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PRACTICE PAPER 04 (2023-24)

CHAPTER 04 QUADRATIC EQUATIONS (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If a and b are the roots of the equation $x^2 + ax – b = 0$, then find a and b.

(a) $a = – 1$ and b = 2 (b) $a = 1$ and b = 2

(c) $a = – 2$ and b = 1 (d) $a = 2$ and b = – 1

Ans: (a) $a = – 1$ and b = 2

Sum of the roots = $a + b = \frac{-B}{A} = – a$

Product of the roots = $ab = \frac{C}{A} = – b$

$\Rightarrow a + b = – a$ and $ab = – b$

$\Rightarrow 2a = – b$ and $a = –1 \Rightarrow b = 2$ and $a = – 1$

2. Which of the following are the roots of the quadratic equation, $x^2 – 9x + 20 = 0$?

(a) 3, 4 (b) 4, 5 (c) 5, 6 (d) 6, 7

Ans: (b) Given equation is $x^2 – 9x + 20 = 0$

$\Rightarrow x^2 – 5x – 4x + 20 = 0 \Rightarrow x(x – 5) – 4(x – 5) = 0$

$\Rightarrow (x – 5)(x – 4) = 0 \Rightarrow$ either $x – 5 = 0$ and $x – 4 = 0$

$\Rightarrow x = 5$ and $x = 4$

$\therefore x = 4$ and 5 are the roots/solution of the given quadratic equation.

3. If $(1 – p)$ is a root of the equation $x^2 + px + 1 – p = 0$, then roots are

(a) 0, 1 (b) – 1, 1 (c) 0, – 1 (d) – 1, 2

Ans: (c) $(1 – p)$ is a root

$\therefore (1 – p)^2 + p(1 – p) + 1 – p = 0$

$\Rightarrow (1 – p)[1 – p + p + 1] = 0$

$\Rightarrow (1 – p)(2) = 0 \Rightarrow p = 1$

$x^2 + x = 0$

One root = 0 and another root = – 1

$\therefore$ roots are 0 and –1.

4. Which of the following equations has two distinct real roots?

(a) $2x^2 – 3\sqrt{2}\,x + \frac{9}{4} = 0$ (b) $x^2 + x – 5 = 0$

(c) $x^2 + 3x + 2\sqrt{2} = 0$ (d) $5x^2 – 3x + 1 = 0$

Ans: (b) $x^2 + x – 5 = 0$ as $D > 0$

5. Which of the following equations has no real roots ?

(a) $x^2 - 4x + 3\sqrt{2} = 0$ (b) $x^2 + 4x - 3\sqrt{2} = 0$

(c) $x^2 - 4x - 3\sqrt{2} = 0$ (d) $3x^2 + 4\sqrt{3}x + 4 = 0$

Ans: (a) $x^2 - 4x + 3\sqrt{2} = 0$ as $D < 0$

6. If the roots of $ax^2 + bx + c = 0$ are equal in magnitude but opposite in sign, then

(a) $a = 0$ (b) $b = 0$ (c) $c = 0$ (d) none of these

Ans: (b) ∵ sum of roots = 0 $\Rightarrow \frac{-b}{a} = 0 \Rightarrow b = 0$

7. If the roots of equation $3x^2 + 2x + (p + 2)(p - 1) = 0$ are of opposite sign then which of the following cannot be the value of p?

(a) 0 (b) – 1 (c) $\frac{1}{2}$ (d) – 3

Ans: (d) ∵ roots are of opposite sign
∴ product of the roots is negative
⇒ (p + 2)(p – 1) should be negative.
Clearly when p = – 3, (p + 2) (p – 1) is not negative.

8. The value of k for which the equation $x^2 + 2(k + 1)x + k^2 = 0$ has equal roots is

(a) – 1 (b) $-\frac{1}{2}$ (c) 1 (d) none of these

Ans: (b) For equal roots, D = 0
⇒ $[2(k + 1)]^2 - 4 \times k^2 = 0$
⇒ $4(k + 1)^2 - 4k^2 = 0$
⇒ $4(k^2 + 2k + 1) - 4k^2 = 0$
⇒ $8k + 4 = 0 \Rightarrow k = -\frac{1}{2}$.

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): If one root of the quadratic equation $6x^2 - x - k = 0$ is $\frac{2}{3}$, then the value of k is 2.

Reason (R): The quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has almost two roots.
Ans: (a) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

10. Assertion (A): The roots of the quadratic equation $x^2 + 2x + 2 = 0$ are imaginary
Reason (R): If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.
Ans (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Solve for x : $4x^2 - 2(a^2 + b^2)x + a^2 b^2 = 0$.

