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SAMPLE PAPER TEST 03 (2019-20) (SAMPLE ANSWERS)

SUBJECT: MATHEMATICS
CLASS : X

MAX. MARKS : 80
DURATION : 3 HRS

General Instruction:
(i) All the questions are compulsory.
(ii) The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION – A
Questions 1 to 20 carry 1 mark each.

1. Two alarm clocks ring their alarms at regular intervals of 50 seconds and 48 seconds. If they first beep together at 12 noon, at what time will they beep again for the first time?
   (a) 12.20 pm   (b) 12.12 pm   (c) 12.11 pm   (d) none of these
   Ans: (a)

2. The decimal expansion of the rational number \( \frac{14587}{1250} \) will terminate after
   (a) one decimal place   (b) two decimal places   (c) three decimal places   (d) four decimal places
   Ans: (d)

3. If \( x = a, \ y = b \) is the solution of the pair of equations \( x – y = 2 \) and \( x + y = 4 \), then the respective values of \( a \) and \( b \) are
   (a) 3, 5   (b) 5, 3   (c) 3, 1   (d) -1, -3
   Ans: (c)

4. If \( P(1, 2), Q(4, 6), R(5, 7) \) and \( S(a, b) \) are the vertices of a parallelogram \( PQRS \), then
   (a) \( a = 2, \ b = 4 \)   (b) \( a = 3, \ b = 4 \)   (c) \( a = 2, \ b = 3 \)   (d) \( a = 3, \ b = 5 \)
   Ans: (c)

5. The modal class for the following distribution is

<table>
<thead>
<tr>
<th>Marks</th>
<th>Below 10</th>
<th>Below 20</th>
<th>Below 30</th>
<th>Below 40</th>
<th>Below 50</th>
<th>Below 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of students</td>
<td>3</td>
<td>12</td>
<td>27</td>
<td>57</td>
<td>75</td>
<td>80</td>
</tr>
</tbody>
</table>
   (a) 20 – 30   (b) 30 – 40   (c) 40 – 50   (d) 50 – 60
   Ans: (b)

6. If \( \cot \theta = 7/8 \), evaluate \( \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} \)
   (a) 7/8   (b) 64/49   (c) 49/64   (d) none of these
   Ans: (c)

7. If \( \cos \theta = 1/3 \), then find the value of \( 2 \cot^2 \theta + 2 \)
   (a) 18   (b) 12   (c) 9   (d) none of these
   Ans: (d)
8. In the below left figure, AD = 4 cm, BD = 3 cm and CB = 12 cm, find \( \cot \theta \).
   
   (a) \( \frac{12}{5} \)  
   (b) \( \frac{17}{12} \)  
   (c) \( \frac{7}{12} \)  
   (d) none of these

   Ans: (a)

9. If A and B are the points \((-6, 7)\) and \((-1, -5)\) respectively then find the distance 2AB.
   
   (a) 18  
   (b) 26  
   (c) 13  
   (d) none of these

   Ans: (b)

10. A straight line is drawn joining the points \((3, 4)\) and \((5, 6)\). If the line is extended, the ordinate of the point on the line, whose abscissa is \(-1\) is ________.
    
    (a) 1  
    (b) -1  
    (c) 0  
    (d) none of these

   Ans: (c)

11. A cylinder and a cone are of same base radius and of same height. The ratio of the volume of cylinder to that of the cone is 3 : 1.

12. 7th term of an AP is 40. The sum of its first 13th terms is 520.

13. If the roots of the equation \(12x^2 + mx + 5 = 0\) are in the ratio 3 : 2, then \(m\) equals \(5\sqrt{10}\).
    
    OR
    
    If zeroes of \(p(x) = 2x^2 - 7x + k\) are reciprocal of each other, then value of \(k\) is 2.

14. Two coins are tossed simultaneously, then the probability of getting exactly one head is \(\frac{1}{2}\).

15. In the below left figure, if \(\triangle ABC \sim \triangle PQR\). The value of \(x\) is 3 cm.

16. The two tangents from an external point \(P\) to a circle with centre \(O\) are \(PA\) and \(PB\). If \(\angle APB = 70^\circ\), what is the value of \(\angle AOB\)?
    
