SUBJECT: MATHEMATICS
CLASS : X
MAX. MARKS : 80
DURATION : 3 HRS

General Instruction:
(i) All the questions are compulsory.
(ii) The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION – A
Questions 1 to 20 carry 1 mark each.

1. \( n^2 - 1 \) is divisible by 8, if \( n \) is:
   (a) an integer.  
   (b) a natural number.  
   (c) an odd integer.  
   (d) an even integer.
   Ans: (c) an odd integer.

2. The decimal expansion of the rational number \( \frac{33}{27.5} \) will terminate after
   (a) one decimal place  
   (b) two decimal places  
   (c) three decimal places  
   (d) more than 3 decimal places
   Ans: (a) one decimal place

3. The value of \( \tan^0 \tan^0 \tan^3 \ldots \tan^89^0 \) is
   (a) 0  
   (b) 1  
   (c) 3  
   (d) None of these
   Ans: 1

4. \( (\cos^4 A - \sin^4 A) \) is equal to
   (a) 1 - 2 \( \cos^2 A \)  
   (b) 2 \( \sin^2 A - 1 \)  
   (c) \( \sin^2 A - \cos^2 A \)  
   (d) 2 \( \cos^2 A - 1 \)
   Ans: (d) 2 \( \cos^2 A - 1 \)

5. If \( \tan 2A = \cot (A - 18^\circ) \), where 2A is an acute angle, then the value of A is
   (a) 12^\circ  
   (b) 18^\circ  
   (c) 36^\circ  
   (d) 48^\circ
   Ans: (c) 36^\circ

6. Graphically, the pair of equations \( 6x - 3y + 10 = 0 \), \( 2x - y + 9 = 0 \) represents two lines which are
   (a) intersecting at exactly one point  
   (b) intersecting at exactly two points  
   (c) coincident  
   (d) parallel
   Ans: (d) parallel

7. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is:
   (a) 5  
   (b) 12  
   (c) 11  
   (d) 7 + \sqrt{5}
   Ans: (b) 12

8. The point which divides the line segment joining the points (7, -6) and (3, 4) in ratio 1 : 2 internally lies in the:
   (a) I quadrant  
   (b) II quadrant  
   (c) III quadrant  
   (d) IV quadrant
   Ans: (d) IV quadrant
9. The ratio in which the point (2, y) divides the join of (-4, 3) and (6, 3) is
   (a) 2 : 3  (b) 3 : 5  (c) 3 : 2  (d) 5 : 2
   Ans: (c) 3 : 2

10. Consider the following frequency distribution:

<table>
<thead>
<tr>
<th>Class</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5</td>
<td>13</td>
</tr>
<tr>
<td>6-11</td>
<td>10</td>
</tr>
<tr>
<td>12-17</td>
<td>15</td>
</tr>
<tr>
<td>18-23</td>
<td>8</td>
</tr>
<tr>
<td>24-29</td>
<td>11</td>
</tr>
</tbody>
</table>

   The lower limit of the median class is
   (a) 5.5  (b) 11.5  (c) 17.5  (d) none of these
   Ans: (b) 11.5

11. A cylinder, a cone and a hemisphere have same base and same height then the ratio of their volumes is ______
   Ans: 3 : 1 : 2

12. If \( x = \frac{-1}{2} \), is a solution of the quadratic equation \( 3x^2 + 2kx - 3 = 0 \), then the value of k is ____
   Ans: \( k = \frac{-9}{4} \)

   OR

   If the zeroes of the quadratic polynomial \( ax^2 + bx + c, a \neq 0 \) are equal, then the sign of c and a are ______
   Ans: same

13. The areas of two similar triangles ABC and PQR are in the ratio 9 : 16. If BC = 4.5 cm, then the length of QR is _____
   Ans: 6 cm

14. A card is drawn from a deck of 52 cards. The event E is that card is not a face card. The number of outcomes favourable to E is ______
   Ans: 52 - 12 = 40

