$\mathcal{S U B I} \mathcal{E C T}: \mathcal{M A T \mathcal { H E M A T } I C S}$
MAX. MARKS : 40
CLASS : $X$
DURATION: 1112 hrs

## General Instructions:

(i). All questions are compulsory.
(ii). This question paper contains 20 questions divided into five Sections A, B, C, D and E.
(iii). Section A comprises of $\mathbf{1 0}$ MCQs of $\mathbf{1}$ mark each. Section $\mathbf{B}$ comprises of 4 questions of $\mathbf{2}$ marks each. Section C comprises of 3 questions of $\mathbf{3}$ marks each. Section D comprises of 1 question of $\mathbf{5}$ marks each and Section E comprises of 2 Case Study Based Questions of 4 marks each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

## SECTION - A

## Questions 1 to 10 carry 1 mark each.

1. In the given figure, $\frac{A D}{B D}=\frac{A E}{E C}$ and $\angle \mathrm{ADE}=70^{\circ}, \angle \mathrm{BAC}=50^{\circ}$, then angle $\angle \mathrm{BCA}=$

(a) $70^{\circ}$ (b)
(b) $50^{\circ}$ (c) $80^{\circ}$
(d) $60^{\circ}$

Ans: (d) $\because$ DE || BC
$\therefore \angle \mathrm{ABC}=70^{\circ}$. (Corresponding angles)
Using angle sum property of triangle $\angle \mathrm{ABC}+\angle \mathrm{BCA}+\angle \mathrm{BAC}=180^{\circ}$
$\angle B C A=60^{\circ}$.
2. In given figure, $\mathrm{AD}=3 \mathrm{~cm}, \mathrm{AE}=5 \mathrm{~cm}, \mathrm{BD}=4 \mathrm{~cm}, \mathrm{CE}=4 \mathrm{~cm}, \mathrm{CF}=2 \mathrm{~cm}, \mathrm{BF}=2.5 \mathrm{~cm}$, then

(a) $\mathrm{DE} \| \mathrm{BC}$ (b) $\mathrm{DF} \| \mathrm{AC}$ (c) $\mathrm{EF} \| \mathrm{AB}$ (d) none of these

Ans: (c) $\frac{C F}{F B}=\frac{C E}{A E} \Rightarrow E F \| A B$
3. If $\triangle \mathrm{ABC} \sim \triangle \mathrm{EDF}$ and $\triangle \mathrm{ABC}$ is not similar to $\triangle \mathrm{DEF}$, then which of the following is not true?
(a) $\mathrm{BC} \cdot \mathrm{EF}=\mathrm{AC} . \mathrm{FD}$
(b) $\mathrm{AB} \cdot \mathrm{EF}=\mathrm{AC} \cdot \mathrm{DE}$
(c) $\mathrm{BC} \cdot \mathrm{DE}=\mathrm{AB} \cdot \mathrm{EF}$
(d) $\mathrm{BC} \cdot \mathrm{DE}=\mathrm{AB}$. FD

Ans: (c) $\because \triangle \mathrm{ABC} \sim \Delta \mathrm{EDF}$
Then, $\frac{A B}{E D}=\frac{B C}{D F}=\frac{A C}{E F}$
$\Rightarrow \mathrm{AB} . \mathrm{DF}=\mathrm{ED} . \mathrm{BC}$
or $\mathrm{AB} \cdot \mathrm{EF}=\mathrm{AC} \cdot \mathrm{ED}$
or BC.EF $=$ DF.AC
$\therefore \mathrm{BC} . \mathrm{DE} \neq \mathrm{AB} . \mathrm{EF}$
4. If in two triangles ABC and $\mathrm{PQR}, \frac{A B}{Q R}=\frac{B C}{P R}=\frac{C A}{P Q}$, then
(a) $\triangle \mathrm{PQR} \sim \triangle \mathrm{CAB}$
(b) $\triangle \mathrm{PQR} \sim \triangle \mathrm{ABC}$
(c) $\triangle \mathrm{CBA} \sim \triangle \mathrm{PQR}$
(d) $\triangle \mathrm{BCA} \sim \triangle \mathrm{PQR}$

Ans: (a) $\triangle \mathrm{PQR} \sim \Delta \mathrm{CAB}$
5. If in triangles ABC and $\mathrm{DEF}, \frac{A B}{D E}=\frac{B C}{F D}$, then they will be similar, when
(a) $\angle$ B $=\angle$ E
(b) $\angle \mathrm{A}=\angle \mathrm{D}$
(c) $\angle \mathrm{B}=\angle \mathrm{D}$
(d) $\angle A=\angle F$