    Ans: 110°
    
    OR
    
    In the above right sided figure, \(AOB\) is a diameter of a circle with centre \(O\) and \(AC\) is a tangent to the circle at \(A\). If \(\angle BOC = 130^\circ\), then find \(\angle ACO\).
    
    Ans: 40°

17. If the areas of two similar triangles are in ratio 25 : 64, write the ratio of their corresponding sides.

    Ans: 5 : 8

18. The HCF and LCM of two numbers are 9 and 360 respectively. If one number is 45, write the other number.

    Ans: 72

19. If the sum of first \(m\) terms of an AP is \(2m^2 + 3m\), then what is its second term?

    Ans: \(a_2 = 9\)
20. For what value of k, are the roots of the quadratic equation $3x^2 + 2kx + 27 = 0$ real and equal.
Ans: $k = \pm 9.$

SECTION – B
Questions 21 to 26 carry 2 marks each.

21. A bag contains 5 white balls, 7 red balls, 4 black balls and 2 blue balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is (i) white or blue, (ii) neither white nor black.
Ans:
Total number of balls = 18
$P($White or blue$) = \frac{5 + 2}{18} = \frac{7}{18}$
$P($Neither white nor black$) = \frac{9 + 1}{18} = \frac{2}{3}$

OR
A number is selected at random from the first 50 natural numbers. Find the probability that it is a multiple of 3 and 4.
Ans:
Multiples of 3 and 4 are 12, 24, 36, 48
$P($Multiples of 3 and 4$) = \frac{4}{50} = \frac{2}{25}$

22. The first and the last terms of an A.P. are 17 and 350 respectively. If its common difference is 9, find the number of terms in the A.P. and find their sum.
Ans:
Here $a = 17, a_n = 350, d = 9$
Let the number of terms be n.
\[ 17 + (n - 1)9 = 350 \]
\[ \Rightarrow n = 38 \]
\[ \therefore S_{38} = \frac{38}{2} (17 + 350) = 6973 \]

23. If AD and PM are medians of $\triangle ABC$ and $\triangle PQR$ respectively where $\triangle ABC \sim \triangle PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.
Ans:
\[ \triangle ABC \sim \triangle PQR \]
\[ \therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \]
and $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R.$
Now $BC = 2BD$ and $QR = 2QM.$
\[ \therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{2BD}{2QM} \]
i.e. $\frac{BD}{QM} = \frac{AB}{PQ} \Rightarrow \triangle ABD \sim \triangle PQM \Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$
Prove that the sum of the squares of the sides of a rhombus is equal to sum of the squares of its diagonals.

Ans:

OR

24. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Ans:

25. The origins of the fez, called the "tarboosh" by the Moroccans, are in dispute. Some claim that its origins are ancient Greece; others claim it comes from the Balkans. The name "fez" comes from Moroccan city of the same name. Fez, Morocco produced the dye, made from crimson berries, to colour the hat. A fez, the cap used by the Turks, is shaped like the frustum of a cone (see the below left figure). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.

Ans: NCERT Exercise 13.4 Q3 p-257

26. An observer 1.5 m tall is 28.5 m away from a chimney (see above right sided figure). The angle of elevation of the top of the chimney from her eyes is \( \alpha \) such that \( \cos 2\alpha = \sin(45^0 - \alpha) \). What is the height of the chimney?

Ans: \( \cos 2\alpha = \sin(45^0 - \alpha) \Rightarrow \sin(90^0 - 2\alpha) = \sin(45^0 - \alpha) \Rightarrow \alpha = 45^0 \)
\( \tan 45^0 = \frac{AE}{DE} \Rightarrow 1 = \frac{AE}{28.5} \)
Therefore, \( AE = 28.5 \)
So the height of the chimney \( (AB) = (28.5 + 1.5) \) m = 30 m.
SECTION – C
Questions 13 to 22 carry 3 marks each.

27. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find (i) the length of the arc and (ii) area of sector formed by the arc.

\[ \text{Ans:} \]
\[ \text{Length of arc} = 2\pi r \cdot \frac{\theta}{360°} = 2 \times \frac{22}{7} \times 21 \times \frac{60°}{360°} = 22 \text{ cm} \]
\[ \text{Area of sector} = \pi r^2 \cdot \frac{\theta}{360°} = \frac{22}{7} \times 21 \times 21 \times \frac{60°}{360°} = 231 \text{ cm}^2 \]

28. In a classroom, three friends are seated at the points A, B and C as shown in below left figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, “Don’t you think ABC forms a triangle?” Chameli agrees. Both are trying to calculate the area of the triangle ABC formed. Find the coordinates and how much area they calculated.

\[ \text{Ans: A}(3, 4), \text{ B}(6, 7) \text{ and C}(9, 4). \text{ Area of DABC} = \left(\frac{1}{2}\right) \times 6 \times 3 = 9 \text{ sq. units} \]

29. A teacher asks the student to collect the marks of the students in Maths Periodic Test out of 50 marks. A student collected the marks and prepares the frequency distribution table as per the instructions of the teacher. From the frequency table, again student prepare less than cf distribution and then he draws less than ogive as shown in above right sided figure. Construct the frequency distribution table using the less than ogive and then find the median marks.

\[ \text{Ans:} \]
\[ \begin{array}{c|c|c|c|c|c|c}
\hline
\text{Class} & 0 – 10 & 10 – 20 & 20 – 30 & 30 – 40 & 40 – 50 \\
\hline
\text{Frequency} & 8 & 12 & 10 & 11 & 9 \\
\hline
\end{array} \]

\[ \text{Here } \frac{n}{2} = 25, \ l = 20, \ cf = 20, \ f = 10, \ h = 10 \]

\[ \text{Median} = 1 + \frac{n - c}{f} \times h = 20 + \frac{25 - 20}{10} \times 10 = 25 \]
30. Show that any positive even integer is of the form 4q or 4q + 2, where q is some integer.
   Ans: Let x be a positive odd integer and b = 4 then by Euclid’s division lemma
   \[ x = 4q + r, \ 0 \leq r < 4. \] So x can be 4q, 4q + 1, 4q + 2 or 4q + 3.

   Now \( 4q = 2 \times 2q \Rightarrow 4q \) is an even integer.

   We know that the sum of the two even positive integers is always an even integer and the sum of even and odd integer is always an odd integer. Therefore 4q + 1, 4q + 3 are odd integers and 4q + 2 is an even integer.

   Hence, any even integer is of the form 4q and 4q + 2.

OR

If the HCF of 65 and 117 can be written as (65 m – 117), find the value of m.
   Ans:
   \[ \begin{align*}
   117 &= 65 \times 1 + 52 \\
   65 &= 52 \times 1 + 13 \\
   52 &= 13 \times 4 + 0 \\
   13 &= 65 - 52
   \end{align*} \]
   \[ \Rightarrow 65 - (117 - 65) = 2 \times 65 - 117 \Rightarrow m = 2 \]

31. A number consists of two digits whose sum is 9. If 27 is added to the number, the digits change their places. Find the number.
   Ans:
   Let the digit at units place be x
   \[ \therefore \text{The digit at tens place} = 9 - x \]
   As per question, \(10(9 - x) + x + 27 = 10x + (9 - x)\)
   \[ x = 6 \]
   \[ \therefore \text{Required number is 36} \]

OR

Solve the following pair of linear equations: 6x + 3y = 6xy and 2x + 4y = 5xy
   NCERT Exercise 3.6 Q1(vi) p-67