Ans: $4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$
$\Rightarrow 4x^2 - 2a^2x - 2b^2x + a^2b^2 = 0$
$\Rightarrow 2x(2x - a^2) - b^2(2x - a^2) = 0$
$\Rightarrow (2x - a^2)(2x - b^2) = 0$
$\Rightarrow x = \frac{a^2}{2}, x = \frac{b^2}{2}$

12. The sum of the squares of three consecutive positive integers is 50. Find the integers.
Ans: Let three consecutive positive integers be x, $x + 1$ and $x + 2$.
According to the question, $x^2 + (x + 1)^2 + (x + 2)^2 = 50$
$\Rightarrow x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 50$
$\Rightarrow 3x^2 + 6x - 45 = 0$
$\Rightarrow x^2 + 2x - 15 = 0$
$\Rightarrow (x + 5)(x - 3) = 0$
$\Rightarrow x = -5$ or $x = 3$
But x is positive integer, $\therefore\ x \neq -5$
Hence $x = 3$; when $x = 3$ integers are 3, 3 + 1, 3 + 2 i.e. 3, 4 and 5.

13. Find the value of α such that the quadratic equation $(\alpha - 12)x^2 + 2(\alpha - 12)x + 2 = 0$, has equal roots.
Ans: Here, $a = \alpha - 12$, $b = 2(\alpha - 12)$, $c = 2$
For equal roots, $D = 0 \Rightarrow b^2 - 4ac = 0$
$\Rightarrow [2(\alpha - 12)]^2 - 4 \times [2(\alpha - 12)] = 0$
$2(\alpha - 12)[2(\alpha - 12) - 4] = 0$
$\Rightarrow (\alpha - 12)(2\alpha - 28) = 0$
$\Rightarrow \alpha = 12, 14$
$\alpha = 12$ not possible, take $\alpha = 14$

14. Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.
Ans: Let the root be α then other root will be 6α
$\Rightarrow$ Sum of the roots = $-b/a$
$\Rightarrow \alpha + 6\alpha = \frac{-(-14)}{p} \Rightarrow 7\alpha = \frac{14}{p} \Rightarrow \alpha = \frac{2}{p}$ -------- (1)
$\Rightarrow$ Product of the roots = c/a
$\Rightarrow \alpha.6\alpha = \frac{8}{p} \Rightarrow 6\alpha^2 = \frac{8}{p} \Rightarrow 6\left(\frac{2}{p}\right)^2 = \frac{8}{p}$ [From (1)]
$\Rightarrow 6 \times \frac{2}{p^2} = \frac{8}{p} \Rightarrow p^2 - 3p = 0$
$\Rightarrow p(p - 3) = 0 \Rightarrow p = 0, 3$
If p = 0, then the given equation will not remain quadratic.
So, p = 0 is rejected. Hence, the value of p is 3.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.
Ans: Since, -5 is a root of equation
$2x^2 + px - 15 = 0$
$\therefore\ 2(-5)^2 + p(-5) - 15 = 0$
$\Rightarrow 50 - 5p - 15 = 0 \Rightarrow p = 7$

$\therefore\ p(x^2 + x) + k = 0$ becomes $7(x^2 + x) + k = 0$
$\Rightarrow 7x^2 + 7x + k = 0$
$\because\ 7x^2 + 7x + k = 0$ has equal roots
$\therefore\ D = 0$
$\Rightarrow (7)^2 - 4 \times 7 \times k = 0$
$\Rightarrow 28k = 49 \Rightarrow k = \frac{7}{4}$

16. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, then show that $c^2 = a^2(1 + m^2)$.
Ans: The given equation is
$(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$
Here $A = (1 + m^2)$, $B = 2mc$, $C = c^2 - a^2$
We know that For equal roots, $D = 0$
$\Rightarrow B^2 - 4AC = 0$
$\Rightarrow (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$
$\Rightarrow 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$
$\Rightarrow 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$
$\Rightarrow -4c^2 + 4a^2 + 4m^2a^2 = 0$
$\Rightarrow 4c^2 - 4a^2 - 4m^2a^2 = 0$
$\Rightarrow c^2 - a^2 - m^2a^2 = 0$
$\Rightarrow c^2 = a^2 + m^2a^2$
$\Rightarrow c^2 = a^2(1 + m^2)$

17. Solve the following for x : $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$
Ans: $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x} \Rightarrow \frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$
$\Rightarrow \frac{2x-2a-b-2x}{(2a+b+2x)(2x)} = \frac{b+2a}{2ab}$
$\Rightarrow \frac{-(2a+b)}{(2a+b+2x)(2x)} = \frac{2a+b}{2ab}$
$\Rightarrow \frac{-1}{4ax+2bx+4x^2} = \frac{1}{2ab}$
$\Rightarrow 4x^2 + 2bx + 4ax = -2ab$
$\Rightarrow 4x^2 + 2bx + 4ax + 2ab = 0 \Rightarrow 2x(2x + b) + 2a(2x + b) = 0$
$\Rightarrow (2x + b)(2x + 2a) = 0 \Rightarrow x = -\frac{b}{2}$ or $x = -a$

SECTION – D

Questions 18 carry 5 marks.