Ans: (c) $\angle \mathrm{B}=\angle \mathrm{D}$
6. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm ., what is the corresponding side of the other triangle?
(a) 5.4
(b) 3.5
(c) 5.5
(d) 4.5

Ans: (a) 5.4
Let corresponding sides of two similar $\Delta$ 's are $a, b, c$ and $d, e, f$ respectively, let a $=9 \mathrm{~cm}$.
$\therefore \Delta$ 's are similar
$\frac{a}{d}=\frac{b}{e}=\frac{c}{f} \Rightarrow \frac{a+b+c}{d+e+f}=\frac{a}{d}$ (Using property of proportion)
$\Rightarrow \frac{25}{15}=\frac{9}{d} \Rightarrow d=\frac{9 \times 15}{25}=5.4 \mathrm{~cm}$
7. In figure $\mathrm{DE} \| \mathrm{BC}$. If $\mathrm{BD}=x-3, \mathrm{AB}=2 x$. $\mathrm{CE}=x-2$ and $\mathrm{AC}=2 x+3$. Find $x$.

(a) 3
(b) 4
(c) 9
(d) none of these

Ans: (c) 9
In $\triangle \mathrm{ABC}, \mathrm{DE} \| \mathrm{BC}$

$$
\begin{aligned}
& \frac{\mathrm{AD}}{\mathrm{BD}}=\frac{\mathrm{AE}}{\mathrm{CE}} \Rightarrow \frac{\mathrm{AB}-\mathrm{BD}}{\mathrm{BD}}=\frac{\mathrm{AC}-\mathrm{CE}}{\mathrm{CE}} \\
\therefore & \frac{2 x-(x-3)}{x-3}=\frac{2 x+3-(x-2)}{x-2} \Rightarrow \frac{x+3}{x-3}=\frac{x+5}{x-2} \\
\Rightarrow & (x-2)(x+3)=(x+5)(x-3) \Rightarrow x^{2}+x-6=x^{2}+2 x-15 \\
\Rightarrow & x=9 \therefore x=9 \mathrm{~cm}
\end{aligned}
$$

8. In the figure, $\mathrm{AP}=3 \mathrm{~cm}, \mathrm{AR}=4.5 \mathrm{~cm}, \mathrm{AQ}=6 \mathrm{~cm}, \mathrm{AB}=5 \mathrm{~cm}$ and $\mathrm{AC}=10 \mathrm{~cm}$. Find the length of $A D$.

(a) 6.5
(b) 7.5
(c) 5.5
(d) 4.5

Ans: (b) 7.5
In $\triangle \mathrm{ABC}, \frac{A P}{A B}=\frac{3}{5}$
$\frac{A Q}{A C}=\frac{6}{10}=\frac{3}{5}$
From (i) and (ii),
$\frac{A P}{A B}=\frac{A Q}{A C} \Rightarrow \mathrm{PQ} \| \mathrm{BC}$
(By converse of BPT)
In $\triangle \mathrm{ABD}, \mathrm{PR} \| \mathrm{BD}$
$\Rightarrow \frac{A P}{A B}=\frac{A R}{A D}(\mathrm{By} \mathrm{BPT}) \Rightarrow \frac{3}{5}=\frac{4.5}{A D} \Rightarrow A D=7.5 \mathrm{~cm}$
In the following questions 9 and 10 , a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a)Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b)Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c)Assertion (A) is true but reason (R) is false.
(d)Assertion (A) is false but reason (R) is true.
9. Assertion (A): $D$ and $E$ are points on the sides $A B$ and $A C$ respectively of a $\triangle A B C$ such that $D E \| B C$ then the value of $x$ is 11 , when $A D=4 \mathrm{~cm}, D B=(x-4) c m, A E=8 \mathrm{~cm}$ and $E C=(3 x-$ 19) cm .

Reason (R): If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side.
Ans: (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
10. Assertion (A): $D$ and $E$ are points on the sides $A B$ and $A C$ respectively of a $\triangle A B C$ such that $A D$ $=5.7 \mathrm{~cm}, \mathrm{DB}=9.5 \mathrm{~cm}, \mathrm{AE}=4.8 \mathrm{~cm}$ and $\mathrm{EC}=8 \mathrm{~cm}$ then DE is not parallel to BC .
Reason (R): If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side.
Ans: (d) Assertion (A) is false but reason (R) is true.