32. If two zeroes of the polynomial \(x^4 - 6x^3 + 26x^2 - 138x - 35\) are \(2 \pm \sqrt{3}\), find other zeroes.
   Ans:
   The polynomial having two zeroes \(2 + \sqrt{3}\) and \(2 - \sqrt{3}\) is
   \[ (x - 2 + \sqrt{3})(x - 2 - \sqrt{3}) = (x - 2)^2 - 3 \]
   \[ = x^2 - 4x + 1 \]

   Now \((x^4 - 6x^3 + 26x^2 + 138x - 35) \div (x^2 - 4x + 1) = x^2 - 2x - 35\)
   \[ x^2 - 2x - 35 = (x - 7)(x + 5) \]
   \[ \therefore \text{Other zeroes are 7 and } -5 \]

33. Prove that: \(\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \csc \theta\)
   Ans:
   \[ \text{L.H.S.} = \frac{\sin^2 \theta + (1 + \cos \theta)^2}{\sin \theta (1 + \cos \theta)} = \frac{2 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)} = \frac{2(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)} = 2 \csc \theta \]
If \( \cos \theta + \sin \theta = \sqrt{2} \cos \theta \), show that \( \cos \theta - \sin \theta = \sqrt{2} \sin \theta \).

**Ans:**

\[
\cos \theta + \sin \theta = \sqrt{2} \cos \theta \quad \Rightarrow \quad \sin \theta = (\sqrt{2} - 1) \cos \theta \\
\Rightarrow \quad \sin \theta = \frac{(\sqrt{2} - 1)(\sqrt{2} + 1)}{\sqrt{2} + 1} \cos \theta \quad \Rightarrow \quad \sin \theta = \frac{\cos \theta}{\sqrt{2} + 1} \]

\[
\Rightarrow \quad \sqrt{2} \sin \theta = \cos \theta - \sin \theta
\]

**34.** Find three numbers in A.P. whose sum is 15 and whose product is 105.

**Ans:**

Let the three numbers in A.P. be \( x - d \), \( x \), and \( x + d \).

\[
\therefore \quad x - d + x + x + d = 15 \Rightarrow x = 5 \\
x(x - d) (x + d) = 105 \\
\Rightarrow \quad 5(5^2 - d^2) = 105 \Rightarrow 25 - d^2 = 21 \\
\Rightarrow \quad d^2 = 4 \Rightarrow d = \pm 2 \\
\therefore \quad \text{Required number are 3, 5, 7}
\]

**SECTION D**

Questions 23 to 30 carry 4 marks each.

**35.** Two pipes together can fill a tank in 12 hours. If first pipe can fill the tank 10 hours faster than the second, then how many hours will the second pipe take to fill the tank?

**Ans:**

Let the time taken by first pipe to fill a tank = \( x \) hour

and the time taken by second pipe to fill the tank = \( x + 10 \) hours.

As per question \[
\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12} \quad \Rightarrow \quad \frac{(x+10) + x}{x(x+10)} = \frac{1}{12}
\]

\[
\Rightarrow \quad (2x + 10)12 = x(x + 10) \quad \Rightarrow \quad x^2 - 14x - 120 = 0 \\
\Rightarrow \quad (x - 20)(x + 6) = 0 \\
\Rightarrow \quad x = 20 \text{ hours} \Rightarrow \text{Second pipe can fill in 30 hours.}
\]

**OR**

Solve for \( x \): \[
\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}, \quad x \neq -1, -2, -4
\]

**Ans:**

\[
\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4} \quad \Rightarrow \quad \frac{x+2 + 2(x+1)}{x^2 + 3x + 2} = \frac{4}{x+4}
\]

\[
\Rightarrow \quad (3x + 4) (x + 4) = 4(x^2 + 3x + 2) \quad \Rightarrow \quad 3x^2 + 16x + 16 = 4x^2 + 12x + 8 \\
\Rightarrow \quad x^2 - 4x - 8 = 0 \\
\Rightarrow \quad x = \frac{4 \pm \sqrt{16 + 32}}{2} = \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}
\]

Hence, \( x = 2 + 2\sqrt{3} \) or \( 2 - 2\sqrt{3} \)
36. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