18. In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced by 200 km/hr and time of flight increased by 30 minutes. Find the original duration of flight.
Ans: Let original speed of the aircraft be x km/hr
Reduced speed = (x – 200) km/hr
According to given condition, $\frac{600}{x-200} - \frac{600}{x} = \frac{30}{60} = \frac{1}{2}$
$\Rightarrow \frac{600x - 600x + 120000}{x(x-200)} = \frac{1}{2} \Rightarrow \frac{120000}{x^2 - 200x} = \frac{1}{2}$
$\Rightarrow x^2 - 200x = 240000$
$\Rightarrow x^2 - 200x - 240000 = 0$

$\Rightarrow x^2 - 600x + 400x - 240000 = 0$
$\Rightarrow x(x - 600) + 400(x - 600) = 0$
$\Rightarrow (x + 400)(x - 600) = 0$
$\Rightarrow x + 400 = 0$ or $x - 600 = 0$
$\Rightarrow x = -400$ (rejected) or $x = 600$
$\therefore$ original speed = 600 km/hr
$\therefore$ original duration of flight = $\frac{600}{600}$ = 1 hour

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Raj and Ajay are very close friends. Both the families decide to go to Ranikhet by their own cars. Raj's car travels at a speed of x km/h while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete he journey of 400 km.

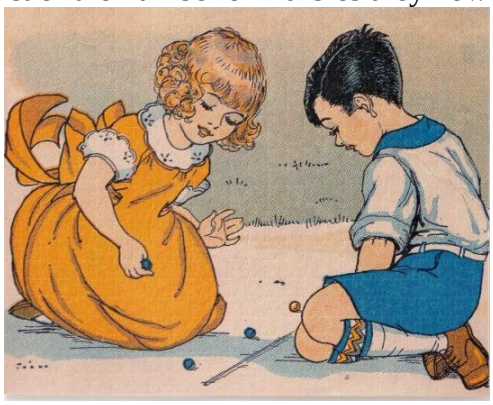
(a) What will be the distance covered by Ajay's car in two hours? (1)
(b) Which of the following quadratic equation describe the speed of Raj's car? (2)
(c) What is the speed of Raj's car? (1)
Ans: (a) Given, Raj's car travel at a speed of x km/h.
Then Ajay's car travels a distance in one hour is (x + 5) km.
Therefore, Ajay's car travels a distance in two hours is 2(x + 5) km.
(b) We know that, Time = $\frac{\text{Distance}}{\text{Speed}}$
Time taken by Ajay = $\frac{400}{x+5}$ and Time taken by Raj = $\frac{400}{x}$
According to the question, we have $\frac{400}{x} - \frac{400}{x+5} = 4$

$$\Rightarrow \frac{100}{x} - \frac{100}{x+5} = 1 \Rightarrow \frac{100x + 500 - 100x}{x(x+5)} = 1$$

$$\Rightarrow \frac{500}{x(x+5)} = 1 \Rightarrow x^2 + 5x = 500 \Rightarrow x^2 + 5x - 500 = 0$$

(c) $x^2 + 5x - 500 = 0$
$\Rightarrow x^2 + 25x - 20x - 500 = 0$
$\Rightarrow x(x + 25) - 20(x + 25) = 0$
$\Rightarrow (x + 25)(x - 20) = 0$
$\Rightarrow x + 25 = 0$ or $x - 20 = 0$
$\Rightarrow x = -25$ or $x = 20$
Since, speed cannot be negative, so we consider only, x = 20.
Hence, speed of Raj's car is 20 km/h.

20. John and Jivanti are playing with the marbles. They together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124.



(a) Find the quadratic equation related to the given problem (2)
(b) Find the Number of marbles John had. (2)
Ans: If John had x number of marbles, then Jivanti had (45 – x) marbles, because there are total 45 marbles.
Number of marbles left with John, when he lost 5 marbles = x – 5
Number of marbles left with Jivanti, when she lost 5 marbles
= (45 – x – 5) = (40 – x)
(a) According to question, (x – 5)(40 – x) = 124
$\Rightarrow -x^2 - 200 + 40x + 5x - 124 = 0$
$\Rightarrow x^2 - 45x + 324 = 0$
(b) $x^2 - 45x + 324 = 0$
$\Rightarrow x^2 - 9x - 36x + 324 = 0 \Rightarrow x(x - 9) - 36(x - 9) = 0$
$\Rightarrow (x - 9)(x - 36) = 0 \Rightarrow$ Either x = 9 or x = 36.
Therefore, the number of marbles John had 9 or 36.