## SECTION - B

## Questions 11 to 14 carry 2 marks each.

11. In figure, $\triangle A B D$ is a right triangle, right angled at $A$ and $A C \perp B D$. Prove that $A B^{2}=B C \cdot B D$.


Ans: In $\triangle \mathrm{DAB}$, and $\triangle \mathrm{ACB}$
$\angle \mathrm{DAB}=\angle \mathrm{ACB}=90^{\circ}$
$\angle \mathrm{B}=\angle \mathrm{B}$ (common)
$\therefore \triangle \mathrm{DAB} \sim \triangle \mathrm{ACB}$
$\Rightarrow \frac{A D}{A C}=\frac{A B}{B C}=\frac{B D}{A B} \Rightarrow \frac{A B}{B C}=\frac{B D}{A B}$
$\Rightarrow \mathrm{AB}^{2}=\mathrm{BC} . \mathrm{BD}$ Hence proved.
12. In $\triangle A B C, D$ and $E$ are points on the sides $A B$ and $A C$ respectively, such that $D E \| B C$. If $A D$ $=x, \mathrm{DB}=x-2, \mathrm{AE}=x+2$ and $\mathrm{EC}=x-1$, Find the value of $x$.
Ans: In $\triangle \mathrm{ABC}, \mathrm{DE} \| \mathrm{BC}$ (Given)

$\frac{A D}{D B}=\frac{A E}{E C}($ By BPT $)$
$\Rightarrow \frac{x}{x-2}=\frac{x+2}{x-1}$
$\Rightarrow x(x-1)=(x+2)(x-2)$
$\Rightarrow x^{2}-x=x_{2}-2^{2} \Rightarrow x^{2}-x=x^{2}-4$
$\Rightarrow x=4$
13. In the figure, $P Q R$ and $Q S T$ are two right triangles, right angled at $R$ and $T$ respectively. Prove that $\mathrm{QR} \times \mathrm{QS}=\mathrm{QP} \times \mathrm{QT}$


Ans: In $\triangle P R Q$ and $\triangle S T Q$
$\angle \mathrm{Q}=\angle \mathrm{Q}$ (common)
$\angle \mathrm{R}=\angle \mathrm{T}\left(\right.$ each $\left.90^{\circ}\right)$
$\therefore \triangle \mathrm{PRQ} \sim$ DSTQ (SS similarity corollary)
$\Rightarrow \frac{Q R}{Q T}=\frac{Q P}{Q S} \Rightarrow \mathrm{QR} \times \mathrm{QS}=\mathrm{QP} \times \mathrm{QT}$
14. In the given figure, $A B C$ is a triangle, right angled at $B$ and $B D \perp A C$. If $A D=4 \mathrm{~cm}$ and $C D=5$ cm , find $B D$ and $A B$.


Ans: Here $\triangle \mathrm{ADB} \sim \Delta \mathrm{BDC}$
$\therefore \frac{A D}{B D}=\frac{B D}{C D} \Rightarrow \mathrm{AD} \times \mathrm{CD}=\mathrm{BD} \times \mathrm{BD}$
$\Rightarrow 4 \times 5=\mathrm{BD}^{2}$
$\Rightarrow \mathrm{BD}=(2 \sqrt{5})^{2} \mathrm{~cm}$
In right $\triangle \mathrm{BDA}$
$\mathrm{AB}^{2}=\mathrm{BD}^{2}+\mathrm{AD}^{2}$
$\Rightarrow \mathrm{AB}^{2}=(2 \sqrt{5})^{2}+4^{2}$
$\Rightarrow \mathrm{AB}^{2}=36 \Rightarrow \mathrm{AB}=6 \mathrm{~cm}$

## SECTION - C

## Questions 15 to 17 carry 3 marks each.

15. In figure, two triangles $A B C$ and $D B C$ lie on the same side of base $B C$. $P$ is a point on $B C$ such that $\mathrm{PQ} \| \mathrm{BA}$ and $\mathrm{PR} \| \mathrm{BD}$. Prove that $\mathrm{QR} \| \mathrm{AD}$.


