( $\mathcal{A N S}$ WERS)
$\mathcal{S U B I} E C T: ~ \mathscr{M A T H E M A T I C S}$
MAX. $\mathcal{M A R K S}: 80$
CLASS : $X$
$\mathcal{D U R A \mathcal { A T }} \boldsymbol{O} \mathcal{N}: 3 \mathcal{H R S}$

## General Instruction:

1. This Question Paper has 5 Sections A-E.
2. Section $\mathbf{A}$ has 20 MCQs carrying 1 mark each.
3. Section $\mathbf{B}$ has 5 questions carrying 02 marks each.
4. Section $\mathbf{C}$ has 6 questions carrying 03 marks each.
5. Section $\mathbf{D}$ has 4 questions carrying 05 marks each.
6. Section $\mathbf{E}$ has 3 case based integrated units of assessment ( 04 marks each) with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi=22 / 7$ wherever required if not stated.

## SECTION - A

Questions 1 to 20 carry 1 mark each.

1. If two positive integers $p$ and $q$ can be expressed as $p=a b^{2}$ and $q=a^{3} b ; a, b$ being prime numbers, then $\operatorname{LCM}(p, q)$ is
(a) ab
(b) $a^{2} b^{2}$
(c) $a^{3} b^{2}$
(d) $a^{3} b^{3}$

Ans: (c) $a^{3} b^{2}$
2. The perimeter of a triangle with vertices $(0,4),(0,0)$ and $(3,0)$ is
(a) 5 units
(b) 12 units
(c) 11 units
$(d)(7+\sqrt{ } 5)$ units

Ans: (b) 12 units
3. The zeroes of the polynomial $x^{2}-3 x-m(m+3)$ are
(a) $\mathrm{m}, \mathrm{m}+3$
(b) $-m, m+3$
(c) $m,-(m+3)$
(d) $-\mathrm{m},-(\mathrm{m}+3)$

Ans: (b) $-\mathrm{m}, \mathrm{m}+3$
Let $p(x)=x^{2}-3 x-m(m+3)$
$\Rightarrow \mathrm{p}(\mathrm{x})=\mathrm{x}^{2}-(\mathrm{m}+3) \mathrm{x}+\mathrm{mx}-\mathrm{m}(\mathrm{m}+3)$
$=x\{x-(m+3)\}+m\{x-(m+3)\}$
For zeros of $p(x)$
$\Rightarrow \mathrm{p}(\mathrm{x})=(\mathrm{x}+\mathrm{m})\{(\mathrm{x}-(\mathrm{m}+3)\}=0$
$\Rightarrow \mathrm{x}=-\mathrm{m}, \mathrm{m}+3$
$\therefore$ Its zeros are $-\mathrm{m}, \mathrm{m}+3$.
4. The area of a quadrant of a circle, whose circumference is 22 cm , is
(a) $\frac{11}{8} \mathrm{~cm}^{2}$
(b) $\frac{77}{8} \mathrm{~cm}^{2}$
(c) $\frac{77}{2} \mathrm{~cm}^{2}$
(d) $\frac{77}{4} \mathrm{~cm}^{2}$

Ans: (b) $\frac{77}{8} \mathrm{~cm}^{2}$
5. The pair of linear equations $2 x+3 y=5$ and $4 x+6 y=10$ is
(a) inconsistent
(b) consistent
(c) dependent consistent
(d) none of these

Ans: (c) dependent consistent
6. If the circumference of a circle and the perimeter of a square are equal, then
(a) Area of the circle $=$ Area of the square
(b) Area of the circle > Area of the square
(c) Area of the circle < Area of the square
(d) Nothing definite can be said about the relation between the areas of the circle and square.

Ans: (b) Area of the circle > Area of the square
7. The sum of the lower limit of median class and the upper limit of the modal class of the following data is:

| Marks | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students | 8 | 10 | 12 | 22 | 30 | 18 |

(a) 70
(b) 80
(c) 90
(d) 100

Ans: (b) 80
8. A card is selected at random from a well shuffled deck of 52 cards. The probability of its being a face card is
(a) $3 / 26$
(b) $3 / 13$
(c) $2 / 13$
(d) $1 / 2$

Ans: (b) 3/13
9. In $\Delta \mathrm{ABC}$ right angled at $\mathrm{B}, \sin \mathrm{A}=\frac{7}{25}$, then the value of $\cos \mathrm{C}$ is $\qquad$
(a) $\frac{7}{25}$
(b) $\frac{24}{25}$
(c) $\frac{7}{24}$
(d) $\frac{24}{7}$

Ans: (a) $\frac{7}{25}$
10. The radius of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is
(a) 2.1 cm
(b) 4.2 cm
(c) 3.1 cm
(d) 2.2 cm

