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PRACTICE PAPER 10 (2023-24)

CHAPTER 11 AREAS RELATED TO CIRCLES

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

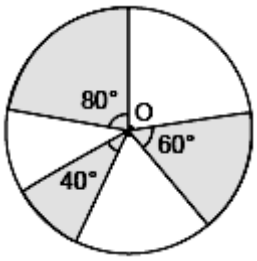
General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. In the given figure, three sectors of a circle of radius 7 cm, making angles of 60°, 80° and 40° at the centre are shaded. The area of the shaded region (in cm^2) is [Use $\pi = \frac{22}{7}$]



(a) 77 (b) 154 (c) 44 (d) 22

Ans: (a) 77

Area of shaded region = area of sector with angle (60° + 80° + 40°) = $\frac{180^0}{360^0} \times \frac{22}{7} \times 7 \times 7 = 77cm^2$

2. The ratio of the areas of the incircle and circumcircle of a square is

(a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) 1: $\sqrt{2}$

Ans: (a) 1 : 2

Let side of square = x units

∴ Diagonal of the square = $\sqrt{2}$ x units

Diameter of the incircle = x units

Diameter of the circumcircle = $\sqrt{2}$ x units

$$Now, \frac{Area\ of\ incircle}{Area\ of\ Circumcircle} = \frac{\pi\left(\frac{x}{2}\right)^2}{\pi\left(\frac{\sqrt{2}x}{2}\right)^2} = \frac{1}{2}$$

3. A circular wire of radius 42 cm is cut and bent into the form of a rectangle whose sides are in the ratio of 6 : 5. The smaller side of the rectangle is

(a) 30 cm (b) 60 cm (c) 70 cm (d) 80 cm

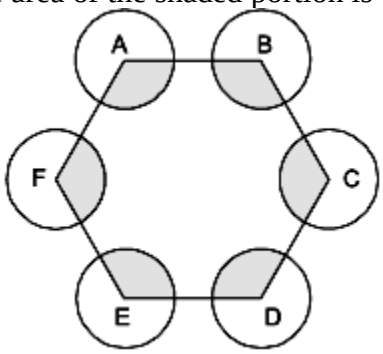
Ans: (b) 60 cm

Length of wire = $2\pi r = 2\pi \times 42 = 2 \times \frac{22}{7} \times 42 = 264$ cm

Let sides of rectangle are 6x and 5x

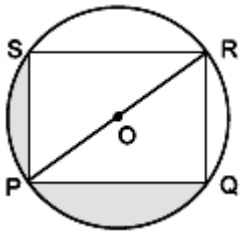
$\Rightarrow 2(6x + 5x) = 264 \Rightarrow 11x = 132 \Rightarrow x = 12$
$\therefore$ Smaller side $= 12 \times 5 = 60$ cm

4. ABCDEF is any hexagon with different vertices A, B, C, D, E and F as the centres of circles with same radius r are drawn. The area of the shaded portion is



(a) πr^2 (b) $2\pi r^2$ (c) $3\pi r^2$ (d) $4\pi r^2$
Ans: (b) $2\pi r^2$
Internal angles of a regular Hexagon $= 120^0$
Area of one sector $= \frac{120^0}{360^0}\pi r^2 = \frac{1}{3}\pi r^2$
$\therefore$ Area of 6 sectors $= 6 \times \frac{1}{3}\pi r^2 = 2\pi r^2$

5. In the figure, PQRS is a square and O is centre of the circle. If RS $= 10\sqrt{2}$, then area of shaded region is



(a) $90\pi - 90$ (b) $80\pi - 80$ (c) $50\pi - 100$ (d) $100\pi - 100$
Ans: (c) $50\pi - 100$
Diagonal of square $= \sqrt{2} \times 10\sqrt{2} = 20$ units
$\therefore$ Diameter of circle $= 20$ units
Area of circle $= \pi \times (10)^2 = 100\pi$ sq.units
Area of square $= \left(10\sqrt{2}\right)^2 = 200$ sq. units
Area of circle not included in the square $= (100\pi - 200)$ sq.units
$\therefore$ Area of shaded portion $= \frac{1}{2}(100\pi - 200) = 50\pi - 100$.

6. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then
(a) $R_1 + R_2 = R$ (b) $R_1 + R_2 > R$ (c) $R_1 + R_2 < R$
(d) nothing definite can be said about the relation among R_1, R_2 and R.
Ans: (a) $R_1 + R_2 = R$
$2\pi R_1 + 2\pi R_2 = 2\pi R$
$\Rightarrow R_1 + R_2 = R$.