Ans:
Given : In $\triangle \mathrm{ABC}, \mathrm{PQ} \| \mathrm{AB}$ and $\mathrm{PR} \| \mathrm{BD}$
To prove : $\mathrm{QR} \| \mathrm{AD}$
Proof : By BPT $\frac{C P}{B P}=\frac{C Q}{A Q}$
Now in $\triangle B C D, P R \| B D$
$\Rightarrow$ By using BPT $\frac{C P}{B P}=\frac{C R}{R D}$
From (i) and (ii), $\frac{C Q}{A Q}=\frac{C R}{R D} \Rightarrow$ By converse of BPT, $\mathrm{QR} \| \mathrm{AD}$
16. P and Q are points on the sides AB and AC respectively of a triangle ABC . If $\mathrm{AP}=2 \mathrm{~cm}, \mathrm{~PB}=4$ $\mathrm{cm}, \mathrm{AQ}=3 \mathrm{~cm}, \mathrm{QC}=6 \mathrm{~cm}$, prove that $\mathrm{BC}=3 \mathrm{PQ}$.
Ans: $\frac{A P}{P B}=\frac{2}{4}=\frac{1}{2} \ldots$ (i) $\frac{A Q}{Q C}=\frac{3}{6}=\frac{1}{2}$
From (i) and (ii) $\frac{A P}{P B}=\frac{A Q}{Q C} \Rightarrow \mathrm{PQ} \| \mathrm{BC}$
In $\triangle \mathrm{ABC}$ and $\triangle \mathrm{APQ}$.
$\frac{A B}{A P}=\frac{A C}{A Q}(\because \mathrm{PQ} \| \mathrm{BC})$
$\angle \mathrm{A}=\angle \mathrm{A}$ (Common)
$\therefore \triangle \mathrm{ABC} \sim \triangle \mathrm{APQ}$, (SAS similarity)
$\Rightarrow \frac{A B}{A P}=\frac{B C}{P Q} \Rightarrow \frac{A P+P B}{A P}=\frac{B C}{P Q}$
$\Rightarrow \frac{2+4}{2}=\frac{B C}{P Q} \Rightarrow \frac{6}{2}=\frac{B C}{P Q}=\frac{1}{3} \Rightarrow \mathrm{BC}=3 \mathrm{PQ}$. Hence proved.
17. In figure, D and E are points on AB and AC respectively, such that $\mathrm{DE} \| \mathrm{BC}$. If $\mathrm{AD}=\frac{1}{3} \mathrm{BD}, \mathrm{AE}$ $=4.5 \mathrm{~cm}$, find AC .


Ans: Here $\mathrm{AD}=\frac{1}{3} \mathrm{BD}$,
$\mathrm{AE}=4.5 \mathrm{~cm}, \mathrm{DE} \| \mathrm{BC}$
$\therefore \frac{A D}{B D}=\frac{A E}{E C}$ (using B.P.T.)
$\Rightarrow \frac{\frac{1}{3} B D}{B D}=\frac{4.5}{E C}$
$\Rightarrow \frac{1}{3}=\frac{4.5}{E C}$
$\Rightarrow \mathrm{EC}=4.5 \times 3 \mathrm{~cm}$
$\Rightarrow \mathrm{EC}=13.5 \mathrm{~cm}$
Now $\mathrm{AC}=\mathrm{AE}+\mathrm{EC}=4.5+13.5=18 \mathrm{~cm}$

## SECTION - D

## Questions 18 carry 5 marks.

18. If a line is drawn parallel to one side of a triangle, the other two sides are divided in the same ratio, prove it. Use this result to prove the following :
In the given figure, if ABCD is a trapezium in which $\mathrm{AB}\|\mathrm{DC}\| \mathrm{EF}$, then $\frac{A E}{E D}=\frac{B F}{F C}$


Ans: Given - $1 / 2$ mark
To prove - $1 / 2$ mark
Figure - $1 / 2$ mark
Construction-1/2 mark
Proof - 2 marks


Second part - 1 mark
Join BD intersecting EF at G.
In $\triangle \mathrm{DAB}, \mathrm{EG} \| \mathrm{AB}$
$\therefore \frac{A E}{E D}=\frac{B G}{G D}$ (Using B.P.T.) ...(i)

In $\triangle \mathrm{DBC}, \mathrm{GF}| | \mathrm{DC}$
$\therefore \frac{B G}{G D}=\frac{B F}{F C} \ldots$ (ii)
From (i) and (ii) $\frac{A E}{E D}=\frac{B F}{F C}$

## SECTION - E (Case Study Based Questions)

## Questions 19 to 20 carry 4 marks each.

19. On one day, a poor girl of height 90 cm is looking for a lamp-post for completing her homework as in her area power is not there and she finds the same at some distance away from her home. After completing the homework, she is walking away from the base of a lamp-post at a speed of $1.2 \mathrm{~m} / \mathrm{s}$. The lamp is 3.6 m above the ground (see below figure).