15. In an AP, if a = 3.5, d = 0 and n = 101, then \( a_n \) will be ______
   Ans: 3.5

16. Find the positive root of \( \sqrt{3x^2 + 6} = 9 \).
   \[
   \sqrt{3x^2 + 6} = 9 \\
   \Rightarrow 3x^2 + 6 = 81 \\
   \Rightarrow 3x^2 = 81 - 6 = 75 \\
   \Rightarrow x^2 = \frac{75}{3} = 25 \\
   \Rightarrow x = \pm 5 \\
   \therefore \text{Positive root} = 5
   \]

17. The L.C.M. and H.C.F. of x and 18 are 36 and 2 respectively. What is the number x ?
   Ans: 4
18. If radii of two concentric circles are 4 cm and 5 cm, then find the length of each chord of one circle which is tangent to the other circle.
   Ans: 6 cm

   OR

   If PQ and PR are two tangents to a circle with centre O. If \( \angle QPR = 46^0 \), find \( \angle QOR \).
   Ans: 134^0.

19. A segment AB is divided at point P such that \( \frac{PB}{AB} = \frac{3}{7} \) then find the ratio AP : PB.

   Here, \( AB = 7 \), \( PB = 3 \)
   \[ \therefore \quad AP = AB - PB = 7 - 3 = 4 \]
   \[ \therefore \quad AP : PB = 4 : 3 \]

20. Is series \( \sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots \) an AP? Give reason.

   Common difference,
   \[ d = \sqrt{6} - \sqrt{3} = \sqrt{3} (\sqrt{2} - 1) \]
   \[ a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6} \]
   \[ = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - 3 \]

   The given series is not in A.P. as common difference does not exist.

### SECTION – B

Questions 21 to 26 carry 2 marks each.

21. In \( \triangle ABC \), \( AB = AC \). If the interior circle of \( \triangle ABC \) touches the sides \( AB \), \( BC \) and \( CA \) at \( D \), \( E \) and \( F \) respectively. Prove that \( E \) bisects \( BC \).

   Ans:

   ![Diagram of \( \triangle ABC \) with incircle]

   \( AF = AD \)
   \( BE = BD \),

   (tangents from external points)
   \( CE = CF \)
   \( AB = AC \)

   \( AD + BD = AF + FC \)

   \[ \Rightarrow \quad BD = FC \quad (\because \ AD = AF) \]

   \( BE = EC (\because \ BD = BE, CE = CF) \)

   \[ \therefore \ E \text{ bisects } BC. \]

22. While playing Ravi dropped a sphere of diameter 6 cm in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel?

   Ans:

   Let the water level raised in cylindrical vessel be \( h \) cm

   Volume of Sphere = Volume of water displaced in cylinder

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Prepared by: M. S. KumarSwamy, TGT(Maths)
23. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a card of spade or an ace (ii) a black king

**Ans:**

(i) Cards of spade or an ace = 13 + 3 = 16
Total no. of cards = 52

\[ P(\text{spade or an ace}) = \frac{16}{52} = \frac{4}{13} \]

(ii) Black kings = 2

\[ P(\text{a black king}) = \frac{2}{52} = \frac{1}{26} \]

OR

A piggy bank contains hundred 50p coins, fifty Rs 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 p coin ? (ii) will not be a Rs 5 coin?

(i) Total number of coins = 100 + 50 + 20 + 10

\[ = 180 \]

\[ \therefore \text{Total number of possible outcomes of a coin} \]
\[ \text{will fall out} = 180 \]

Number of 50 p coins = 100

\[ \therefore \text{Number of favourable outcomes relating to} \]
\[ \text{fall out of a 50 p coin} = 100 \]

Now, \[ P(\text{of getting a 50 p coin}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} \]

\[ = \frac{100}{180} = \frac{5}{9} \]

(ii) \[ P(\text{not a 5 p coin}) = 1 - P(\text{5 p coin}) \]

\[ = 1 - \frac{10}{180} = \frac{17}{18} \]

24. If \( \triangle ABC \sim \triangle DEF \), \( AB = 4 \text{ cm} \), \( DE = 6 \text{ cm} \), \( EF = 9 \text{ cm} \) and \( FD = 12 \text{ cm} \), then find the perimeter of \( \triangle ABC \).