7. If the perimeter of a circle is equal to that of a square, then the ratio of the area of circle to the area of the square is
(a) 14: 11 (b) 12: 13 (c) 11:14 (d) 13:12
Ans: (a) 14: 11

Here, it is given that $4s = 2\pi r \Rightarrow s = \frac{\pi r}{2}$

$$Now, \frac{Area\ of\ Circle}{Area\ of\ Square} = \frac{\pi r^2}{s^2} = \frac{\pi r^2}{\frac{\pi^2 r^2}{4}} = \frac{4}{\pi} = \frac{4\times 7}{22} = \frac{14}{11}$$

So, the ratio of the area of the circle to the area of square is 14 : 11.

8. The number of revolutions made by a circular wheel of radius 0.7 m in rolling a distance of 176 m is

(a) 22 (b) 24 (c) 75 (d) 40

Ans: (d) 40

$$\text{Number of revolutions} = \frac{\text{total distance}}{\text{circumference}} = \frac{176}{2\times\frac{22}{7}\times 0.7} = 40$$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): A bicycle wheel makes 5000 revolutions in covering 11 km. Then diameter of the wheel is 70 cm.

Reason (R): Area of segment of a circle is $\frac{\theta}{360^0}\times\pi r^2 - \frac{1}{2}r^2\sin\theta$

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of Assertion (A)
(c) Assertion (A) is true but reason(R) is false.
(d) Assertion (A) is false but reason(R) is true.
Ans: (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of Assertion (A)

We have, $2\pi r = \frac{11000}{5000} = \frac{11}{5}m = \frac{1100}{5}cm \Rightarrow 2r = \frac{1100}{5\pi} = \frac{1100\times 7}{22} \Rightarrow 2r = 70cm$

Diameter = 70 cm
∴ A is true and R is also true but not the correct explanation of A.

10. Assertion (A): The length of the minute hand of a clock is 7 cm. Then the area swept by the minute hand in 5 minute is 77/6 cm^2.

Reason (R): The length of an arc of a sector of angle q and radius r is given by $l = \frac{\theta}{360^0}\times 2\pi r$

Ans: (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of Assertion (A)

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. The minute hand of a clock is $\sqrt{21}$ cm long. Find the area described by the minute hand on the face of the clock between 7.00 am and 7.05 am. [Use $\pi = \frac{22}{7}$]

Ans: Time taken by minute hand to make one circle = 60 minutes.
∴ Angle described in 60 minutes = 360°

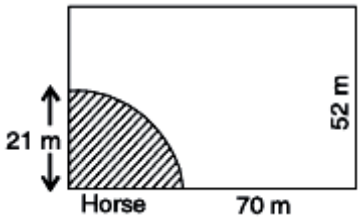
Angle described in 5 minutes [i.e., from 7.00 a.m. to 7.05 a.m.] = $\frac{360^0}{60} \times 5 = 30°$

Radius of circle = length of minute hand = $\sqrt{21}$ cm

Area swept = $\frac{\theta}{360^0} \times \pi r^2 = \frac{30^0}{360^0} \times \frac{22}{7} \times \sqrt{21} \times \sqrt{21} = \frac{1}{12} \times \frac{22}{7} \times 21 = \frac{11}{2} cm^2 = 5.5 cm^2$

12. A horse is placed for grazing inside a rectangular field 70 m by 52 m and is tethered to one corner by a rope 21 m long. On how much area can it graze?

Ans: Area of the portion that horse can graze = area of the shaded portion.



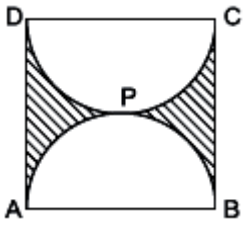
Shaded portion is a sector of radius 21 m = length of the rope

Angle of this sector = angle of the corners of the rectangle = 90°

Area of the shaded portion that horse can graze

$= \frac{\theta}{360^0} \times \pi r^2 = \frac{90^0}{360^0} \times \frac{22}{7} \times (21)^2 = \frac{1}{4} \times \frac{22}{7} \times 21 \times 21 = \frac{11}{2} \times 3 \times 21 m^2 = 346.5 m^2$

13. Find the perimeter of the shaded region in figure, if ABCD is a square of side 14 cm and APB and CPD are semicircles. [Use $\pi = \frac{22}{7}$]



Ans: Perimeter = AD + BC + length of DPC + length of APB

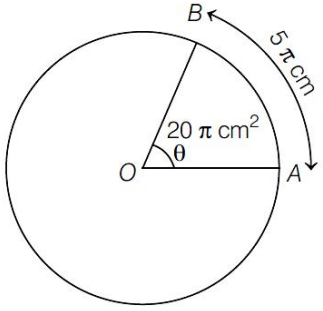
$= 14 + 14 + \pi r + \pi r$

$= 28 + 2 \times \frac{22}{7} \times \frac{14}{2} = 72$ cm

14. An arc of a circle is of length 5π cm and the sector if bounds has an area of 20π cm^2. Find the radius of the circle.

