans: \(-b\)

16. AB and CD are two common tangents to circles which touch each other at a point C. If D lies on AB such that CD = 4 cm then find AB.
   Ans: \( AB = 8 \text{ cm} \)

   OR

   TP is a tangent to the circle with centre O. If \( \angle TOQ = 120^\circ \), find the diameter of the circle when \( OT = 10 \text{ cm} \).

   Ans: diameter = 10 cm
17. Show that $12^n$ cannot end with the digit 0 or 5 for any natural number $n$.
   Ans: If the number $12^n$, for any $n$, were to end with the digit zero, then it would be divisible by 5. That is, the prime factorisation of $12^n$ would contain the prime 5 but $12^n = (2 \times 2 \times 3)^n$; so the only prime in the factorisation of $12^n$ are 2 and 3. So, there is no natural number $n$ for which $12^n$ ends with the digit zero.

18. Find the value(s) of $k$, if the quadratic equation $3x^2 - k\sqrt{3}x + 4 = 0$ has equal roots.
   Ans: $k = \pm 4$

19. Find the eleventh term from the last term of the AP: 27, 23, 19, ..., −65.
   Ans: $a_{11} = -25$

20. A girl walks 200m towards East and then she walks 150m towards North then find the distance of the girl from the starting point.
   Ans: 250 m

SECTION – B

Questions 21 to 26 carry 2 marks each.

21. In an equilateral triangle ABC, AD is drawn perpendicular to BC meeting BC in D. Prove that $AD^2 = 3BD^2$.
   Ans: $AB^2 = AD^2 + BD^2 \Rightarrow BC^2 = AD^2 + BD^2$ (since $AB = BC = AC$)
   Now, $BD = DC = BC/2 \Rightarrow BC = 2BD \Rightarrow (2BD)^2 = AD^2 + BD^2$
   $\Rightarrow 4BD^2 = AD^2 + BD^2 \Rightarrow 4BD^2 - BD^2 = AD^2 \Rightarrow 3BD^2 = AD^2$
   OR

22. If the sides AB, BC and CA of $\Delta ABC$ touch a circle at F, D and E respectively, then prove that $AF + BD + CE = \frac{1}{2} \times \text{Perimeter of } \Delta ABC$.

   Ans: Since lengths of the tangents drawn from an external point to a circle are equal.
   Therefore, $AF = AE$ (From A) ...........(i)
   $BD = BF$ (From B) ............(ii)
   $CE = CD$ (From C) .......(iii)
   Adding equations (i), (ii) and (iii), we get $AF + BD + CE = AE + BF + CD$
   Now, Perimeter of $\Delta ABC = AB + BC + CA$
   Perimeter of $\Delta ABC = (AF + FB) + (BD + CD) + (EC + AE)$
   $= (AF + AE) + (BD + BF) + (EC + CD)$
   $= 2(AF + BD + CE)$
   $\Rightarrow AF + BD + CE = \frac{1}{2} \times \text{Perimeter of } \Delta ABC$. 

23. From a point P on the ground the angle of elevation of the top of a 50 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff.

Ans: In Δ PAB, \(\tan 30^\circ = \frac{AB}{AP} \Rightarrow 1 = \frac{50}{AP} \Rightarrow AP = 50\sqrt{3} m = 50 \times 1.732 = 86.6 m\)

Next, let us suppose DB = x m. Then AD = (50 + x) m.

Now, in right Δ PAD, \(\tan 45^\circ = \frac{AD}{AP} \Rightarrow 1 = \frac{50 + x}{50\sqrt{3}}\)

\(\Rightarrow x = 50(\sqrt{3} - 1) = 50 \times 0.732 = 36.6 m\)

So, the length of the flagstaff is 36.6 m.

24. Two dairy owners A and B sell flavoured milk filled to capacity in mugs of negligible thickness, which are cylindrical in shape with a raised hemispherical bottom. The mugs are 14 cm high and have diameter of 7 cm as shown in given figure. Both A and B sell flavoured milk at the rate of Rs. 80 per litre. The dairy owner A uses the formula \(\pi r^2 h\) to find the volume of milk in the mug and charges Rs. 43.12 for it. The dairy owner B is of the view that the price of actual quantity of milk should be charged. What according to him should be the price of one mug of milk?