**Ans:** Given \( AB = 4\text{ cm} \), \( DE = 6\text{ cm} \) and \( EF = 9\text{ cm} \) and \( FD = 12\text{ cm} \)

Also, \( \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \Rightarrow \frac{4}{6} = \frac{BC}{9} = \frac{AC}{12} \)

On taking first two terms, we get

\[ \frac{4}{6} = \frac{BC}{9} \Rightarrow BC = \frac{4 \times 9}{6} = 6 \text{ cm} \]

\[ AC = \frac{6 \times 12}{9} = 8 \text{ cm} \]

Now, \[ \text{perimeter of } \triangle ABC = AB + BC + AC \]

\[ = 4 + 6 + 8 = 18 \text{ cm} \]
In given figure, if \( \angle ACB = \angle CDA \), \( AC = 8 \) cm and \( AD = 3 \) cm, then find \( BD \).

**Ans:** Given, \( AC = 8 \) cm, \( AD = 3\) cm and \( \angle ACB = \angle CDA \)

From figure, \( \angle CDA = 90^\circ \) and \( \angle ACB = \angle CDA = 90^\circ \)

In right angled \( \triangle ADC \),
\[ AC^2 = AD^2 + CD^2 \]
\[ 64 - 9 = CD^2 \]
\[ CD = \sqrt{55} \text{ cm} \]

In \( \triangle CDB \) and \( \triangle ADC \),
\[ \angle BDC = \angle ADC \]
\[ \angle DBC = \angle DCA \]

Then,
\[ \frac{\triangle CDB - \triangle ADC}{\triangle CD} = \frac{BD}{CD} \]
\[ \frac{BD}{AD} = \frac{\sqrt{55}^2}{3} = \frac{55}{3} \text{ cm} \]

25. Check whether 100 is a term of the AP 25, 28, 31, ........ or not?

**Ans:**

\[ a = 25, d = 3 \]

Let the number of terms be “\( n \”).

\[ 25 + (n - 1) \times 3 = 100 \]

\[ (n - 1) \times 3 = 75 \]

\[ n = 26 \]

Hence, 100 is a term of the given A.P.

26. An airplane or aeroplane (informally plane) is a powered, fixed-wing aircraft that is propelled forward by thrust from a jet engine, propeller or rocket engine. Airplanes come in a variety of sizes, shapes, and wing configurations. The essential components of an airplane are a wing system to sustain it in flight, tail surfaces to stabilize the wings, movable surfaces to control the attitude of the plane in flight, and a power plant to provide the thrust necessary to push the vehicle through the air. Provision must be made to support the plane when it is at rest on the ground and during takeoff and landing. Most planes feature an enclosed body (fuselage) to house the crew, passengers, and cargo; the cockpit is the area from which the pilot operates the controls and instruments to fly the plane. A passenger is travelling in an airplane. An airplane is flying at a height of 3000 m above the level ground. He observes that the angle of depression from the plane to the foot of a tree is \( \alpha \), such that \( \cos 3\alpha = \sin (135^\circ - 4\alpha) \). Find the distance that the airplane must fly to be directly above the tree.

**Ans:**

\[ \cos 3\alpha = \sin (135^\circ - 4\alpha) \Rightarrow \sin (90^\circ - 3\alpha) = \sin (135^\circ - 4\alpha) \Rightarrow 90^\circ - 3\alpha = 135^\circ - 4\alpha \Rightarrow \alpha = 45^\circ \]

In \( \triangle ABC \),
\[ \tan \alpha = \frac{AB}{BC} = \tan 45^\circ \Rightarrow \frac{3000}{x} = 1 \Rightarrow x = 3000 \text{ m} \]
SECTION – C
Questions 27 to 34 carry 3 marks each.