Ans: Given, Arc AB = 5π cm and area of sector OAB = 20π cm^2

$\frac{\theta}{360^0} 2\pi r = 5\pi$ and $\frac{\theta}{360^0} \pi r^2 = 20\pi$

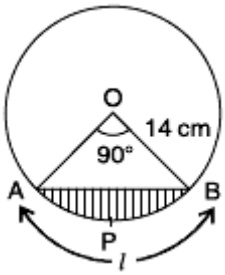


Diving both we get, $\frac{\frac{\theta}{360^0} \pi r^2}{\frac{\theta}{360^0} 2\pi r} = \frac{20\pi}{5\pi} \Rightarrow \frac{r}{2} = 4 \Rightarrow r = 8$ cm

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the area of the segment of a circle of radius 14 cm, if the length of the corresponding arc APB is 22 cm. [Use $\pi = \frac{22}{7}$]



Ans: l = APB = 22 cm

$\frac{\theta}{360^0} \times 2\pi r = 22 \Rightarrow \frac{\theta}{360^0} \times 2 \times \frac{22}{7} \times 14 = 22$

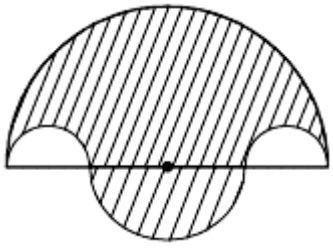
$\Rightarrow \theta = 360^0 \times \frac{7}{22} \times 22 \times \frac{1}{2 \times 14} = \frac{360^0}{4} = 90^0$

Area of the sector = $\frac{lr}{2} = \frac{22 \times 14}{2}$ = 154 cm^2

Area of the triangle AOB = $\frac{1}{2}$ × OA × OB = $\frac{1}{2}$ × 14 × 14 = 98 cm^2

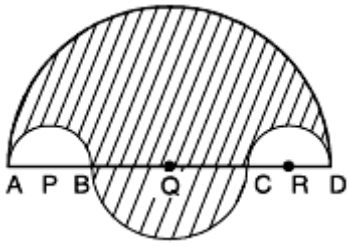
Area of the segment = (154 – 98) cm^2 = 56 cm^2

16. In figure, the boundary of shaded region consists of four semicircular arcs, two smallest being equal. If diameter of the largest is 14 cm and that of the smallest is 3.5 cm, calculate the area of the shaded region. [Use $\pi = \frac{22}{7}$]



Ans: Given AD = 14 cm, AB = CD = 3.5 cm

∴ BC = 7 cm



Area of shaded region = Area of largest semicircle + Area of semicircle on BC – Area of 2 small semicircles

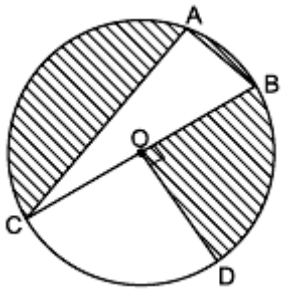
$= \frac{1}{2}\pi(7)^2 + \frac{1}{2}\pi\left(\frac{7}{2}\right)^2 - 2 \times \frac{1}{2}\pi\left(\frac{7}{4}\right)^2$

$= \frac{1}{2}\pi\left[49 + \frac{49}{4} - \frac{49}{8}\right] = \frac{1}{2} \times \frac{22}{7} \times 49 \times \left[1 + \frac{1}{4} - \frac{1}{8}\right]$

$= \frac{1}{2} \times \frac{22}{7} \times 49 \times \frac{9}{8}$

$= 86.63$ sq. cm

17. In the given figure, O is the centre of the circle with AC = 24 cm, AB = 7 cm and $\angle BOD = 90°$. Find the area of the shaded region. [Use $\pi = 3.14$]



Ans: In ΔCAB $\angle CAB = 90°$ (Angle in semicircle)