Ans:

Capacity of mug (actual quantity of milk) = \(\pi r^2 h - \frac{2}{3} \pi r^3\)

\(= \pi r^2 \left( h - \frac{2}{3} r^3 \right)\)

\(= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \left(14 - \frac{2}{3} \times \frac{7}{2} \right)\)

\(= \frac{2695}{6} \text{ cm}^3\)

Amount dairy owner B should charge for one mug of milk

\(= \frac{2695}{6} \times \frac{80}{1000} = ₹ 35.93\)

25. For what value of n, are the nth terms of two APs: 63, 65, 67, . . . and 3, 10, 17, . . . equal?

NCERT Exercise 5.2 Q15 p-106
26. A card is drawn at random from a well shuffled deck of 52 cards. Find the probability of getting (i) neither a red card nor a queen (ii) not a black face card.

Ans: (i) Probability of either a red or a queen card = \( \frac{26 + 2}{52} = \frac{28}{52} \)

\[ P(\text{neither red card nor a queen card}) = 1 - \frac{28}{52} = \frac{24}{52} = \frac{6}{13} \]

(ii) Probability of getting not a black face card = \( \frac{52 - 6}{52} = \frac{46}{52} = \frac{23}{26} \)

OR

Two different dice are tossed together. Find the probability :
(i) of getting a doublet
(ii) of getting a sum 10, of the numbers on the two dice.

Ans: Total number of outcomes = 36

(i) Probability of getting a doublet = \( \frac{6}{36} = \frac{1}{6} \)

(ii) Probability of getting a sum 10 = \( \frac{3}{36} = \frac{1}{12} \)

SECTION – C

Questions 27 to 34 carry 3 marks each.

27. Show that exactly one of the numbers n, n + 2 or n + 4 is divisible by 3.

Ans:
Let \( n = 3k, 3k + 1 \) or \( 3k + 2 \).

(i) When \( n = 3k \): 
\( n \) is divisible by 3. 
\( n + 2 = 3k + 2 \Rightarrow n + 2 \) is not divisible by 3. 
\( n + 4 = 3k + 4 = 3(k + 1) + 1 \Rightarrow n + 4 \) is not divisible by 3.

(ii) When \( n = 3k + 1 \):
\( n \) is not divisible by 3. 
\( n + 2 = (3k + 1) + 2 = 3k + 3 = 3(k + 1) \Rightarrow n + 2 \) is divisible by 3. 
\( n + 4 = (3k + 1) + 4 = 3k + 5 = 3(k + 1) + 2 \Rightarrow n + 4 \) is not divisible by 3.

(iii) When \( n = 3k + 2 \):
\( n \) is not divisible by 3. 
\( n + 2 = (3k + 2) + 2 = 3k + 4 = 3(k + 1) + 1 \Rightarrow n + 2 \) is not divisible by 3. 
\( n + 4 = (3k + 2) + 4 = 3k + 6 = 3(k + 2) \Rightarrow n + 4 \) is divisible by 3. 
Hence exactly one of the numbers \( n, n + 2 \) or \( n + 4 \) is divisible by 3.

OR

Use Euclid’s division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Ans: NCERT Exercise 1.1 Q6

28. Solve the following pair of linear equations:

\( (a - b)x + (a + b)y = a^2 - 2ab - b^2 \)
\( (a + b)(x + y) = a^2 + b^2 \)

Ans: NCERT Exercise 3.7 Q7(iv)

OR

The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Ans: NCERT Exercise 3.7 Q4
29. Find all zeroes of the polynomial \((2x^4 - 9x^3 + 5x^2 + 3x - 1)\) if two of its zeroes are \((2 + \sqrt{3})\) and \((2 - \sqrt{3})\).

Ans:

\[ p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1 \]
\[ 2 + \sqrt{3} \text{ and } 2 - \sqrt{3} \text{ are zeroes of } p(x) \]
\[ \therefore \quad p(x) = (x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) \times g(x) = (x^2 - 4x + 1) \, g(x) \]
\[ (2x^4 - 9x^3 + 5x^2 + 3x - 1) \div (x^2 - 4x + 1) = 2x^2 - x - 1 \]
\[ \therefore \quad g(x) = 2x^2 - x - 1 = (2x + 1)(x - 1) \]
Therefore other zeroes are \(x = -\frac{1}{2}\) and \(x = 1\).