27. Show that any positive even integer can be written in the form 6q, 6q + 2, 6q + 4, where q is any integer.
   Ans: Let \( x = 6q + r \), \( 0 \leq r < 6 \)
   \( \Rightarrow x = 6q \) or \( 6q +1 \) or \( 6q + 2 \) or \( 6q + 3 \) or \( 6q + 4 \) or \( 6q + 5 \)
   Now, \( 6q \) is an even integer being a multiple of 2.
   We know that the sum of two even integers are always even integers.
   Therefore, \( 6q +2 \), \( 6q + 4 \) are even integers.
   Also, we know that sum of even integer and odd integer is always an odd integer.
   Therefore, \( 6q +1 \), \( 6q + 2 \) and \( 6q + 5 \) are odd integers.
   Hence, any positive even integer can be written in the form \( 6q \), \( 6q + 2 \), \( 6q + 4 \), where \( q \) is any integer

   OR

Prove that \( \sqrt{3} + \sqrt{5} \) is an irrational number.
   Ans:
   Let us suppose that \( \sqrt{3} + \sqrt{5} \) is rational.
   Let \( \sqrt{3} + \sqrt{5} = a \), where \( a \) is rational.
   Therefore, \( \sqrt{3} = a - \sqrt{5} \)
   On squaring both sides, we get
   \[ (\sqrt{3})^2 = (a - \sqrt{5})^2 \]
   \[ \Rightarrow 3 = a^2 + 5 - 2a\sqrt{5} \]
   \[ \Rightarrow 2a\sqrt{5} = a^2 + 2 \]
   \[ \therefore \] \( \sqrt{5} = \frac{a^2 + 2}{2a} \) which is contradiction.
   As the right hand side is rational number while \( \sqrt{5} \) is irrational. Since, 3 and 5 are prime numbers. Hence, \( \sqrt{3} + \sqrt{5} \) is irrational.

28. In the given figure, O is the centre of the circle with AC = 24 cm, AB = 7 cm and \( \angle BOD = 90^\circ \).
   Find the area of the shaded region.

   Ans:
   \( \angle CAB = 90^\circ \) (Angle in semi-circle)
   Using pythagoras theorem in \( \triangle CAB \)
   \[ BC^2 = AC^2 + AB^2 = (24)^2 + (7)^2 = 576 + 49 = 625 \]
   \[ BC = 25 \text{ cm} \]
   Radius of circle = \( OB = OD = OC = \frac{25}{2} \) cm
Area of shaded region

\[= \text{Area of semi-circle with diameter } BC - \text{Area of } \triangle CAB + \text{Area of sector } BOD\]

\[= \frac{1}{2} \pi \left(\frac{25}{2}\right)^2 - \frac{1}{2} \times 24 \times 7 + \frac{90}{360} \pi \left(\frac{25}{2}\right)^2\]

\[= \frac{3}{4} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} - 84 = \frac{20625}{56} - 84\]

\[= \frac{20625 - 4704}{56} = \frac{15921}{56} = 284.3 \text{ cm}^2 \text{(approx.)}\]

29. If \( \sec \theta = x + \frac{1}{4x} \), prove that \( \sec \theta + \tan \theta = 2x \text{ or } \frac{1}{2x} \).

Let \( \sec \theta + \tan \theta = \lambda \) \ ...(i)

We know that \( \sec^2 \theta - \tan^2 \theta = 1 \)

\[\Rightarrow (\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1\]

\[\Rightarrow \lambda(\sec \theta - \tan \theta) = 1\]

\[\Rightarrow \sec \theta - \tan \theta = \frac{1}{\lambda} \] \ ...(ii)

Adding eqns. (i) and (ii),

\[2\sec \theta = \lambda + \frac{1}{\lambda}\]

\[\Rightarrow 2\left(x + \frac{1}{4x}\right) = \lambda + \frac{1}{\lambda}\]

\[\Rightarrow 2x + \frac{1}{2x} = \frac{1}{\lambda}\]

Comparing both sides,

\[\lambda = 2x \text{ or } \lambda = \frac{1}{2x}\]

\[\Rightarrow \sec \theta + \tan \theta = 2x \text{ or } \frac{1}{2x}.\]

OR

If \( x\sin^3 \theta + y\cos^3 \theta = \sin \theta \cos \theta \) and \( x\sin \theta = y\sin \theta \), prove that \( x^2 + y^2 = 1 \).