Ans: (a) 2.1 cm
The diameter of the largest right circular cone that can be cut out from a cube of edge 4.2 cm .
$\therefore 2 \mathrm{r}=4.2 \mathrm{~cm} \Rightarrow \mathrm{r}=2.1 \mathrm{~cm}$
$\Rightarrow$ Radius of the largest right circular cone $=2.1 \mathrm{~cm}$.
11. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?
(a) 9 units
(b) 6 units
(c) 4.5 units
(d) 18 units

Ans: (a) 9 units
Volume of hemisphere $=$ Surface area of hemisphere
$\Rightarrow \frac{2}{3} \pi r^{3}=3 \pi r^{2} \Rightarrow r=\frac{9}{2}$ units
$\therefore \mathrm{d}=9$ units
12. In the $\triangle A B C, D E \| B C$ and $A D=3 x-2, A E=5 x-4, B D=7 x-5, C E=5 x-3$, then find the value of $x$
(a) 1
(b) $7 / 10$
(c) both (a) \& (b)
(d) none of these

Ans: (c) both (a) \& (b)


Given that, $\mathrm{AD}=3 \mathrm{x}-2, \mathrm{AE}=5 \mathrm{x}-4, \mathrm{BD}=7 \mathrm{x}-5, \mathrm{CE}=5 \mathrm{x}-3$
By Basic Proportionality theorem, we have $\frac{A D}{B D}=\frac{A E}{E C}$
$\Rightarrow \frac{3 x-2}{7 x-5}=\frac{5 x-4}{5 x-3} \Rightarrow(3 x-2)(5 x-3)=(5 x-4)(7 x-5)$
$\Rightarrow 15 x^{2}-19 x+6=35 x^{2}-53 x+20$
$\Rightarrow 20 x^{2}-34 x+14=0 \Rightarrow 10 x^{2}-17 x+7=0$
$\Rightarrow(x-1)(10 x-7)=0 \Rightarrow x=1, \mathrm{x}=\frac{7}{10}$
13. Two circles touch each other externally at C and AB is common tangent of circles, then $\angle \mathrm{ACB}$ is
(a) $70^{\circ}$
(b) $60^{\circ}$
(c) $100^{\circ}$
(d) $90^{\circ}$

Ans: (d) $90^{\circ}$
Draw CM perpendicular to AB .


Now, $\mathrm{AM}=\mathrm{MC}$ and $\mathrm{MB}=\mathrm{MC}$ (tangents drawn from external point are equal).
$\Rightarrow \mathrm{AM}=\mathrm{MC}$
$\Rightarrow \angle \mathrm{MAC}=\angle \mathrm{MCA}=45^{\circ}$
(Since $\triangle$ AMC is right triangle)
$\therefore$ Also, $\mathrm{MB}=\mathrm{MC} \Rightarrow \angle \mathrm{MBC}=\angle \mathrm{MCB}=45^{\circ}$ (Since $\triangle \mathrm{MBC}$ is right angle triangle)
$\therefore \angle \mathrm{ACB}=\angle \mathrm{MCA}+\angle \mathrm{MCB}=45^{\circ}+45^{\circ}=90^{\circ} \Rightarrow \angle \mathrm{ACB}=90^{\circ}$
14. If $5 \tan \theta=4$, then the value of $\frac{5 \sin \theta-3 \cos \theta}{5 \sin \theta+2 \cos \theta}$ is
(a) $1 / 6$
(b) $1 / 7$
(c) $1 / 4$
(d) $1 / 5$

Ans: (a) 1/6
15. Given that $\sin \alpha=1 / 2$ and $\cos \beta=1 / 2$, then the value of $(\beta-\alpha)$ is
(a) $0^{\circ}$
(b) $30^{\circ}$
(c) $60^{\circ}$
(d) $90^{\circ}$

Ans: (b) $30^{\circ}$
16. Two identical solid hemispheres of equal base radius are stuck along their bases. The total surface area of the combination is
(a) $\pi r^{2}$
(b) $2 \pi r^{2}$
(c) $3 \pi r^{2}$
(d) $4 \pi r^{2}$

Ans: (d) $4 \pi r^{2}$
The resultant solid will be a sphere of radius $r$ whose total surface area is $4 \pi r^{2}$.
17. Nature of roots of quadratic equation $2 x^{2}-4 x+3=0$ is
(a) real
(b) equal
(c) not real
(d) none of them

Ans: (c) not real
$D=b^{2}-4 a c=4^{2}-4 \times 2 \times 3=16-24=-8<0$
Since D < 0

Hence, roots are not real.
18. If $\triangle \mathrm{ABC} \sim \triangle \mathrm{EDF}$ and $\triangle \mathrm{ABC}$ is not similar to $\triangle \mathrm{DEF}$, then which of the following is not true?
(a) $\mathrm{BC} . \mathrm{EF}=\mathrm{AC}$. FD
(b) $\mathrm{AB} \cdot \mathrm{EF}=\mathrm{AC}$. DE
(c) $\mathrm{BC} \cdot \mathrm{DE}=\mathrm{AB}$. EF
(d) $\mathrm{BC} \cdot \mathrm{DE}=\mathrm{AB}$. FD