30. In a classroom, 4 friends are seated at the points A, B, C and D as shown in below figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, “Don’t you think ABCD is a square?” Chameli disagrees. Chameli performed certain calculations and claimed that ABCD is a square. State how did she arrive at this conclusion.

Ans: A(3, 4), B(6, 7), C(9, 4) and D(6, 1)

\[ AB = BC = CD = AD = 3\sqrt{2} \text{ units} \]
\[ BD = AC = 6 \text{ units} \]
Since all sides are equal and diagonals are equal, therefore ABCD is a square.

31. Prove that \(\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}\)

Ans: \(LHS = \frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1}\)

Dividing Numerator and Denominator by \(\cos \theta\), we get

\[ \frac{\tan \theta - 1 + \sec \theta}{\tan \theta + 1 - \sec \theta} = \frac{\tan \theta + \sec \theta - 1}{\tan \theta + 1 - \sec \theta} \]
\[ = \frac{\tan \theta + \sec \theta - (\sec^2 \theta - \tan^2 \theta)}{\tan \theta + 1 - \sec \theta} \]
\[ = \frac{(\tan \theta + \sec \theta)(1 - \sec \theta + \tan \theta)}{\tan \theta + 1 - \sec \theta} \]
\[ = \frac{\tan \theta + \sec \theta}{\tan \theta + 1 - \sec \theta} \]
\[
\frac{\sec \theta + \tan \theta}{1} \times \frac{\sec \theta - \tan \theta}{\sec \theta - \tan \theta} = \frac{\sec^2 \theta - \tan^2 \theta}{\sec \theta - \tan \theta} = \frac{1}{\sec \theta - \tan \theta} = RHS
\]

If \( \sec \theta + \tan \theta = p \), then find the value of cosec \( \theta \).

\textbf{Ans: Given:} \( \sec \theta + \tan \theta = p \) \quad \text{(i)}

We know that \( \sec^2 \theta - \tan^2 \theta = 1 \) \( \Rightarrow \ (\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1 \)

\( \Rightarrow \ (p)(\sec \theta - \tan \theta) = 1 \) \( \Rightarrow \ \sec \theta - \tan \theta = \frac{1}{p} \) \quad \text{(ii)}

On solving (i) & (ii), we get

\[
\sec \theta + \tan \theta + \sec \theta - \tan \theta = p + \frac{1}{p} \Rightarrow 2\sec \theta = \frac{p^2 + 1}{p} \Rightarrow \sec \theta = \frac{p^2 + 1}{2p}
\]

\[
\cos \theta = \frac{1}{\sec \theta} = \frac{2p}{p^2 + 1}
\]

Now, \( \sin^2 \theta = 1 - \cos^2 \theta = 1 - \left( \frac{2p}{p^2 + 1} \right)^2 = \left( \frac{p^2 - 1}{p^2 + 1} \right)^2 \Rightarrow \sin \theta = \frac{p^2 - 1}{p^2 + 1}
\]

Now, we know that \( \cosec \theta = \frac{1}{\sin \theta} = \frac{p^2 + 1}{p^2 - 1} \)

32. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of \( x \) such that the sum of the numbers of the houses preceding the house numbered \( x \) is equal to the sum of the numbers of the houses following it. Find this value of \( x \).

\textbf{Ans: NCERT Exercise 5.4 Q4}

33. In the given figure, \( \triangle ABC \) is right triangle in which \( \angle A = 90^\circ \). Semicircles are drawn on \( AB \), \( AC \) and \( BC \) as diameters. Find the area of the shaded region.