Given: \( x \sin \theta = y \cos \theta \)

\[\Rightarrow x = \frac{y \cos \theta}{\sin \theta} \] \ ...(i)

and \( x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta \) \ ...(ii)

Eliminating \( x \) from eqn. (i) and eqn. (ii),

\[\frac{y \cos \theta}{\sin \theta} \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta \]

\[\Rightarrow y \cos \theta \sin^2 \theta + y \cos^3 \theta = \sin \theta \cos \theta \]

\[\Rightarrow y \cos \theta [\sin^2 \theta + \cos^2 \theta] = \sin \theta \cos \theta \]

\[\Rightarrow y = \sin \theta \] \ ...(iii)

Substituting this value of \( y \) in eqn. (i),

\[x = \cos \theta \] \ ...(iv)

\[\therefore \] Squaring and adding eqn. (iii) and eqn. (iv), we get

\[x^2 + y^2 = \cos^2 \theta + \sin^2 \theta = 1\]

Hence proved.
30. The below figure shows the arrangement of desks in a classroom. Arun, Bharath, Charan and Deepak are seated at A, B, C and D respectively. Deepak suggested to mark their position with triangle shaped as shown in below figure. He also observes that their position forming a quadrilateral ABCD and he wants to find out the area of the quadrilateral. How can you help Deepak to find the same?

Ans:

Area of quadrilateral $ABCD = ar(AABC) + ar(DAD)$

$\text{ar}(\text{quad.}ABCD) = \frac{1}{2} [(x_1y_2-x_2y_1) + (x_2y_3-x_3y_2) + (x_3y_4-x_4y_3) + (x_4y_1-x_1y_4)]$

$\text{ar}(\text{quad.}ABCD) = \frac{1}{2} [5(-1) - (-2)(-3) + (-3)(1) - (-1)(2) + (2 \times 0 - 1 \times 6) + 6(-2) - (0 \times 5)]$

$= \frac{1}{2} [-30] = |-15| = 15 \text{ sq. units}$

31. In an election contested between A and B, A obtained votes equal to twice the no. of persons on the electoral roll who did not cast their votes and this later number was equal to twice his majority over B. If there were 18,000 persons on the electoral roll, how many voted for B.
Let $x$ and $y$ be the no. of votes for $A$ & $B$ respectively.
The no. of persons who did not vote
\[ = (18000 - x - y) \]

Given that,
\[ x = 2(18000 - x - y) \]
\[ \Rightarrow 3x + 2y = 36000 \] ... (i)

& \[ (18000 - x - y) = 2(x - y) \]
\[ \Rightarrow 3x - y = 18000 \] ... (ii)

Subtract equation (ii) from equation (i),
\[ 3x + 2y = 36000 \]
\[ 3x - y = 18000 \]
\[ \underline{+} \]
\[ 3y = 18000 \]
\[ \Rightarrow \]
\[ y = 6000 \]

From (ii), \[ 3x - 6000 = 18000 \]
\[ \Rightarrow \]
\[ 3x = 24000 \]
\[ \therefore \]
\[ x = 8000 \]

We get,
\[ y = 6000 \text{ and } x = 8000 \]
Vote for $B = 6000$

OR

Solve the following pair of equations for $x$ and $y$: \[ 4x + \frac{6}{y} = 15, \ 6x - \frac{8}{y} = 14 \] and also find the value of $p$ such that $y = px - 2$.

Ans:

Let \[ \frac{1}{y} = a, \] the given equations become

\[ 4x + 6a = 15 \] \[ \Rightarrow \]
\[ 6x - 8a = 14 \] \[ \Rightarrow \]

Multiply eqn. (i) by 4 and eqn. (ii) by 3 and adding,
\[ 16x + 24a = 60 \]
\[ 18x - 24a = 42 \]
\[ 34x = 102 \]
\[ \therefore \]
\[ x = \frac{102}{34} = 3 \]

Substitute the value of $x$ in eqn. (i),
\[ 4(3) + 6a = 15 \]
\[ \therefore \]
\[ a = \frac{1}{y} = \frac{1}{2} \]

Hence,
\[ y = 2 \]
\[ x = 3 \text{ and } y = 2 \]

Again,
\[ y = px - 2 \]
\[ \Rightarrow \]
\[ 2 = p(3) - 2 \]
\[ \Rightarrow \]
\[ 3p = 4 \]
\[ \therefore \]
\[ p = \frac{4}{3} \]

32. If the $m$th term of an AP is \[ \frac{1}{n} \] and $n$th term is \[ \frac{1}{m} \] then show that its $(mn)$th term is 1.