37. Construct a right angled ΔABC, whose sides (other than hypotenuse) are 4 cm and 3 cm. Then construct a triangle whose sides are 5/3 of the corresponding sides of ΔABC.  
   (Ans: NCERT Exercise 11.1 Q2)  
   OR  
   Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. (Ans: NCERT Exercise 11.2 Q2)

38. On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. From a point 9 m away from the foot of the tower, the angles of elevation of the top and foot of the flag pole are 60º and 30º respectively. Find the heights of the tower and the flag pole mounted on it.  
   Ans:

   ![Diagram](image)

   Let AB, the vertical tower = x m  
   and BC, the flag pole = y m

   \[
   \frac{x}{9} = \tan 30^\circ = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad x = \frac{9}{\sqrt{3}} = 3\sqrt{3} m
   \]

   Also \[
   \frac{x + y}{9} = \tan 60^\circ = \sqrt{3}
   \]

   \[
   3\sqrt{3} + y = 9\sqrt{3} \quad \Rightarrow \quad y = 9\sqrt{3} - 3\sqrt{3} = 6\sqrt{3} m
   \]

   Height of tower = 3\sqrt{3} m  
   Height of flag pole = 6\sqrt{3} m

39. Four circular cylindrical vessels, each having a diameter 21 cm and height 38 cm are full of ice-cream. This ice-cream is to be filled in cones each of height 12 cm and diameter 7 cm, having a hemispherical shape on the top. Find the total number of cones that can be filled with the ice-cream.  
   Ans:

   Volume of ice-cream in 4 cylinders  
   \[
   = 4 \times \pi r^2 \times h
   \]

   \[
   = 4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 38 = 66 \times 21 \times 38 \text{ cm}^3
   \]

   Volume of a cone with hemispherical top  
   \[
   = \frac{1}{3} \pi r^2 (h + 2r) = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} (12 + 7) = \frac{11 \times 7 \times 19}{6} \text{ cm}^3
   \]

   Total number of cones  
   \[
   = \frac{66 \times 21 \times 38 \times 6}{11 \times 7 \times 19} = 216
   \]

   OR  

   A milkman uses a container, in the shape of frustum of a cone, to store milk. The container open from the top, is of height 40 cm with radii of its lower and upper circular ends as 14 cm and 35 cm respectively. Find the volume of milk (in litres) which can completely fill the container. If he sells the milk at Rs. 35 per litre, for how much amount he can sell the whole milk ?  
   Ans:
The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50. Find the missing frequencies \( f_1 \) and \( f_2 \).

<table>
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<tr>
<th>Class</th>
<th>0 - 20</th>
<th>20 - 40</th>
<th>40 - 60</th>
<th>60 - 80</th>
<th>80 - 100</th>
<th>100 - 120</th>
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<tr>
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<td>10</td>
<td>( f_2 )</td>
<td>7</td>
<td>8</td>
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</table>

**Ans:**

<table>
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<tr>
<th>Classes</th>
<th>( f_i )</th>
<th>( x_i )</th>
<th>( d_i = x_i - 50 )</th>
<th>( u_i = \frac{d_i}{20} )</th>
<th>( f_i u_i )</th>
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<tr>
<td>0-20</td>
<td>5</td>
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<td>-40</td>
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<td>20-40</td>
<td>( f_1 )</td>
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<td>-1</td>
<td>( -f_1 )</td>
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<td>0</td>
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<tr>
<td>60-80</td>
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<td>2</td>
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<td>8</td>
<td>110</td>
<td>60</td>
<td>3</td>
<td>24</td>
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</table>

\[
30 + f_1 + f_2 \quad 28 - f_1 + f_2
\]

Here \( 30 + f_1 + f_2 = 50 \) \( \Rightarrow \) \( f_1 + f_2 = 20 \)

Also \( 62.8 = 50 + \frac{28 - f_1 + f_2}{50} \times 20 \) \( \Rightarrow \) \( f_2 - f_1 = 4 \)

Solving \( f_1 = 8, f_2 = 12 \)