\textbf{Ans:}

\[
\text{In right } \triangle ABC, \\
BC^2 = 3^2 + 4^2 = 25 \\
[\text{Pythagoras theorem}] \\
: BC = 5 \text{ cm}
\]

Area of the shaded region
\[
= \ar (\text{semicircle } ABP) + \ar (\text{semicircle } ACQ) + \ar (\triangle ABC) - \ar (\text{semicircle } ABC)
\]
\[
= \frac{1}{2} \pi \times \left( \frac{1}{2} AB \right)^2 + \frac{1}{2} \pi \times \left( \frac{1}{2} AC \right)^2 + \frac{1}{2} \times AB \times AC - \frac{1}{2} \pi \times \left( \frac{1}{2} BC \right)^2
\]
\[
= \frac{1}{2} \left( \pi \times \frac{1}{4} AB^2 + \pi \times \frac{1}{4} AC^2 - \pi \times \frac{1}{4} BC^2 \right) + \frac{1}{2} \times 3 \times 4
\]
\[
= \pi \times \frac{1}{8} \left( AB^2 + AC^2 - BC^2 \right) + 6 = \pi \times \frac{1}{8} (9 + 16 - 25) + 6 = \pi \times \frac{1}{8} \times 0 + 6 = 6 \text{ cm}^2
\]
34. Anil selected a project to prepare more than and less than ogive of the marks obtained by the 100 students in Mathematics Board Examination of the class. After collecting the data, he analyzed the data and prepared a report on the marks of the class. Using the report, he drew the following graph as given below:

Based on the above graph, answer the following questions:
(i) Identify less than type ogive and more than type ogive from the given graph.
(ii) Find the median marks of the class.
(iii) Obtain the Mode marks of the data if mean marks of the class is 57.5.

Ans: (i) Dotted line represents less than ogive and dark line represents more than ogive.
(ii) Median marks of the class is 57 approximately
(iii) Given that Median = 57 and Mean = 57.5
Mode = 3 Median - 2 Mean
Mode = 3 x 57 – 2 x 57.5 = 171 – 115 = 56

SECTION – D

Questions 35 to 40 carry 4 marks each.

35. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.
NCERT Exercise 9.1 Q15 p-205

36. A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Take \(\pi =3.14\))
NCERT Exercise 13.5 Q2 p-258
OR
A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at
the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a
wire of diameter \( \frac{1}{16} \) cm, find the length of the wire.

NCERT Exercise 13.4 Q5 p-257

37. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of
their corresponding sides.

Given, To Prove, Diagram, Construction and Proof

38. Two water taps together can fill a tank in \( \frac{3}{8} \) hours. The tap of larger diameter takes 10 hours
less than the smaller one to fill the tank separately. Find the time in which each tap can
separately fill the tank.

NCERT Exercise 4.3 Q9 p-88

OR
A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the
same distance if its speed were 5 km/h more. Find the original speed of the train.

Ans:
Let original speed of the train be \( x \) km/h.

Time taken at original speed = \( \frac{360}{x} \) hours

Time taken at increased speed = \( \frac{360}{x + 5} \) hours

Now, \( \frac{360}{x} - \frac{360}{x + 5} = \frac{48}{60} \Rightarrow 360 \left[ \frac{1}{x} - \frac{1}{x + 5} \right] = \frac{4}{5} \)

\( \Rightarrow x^2 + 5x - 2250 = 0 \)

\( \Rightarrow x = 45 \) or \(-50\) (as speed cannot be negative)

\( \Rightarrow x = 45 \) km/h

39. Draw a triangle \( ABC \) with side \( BC = 7 \) cm, \( \angle B = 45^0 \), \( A = \angle 105^0 \). Then, construct a triangle
whose sides are \( \frac{7}{4} \) times the corresponding sides of \( \triangle ABC \).

OR
Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a
distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

40. The following data indicates the marks of 54 students in Mathematics.

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<thead>
<tr>
<th>Marks</th>
<th>0 – 10</th>
<th>10 – 20</th>
<th>20 – 30</th>
<th>30 – 40</th>
<th>40 – 50</th>
<th>50 – 60</th>
<th>60 – 70</th>
<th>70 – 80</th>
<th>80 – 90</th>
<th>90 – 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Students</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>9</td>
<td>7</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

Draw more than type ogive for the data above and hence find the median.

Ans:
More than type cf distribution table:
<table>
<thead>
<tr>
<th>Marks</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

Median = 66