$\therefore BC^2 = AC^2 + AB^2$

$\Rightarrow BC^2 = (24)^2 + (7)^2$

$BC^2 = 625 \Rightarrow BC = 25$ cm

$\Rightarrow$ Diameter of circle = 25 cm

$\Rightarrow$ Radius = $\frac{25}{2}$ cm

Area of $\Delta DACB = \frac{1}{2} AB \times AC = \frac{1}{2} \times 7 \times 24 = 84$ cm^2

$\because \angle BOD = 90°$

$\therefore \angle COD = 90°$

Area of shaded part = area of circle – Area of ΔCAB – area of quadrant COD

$= \pi r^2 - \frac{1}{2} \times 24 \times 7 - \frac{1}{4} \pi r^2 = \pi r^2 - \frac{1}{4} \pi r^2 - 84$

$= \frac{3}{4} \pi r^2 - 84 = \frac{3}{4} \times 3.14 \times \frac{25}{2} \times \frac{25}{2} - 84$

$= 367.96875 - 84 = 283.96875$ cm^2 $= 283.97$ cm^2

SECTION – D

Questions 18 carry 5 marks.

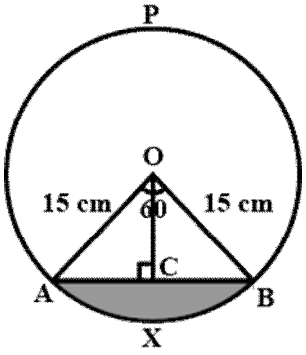
18. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Ans: Here, O is the centre of circle, AB is a chord

AXB is a major arc, OA = OB = radius = 15 cm

Arc AXB subtends an angle 60º at O.

Area of sector $AOB = \frac{60}{360} \times \pi \times r^2 = \frac{60}{360} \times 3.14 \times (15)^2 = 117.75 cm^2$



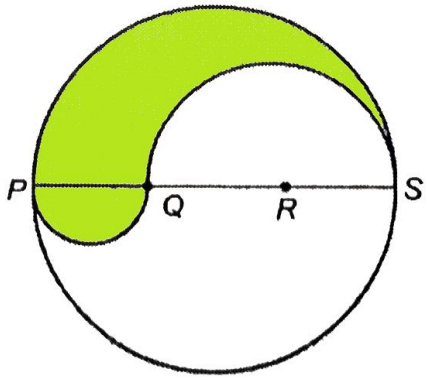
Area of minor segment (Area of Shaded region) = Area of sector AOB − Area of △ AOB
By trigonometry, AC = 15sin30º and OC = 15cos30º
Also, AB = 2AC
∴ AB = 2 × 15sin30º = 15 cm

$$\therefore OC = 15\cos 30^0 = 15\frac{\sqrt{3}}{2} = 15\times\frac{1.73}{2} = 12.975$$

∴ Area of △AOB = 0.5×15×12.975 = 97.3125cm^2
∴ Area of minor segment (Area of Shaded region) =117.75 − 97.3125 = 20.4375 cm^2
Area of major segment = Area of circle − Area of minor segment
= (3.14×15×15) − 20.4375 = 686.0625 cm^2

OR

PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in below figure. Find the perimeter and area of the shaded region (Use π = 3.14)



Ans: Here, PS = 12 cm
as PQ = QR = RS = $\frac{1}{3}$ × PS = $\frac{1}{3}$ ×12 = 4 cm
and QS = 2PQ ⇒ QS = 2 × 4 = 8 cm
Area of shaded region: A = area of a semicircle with PS as diameter + area of a semicircle with PQ as diameter – the area of a semicircle with QS as diameter;

$= \frac{1}{2}$ [3.14 × 6² + 3.14 × 2² - 3.14 × 4²]

$= \frac{1}{2}$ [3.14 × 36 + 3.14 ×4 – 3.14 ×16]

$= \frac{1}{2}$ [3.14 (36 + 4 – 16)]

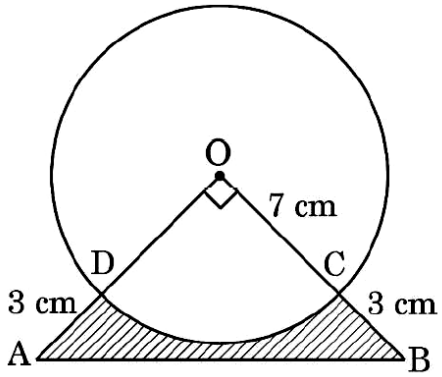
$= \frac{1}{2}$ (3.14 × 24) = $\frac{1}{2}$ × 75.36 = 37.68 cm²

The area of shaded region = 37.68 cm².
The perimeter of the shaded region = Arc of the semicircle of radius 6 +Arc of the semicircle of radius 4 cm + Arc of the semicircle of radius 2 cm
= (6π + 4π + 2π) = 12π
= 12 × 3.14 = 37.68 cm

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. In an annual day function of a school, the organizers wanted to give a cash prize along with a memento to their best students. Each memento is made as shown in the figure and its base ABCD is shown from the front side. The rate of silver plating is 20 per cm^2.