(i) Find her distance from the base of the lamp post. (2)
(ii) Find the length of her shadow after 4 seconds. (2)

## OR

(ii) Find the ratio AC:CE. (2)

Ans:
(i) Let AB denote the lamp-post and CD the girl after walking for 4 seconds away from the lamppost. From the figure, DE is the shadow of the girl.
Let DE be $x$ metres.
Now, her distance from the base of the lamp $=$
$\mathrm{BD}=1.2 \mathrm{~m} \times 4=4.8 \mathrm{~m}$.
(ii) $\triangle A B E \sim \triangle C D E$
$\Rightarrow \frac{B E}{D E}=\frac{A B}{C D} \Rightarrow \frac{4.8+x}{x}=\frac{3.6}{0.9} \Rightarrow 4.8+x=4 x \Rightarrow 3 x=4.8 \Rightarrow x=1.6 m$
OR
$\frac{A E}{C E}=\frac{B E}{D E} \Rightarrow \frac{4.8+1.6}{1.6}=\frac{6.4}{1.6}=4$
$\Rightarrow A E=4 C E \Rightarrow A C+C E=4 C E \Rightarrow A C=3 C E$
$\Rightarrow \frac{A C}{C E}=\frac{3}{1}$
20. Vijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Vijay's house if 20 m when Vijay's house casts a shadow 10 m long on the ground. At the same time, the tower casts a shadow 50 m long on the ground and the house of Ajay casts 20 m shadow on the ground.

(a) What is the height of the tower? (1)
(b) What is the height of Ajay's house? (1)
(c) What will be the length of the shadow of the tower when Vijay's house casts a shadow of 12 m ? (2)

## OR

(c) When the tower casts a shadow of 40 m , same time what will be the length of the shadow of Ajay's house? (2)
Ans:
(a) When two corresponding angles of two triangles are similar, then ratio of sides are equal.

Height of Vijay's house/Length of Shadow $=$ Height of Tower/Length of Shadow
$\Rightarrow 20 \mathrm{~m} / 10 \mathrm{~m}=$ Height of Tower $/ 50 \mathrm{~m}$
$\Rightarrow$ Height of Tower $=20 \times 50 / 10=100 \mathrm{~m}$
(b) Height of Vijay's house/Length of Shadow $=$ Height of Ajay's house/Length of Shadow
$20 \mathrm{~m} / 10 \mathrm{~m}=$ Height of Ajay's house $/ 20 \mathrm{~m}$
Height of Ajay's house $=20 \times 20 / 10=400 / 10=40 \mathrm{~m}$
(c) The height of Vijay's house is $\mathrm{AC}=20 \mathrm{~m}$.

The height of the tower is $A^{\prime} \mathrm{C}^{\prime}=100 \mathrm{~m}$.
The length of shadow of Vijay's house is $\mathrm{AB}=12 \mathrm{~cm}$.
$\because \mathrm{A}^{\prime} \mathrm{B}^{\prime} / \mathrm{A} \mathrm{B}=\mathrm{A}^{\prime} \mathrm{C}^{\prime} / \mathrm{A} \mathrm{C}$
$\Rightarrow \mathrm{A}^{\prime} \mathrm{B}^{\prime}=\mathrm{AB} / \mathrm{AC} \times \mathrm{A}^{\prime} \mathrm{C}^{\prime}=12 / 20 \times 100=12 \times 5=60 \mathrm{~m}$
OR
(c) The height of tower is $\mathrm{A}^{\prime} \mathrm{C}^{\prime}=100 \mathrm{~m}$.

And the height of Ajay's house is $\mathrm{PR}=40 \mathrm{~m}$
The length of shadow of tower is $A^{\prime} B^{\prime}=40 \mathrm{~m}$.
The length of shadow of Ajay's house is PQ .
$\because \triangle \mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime}$ and $\triangle \mathrm{PQR}$ are similar triangles.
$\therefore \mathrm{PQ} / \mathrm{A}^{\prime} \mathrm{B}^{\prime}=\mathrm{PR} / \mathrm{A}^{\prime} \mathrm{C}^{\prime}$
$\Rightarrow \mathrm{PQ}=\left(\mathrm{PR} / \mathrm{A}^{\prime} \mathrm{C}^{\prime}\right) \times \mathrm{A}^{\prime} \mathrm{B}^{\prime}=40 \times 40 / 100=16 \mathrm{~m}$