Ans:

\[ a_m = \frac{1}{n} \Rightarrow a + (m - 1)d = \frac{1}{n} \] \[ \Rightarrow \]
\[ a_n = \frac{1}{m} \Rightarrow a + (n - 1)d = \frac{1}{m} \] \[ \Rightarrow \]
33. Aditya decided to collect the daily income details of all the Employees working in a small company where 50 employees are working. After collecting the data, he analyzed the data and prepared a report on the same. Using this report, he drew the following graph as given below. Prepare the frequency distribution table from the graph. Calculate the average daily income of the employees working in the company.

Ans: The frequency distribution from the graph is:

<table>
<thead>
<tr>
<th>Class</th>
<th>$x_i$ (class mark)</th>
<th>$f_i$</th>
<th>$f_i x_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 100</td>
<td>50</td>
<td>12</td>
<td>600</td>
</tr>
<tr>
<td>100 – 200</td>
<td>150</td>
<td>16</td>
<td>2400</td>
</tr>
<tr>
<td>200 – 300</td>
<td>250</td>
<td>6</td>
<td>1500</td>
</tr>
<tr>
<td>300 – 400</td>
<td>350</td>
<td>7</td>
<td>2450</td>
</tr>
<tr>
<td>400 – 500</td>
<td>450</td>
<td>9</td>
<td>4050</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>Σf_i = 50</strong></td>
<td><strong>Σf_i x_i = 11,000</strong></td>
</tr>
</tbody>
</table>

Mean = $\frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{11000}{50} = 220$

∴ Average daily income = ₹ 220
34. Find the value of $a$ and $b$ so that $8x^4 + 14x^3 - 2x^2 + ax + b$ is exactly divisible by $4x^2 + 3x - 2$.

\[
\begin{align*}
4x^2 + 3x - 2 & \mid 8x^4 + 14x^3 - 2x^2 + ax + b \\
8x^4 + 6x^2 - 4x & \quad - \quad + \\
& \quad - \quad + \\
8x^3 + 2x^2 + ax & \quad - \quad + \\
8x^3 + 6x^2 - 4x & \quad - \quad + \\
-4x^2 + (a + 4)x + b & \quad + \quad - \\
-4x^2 - 3x + 2 & \quad + \quad - \\
(a + 7)x + b - 2 &
\end{align*}
\]

For exact division, remainder is zero, then

\[(a + 7)x + b - 2 = 0\]

\[\Rightarrow \quad a + 7 = 0, b - 2 = 0\]

\[\Rightarrow \quad a = -7, b = 2\]

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SECTION – D

Questions 35 to 40 carry 4 marks each.

35. The angle of elevation of an aeroplane from a point on the ground is $60^0$. After flight of 30 seconds the angle of elevation becomes $30^0$. If the aeroplane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane.

**Ans:**

\[
\angle AED = 60^0, \quad \angle BEC = 30^0
\]

\[AD = BC = 3000\sqrt{3} \text{ m}\]

Let the speed of the aeroplane = $x$ m/s

\[
\begin{align*}
\Delta EDC & \text{ is right angled} \\
\tan 30^0 & = \frac{BC}{EC} \\
\frac{1}{\sqrt{3}} & = \frac{3000\sqrt{3}}{DE + CD} \\
DE + CD & = 3000 \times 3 \\
3000 + 30x & = 9000 \\
[\text{from (i) and (ii)}] \\
30x & = 6000 \\
x & = 200 \text{ m/s}
\end{align*}
\]

Speed of plane is 200 m/s.
36. The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is \( \frac{29}{20} \). Find the original fraction.