Based on the above, answer the following questions :

(i) What is the area of the quadrant ODCO? (1)

(ii) Find the area of ΔAOB. (1)

(iii) What is the total cost of silver plating the shaded part ABCD? (2)

OR

(iii) What is the length of arc CD? (2)

Ans:

(i)Area of sector ODCO = $\frac{90^0}{360^0}\times\frac{22}{7}\times 7\times 7=\frac{77}{2}=38.5cm^2$

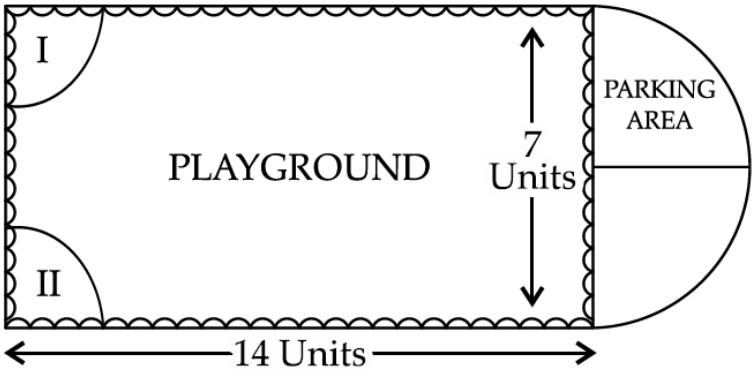
(ii) ar (ΔAOB) = $\frac{1}{2}\times 10\times 10=50cm^2$

(iii) Required cost = (50 – 38·5) × 20 = Rs. 230

OR

(iii) Length of arc CD = $\frac{90^0}{360^0}\times 2\times\frac{22}{7}\times 7=11cm^2$

20. Governing council of a local public development authority of Dehradun decided to build an adventurous playground on the top of a hill, which will have adequate space for parking.



After survey, it was decided to build rectangular playground, with a semi-circular are allotted for parking at one end of the playground. The length and breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants of radius 2 units on one side for special seats.

Based on the above information, answer the following questions:

(i) What is the total perimeter of the parking area?

(ii) (a) What is the total area of parking and the two quadrants?

OR

(b) What is the ratio of area of playground to the area of parking area?

(iii) Find the cost of fencing the playground and parking area at the rate of ` 2 per unit.

Ans: (i) Radius of semi-circle (r) = 7/2 = 3.5 units

Circumference of semi-circle = $\pi r=\frac{22}{7}\times 3.5=11\,\text{units}$

∴ Perimeter of parking area = circumference of semi-circle + diameter of semi-circle

= 11 + 7 = 18 units

(ii) (a) Area of parking = $\frac{1}{2}\pi r^2 = \frac{1}{2}\times\frac{22}{7}\times 3.5\times 3.5 = 11\times 0.5\times 3.5 = 19.25\,\text{unit}^2$

Area of quadrants = 2 × area of one quadrant

= $2\times\frac{1}{4}\pi r_1^2 = \frac{1}{2}\times\frac{22}{7}\times 2\times 2 = 6.285$ unit^2 [$\because r_1 = 2$ units]

Thus, total area = 19.25 + 6.285 = 25.535 unit^2

OR

(b) Area of playground = length × breadth

= 14 × 7

= 98 unit^2

Area of parking = 19.25 unit^2 [from part (ii) a]

∴ Ratio of playground : Ratio of parking area = 98 : 19.25 = 9800/1925 = 56/11

Thus, required ratio is 56 : 11.

(iii) We know that,

Perimeter of parking area = 18 units

Also, Perimeter of playground = 2*(l* + *b*)

= 2(14 + 7) = 2 × 21 = 42 units

Thus, total perimeter of parking area and playground

= 18 + 42 – 7 = 53 units

Hence, total cost = Rs. 2 × 53 = Rs. 106