Let the fraction be \( \frac{x-3}{x} \)

By the given condition, new fraction = \( \frac{x-3+2}{x+2} \)

\[
= \frac{x-1}{x+2}
\]

\[
\frac{x-3}{x} + \frac{x-1}{x+2} = \frac{29}{20}
\]

\[
\Rightarrow 20[(x-3)(x+2) + x(x-1)] = 29(x^2 + 2x)
\]

\[
= 20(x^2 - x - 6 + x^2 - x) = 29x^2 + 58x
\]

or

\[
11x^2 - 98x - 120 = 0
\]

or

\[
11x^2 - 110x + 12x - 120 = 0
\]

\[
(11x + 12) (x - 10) = 0 \Rightarrow x = 10
\]

\[
\therefore \text{The fraction is } \frac{7}{10}.
\]

OR

A train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train.

Let the speed of train = \( x \) km/hr.

\[
\frac{300}{x} - \frac{300}{x+5} = 2
\]

\[
\Rightarrow x^2 + 5x - 750 = 0
\]

\[
\Rightarrow (x + 30)(x - 25) = 0
\]

\[
\Rightarrow x = -30 \text{ or } x = 25
\]

Since, speed cannot be negative.

Hence \( x \neq -30 \), \( x = 25 \) km/hr.

\[
\therefore \text{Speed of train } = 25 \text{ km/hr.}
\]

37. A metallic solid sphere of radius 10.5 is melted and recasted into smaller solid cones, each of radius 3.5 cm and height 3 cm. How many cones will be made?

Let the number of cones be \( n \)

Volume of solid sphere

\[
= \text{Volume of } n \text{ solid cones}
\]

\[
\Rightarrow \frac{4}{3} \pi \times 10.5 \times 10.5 \times 10.5
\]

\[
= n \times \frac{1}{3} \times \pi \times 3.5 \times 3.5 \times 3
\]

\[
\Rightarrow n = \frac{4 \times 10.5 \times 10.5 \times 10.5}{3.5 \times 3.5 \times 3}
\]

\[
= 126
\]

OR
The rainwater from 22m x 20m roof drains into cylindrical vessel of diameter 2m and height 3.5m. If the rainwater collected from the roof fills of \(\frac{4}{5}\) th of cylindrical vessel then find the rainfall in cm.

Volume of water collected in cylindrical vessel

\[
\text{Volume} = \frac{4}{5} \times \pi \times (1)^2 \times \left(\frac{7}{2}\right) \text{m}^3
\]

\[
= \frac{44}{5} \text{ m}^3
\]

Let the rainfall is \(h\) m.

Rain water from roof = 22 \times 20 \times h \text{ m}^3

\[
\Rightarrow 22 \times 20 \times h = \frac{44}{5}
\]

\[
\Rightarrow h = \frac{44}{5} \times \frac{1}{22 \times 20} = \frac{1}{50} \text{ m}
\]

\[
= \frac{1}{50} \times 100 = 2 \text{ cm}
\]

38. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

39. Draw a triangle ABC with side BC = 7 cm, B = 45°, A = 105°. Then, construct a triangle whose sides are 4/3 times the corresponding sides of \(\Delta\) ABC.

OR

Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

40. Find the values of \(x\) and \(y\), if the median for the following data is 31.

<table>
<thead>
<tr>
<th>Class</th>
<th>Frequency</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 10</td>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>10 – 20</td>
<td>(x)</td>
<td>(y)</td>
</tr>
<tr>
<td>20 – 30</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>30 – 40</td>
<td>(y)</td>
<td></td>
</tr>
<tr>
<td>40 – 50</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>50 – 60</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Here from table, \(N = 22 + x + y = 40\)

\[
\Rightarrow x + y = 18 \quad \ldots(i)
\]

Since, Median = 31,

\(\therefore\) Median class = 30 – 40

\[
\text{Median} = l + \left(\frac{n - c.f.}{f}\right) \times h
\]

\[
\Rightarrow 31 = 30 + \left[\frac{20 - (11 + x)}{y}\right] \times 10
\]

\[
\Rightarrow 1 = \frac{(9 - x) \times 10}{y}
\]

\[
\Rightarrow y = 90 - 10x
\]

From (i), \(10x + y = 90\) \ldots(ii)

\[
\Rightarrow x + y = 18
\]

(On subtraction) \(9x = 72\)

\[
\Rightarrow x = \frac{72}{9} = 8
\]

From (i), \(y = 18 - 8 = 10\)