Ans: (c) BC. DE = AB. EF
Since, $\triangle \mathrm{ABC} \sim \triangle \mathrm{EDF}$
Therefore, the ratio of their corresponding sides are equal.
$\Rightarrow \frac{B C}{E F}=\frac{A B}{D E} \Rightarrow B C . D E \neq A B \cdot E F$

## Direction : In the question number 19 \& 20 , A statement of Assertion (A) is followed by a statement of Reason(R). Choose the correct option

19. Assertion (A): The value of $y$ is 3 , if the distance between the points $P(2,-3)$ and $\mathrm{Q}(10, y)$ is 10 .

Reason (R): Distance between two points is given by $\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of Assertion (A)
(c) Assertion (A) is true but reason(R) is false.
(d) Assertion (A) is false but reason( $R$ ) is true.

Ans: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
20. Assertion (A): $6^{\mathrm{n}}$ never ends with the digit zero, where n is natural number.

Reason (R): Any number ends with digit zero, if its prime factor is of the form $2^{m} \times 5^{n}$, where $m, n$ are natural numbers.
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

Ans: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
$6^{\mathrm{n}}=(2 \times 3)^{\mathrm{n}}=2^{\mathrm{n}} \times 3^{\mathrm{n}}$, Its prime factors do not contain 5 i.e., of the form $2^{\mathrm{m}} \times 5^{\mathrm{n}}$,
where $m, n$ are natural numbers.

## SECTION-B <br> Questions 21 to 25 carry 2M each

21. For what values of k will the following pair of linear equations have infinitely many solutions? kx $+3 \mathrm{y}-(\mathrm{k}-3)=0$ and $12 \mathrm{x}+\mathrm{ky}-\mathrm{k}=0$
Ans: Comparing with $a_{1} x+b_{1} y=c_{1}$ and $a_{2} x+b_{2} y=c_{2}$
$a_{1}=k, a_{2}=12, b_{1}=3, b_{2}-k, c_{1}=k-3, c_{2}-k$
For infinite solutions, $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}$
$\therefore \frac{k}{12}=\frac{3}{k}=\frac{k-3}{k} \Rightarrow \frac{k}{12}=\frac{3}{k} \Rightarrow k^{2}=36 \Rightarrow k=6$
22. In the given figure, $\mathrm{AP}=3 \mathrm{~cm}, \mathrm{AR}=4.5 \mathrm{~cm}, \mathrm{AQ}=6 \mathrm{~cm}, \mathrm{AB}=5 \mathrm{~cm}, \mathrm{AC}=10 \mathrm{~cm}$. Find the length of $A D$


Ans: In $\triangle \mathrm{ABC}, \frac{A P}{A B}=\frac{3}{5}$
$\frac{A Q}{A C}=\frac{6}{10}=\frac{3}{5}$
From (i) and (ii), we get $\frac{A P}{A B}=\frac{A Q}{A C} \Rightarrow \mathrm{PQ} \| \mathrm{BC}$
In $\triangle \mathrm{ABD}, \mathrm{PR} \| \mathrm{BD} \Rightarrow \frac{A P}{A B}=\frac{A R}{A D} \Rightarrow \frac{3}{5}=\frac{4.5}{A D} \Rightarrow A D=7.5 \mathrm{~cm}$
23. Two concentric circles are of radii 5 cm and 3 cm . Find the length of the chord of the larger circle which touches the smaller circle.
Ans: Let $O$ be the centre of the concentric circle of radii 5 cm and 3 cm respectively. Let AB be a chord of the larger circle touching the smaller circle at $P$


Then $\mathrm{AP}=\mathrm{PB}$ and $\mathrm{OP} \perp \mathrm{AB}$
Applying Pythagoras theorem in $\triangle \mathrm{OPA}$, we have

$$
\begin{aligned}
& \mathrm{OA}^{2}=\mathrm{OP}^{2}+\mathrm{AP}^{2} \Rightarrow 25=9+\mathrm{AP}^{2} \\
& \Rightarrow \mathrm{AP}^{2}=16 \Rightarrow \mathrm{AP}=4 \mathrm{~cm} \\
& \quad \therefore \mathrm{AB}=2 \mathrm{AP}=8 \mathrm{~cm}
\end{aligned}
$$

24. If $\sin (A+B)=\sqrt{3} / 2$ and $\sin (A-B)=\frac{1}{2}, 0 \leq A+B \leq 90^{\circ}$ and $A>B$, then find $A$ and $B$.

Ans: $\sin (\mathrm{A}+\mathrm{B})=\sqrt{3} / 2=\sin 60^{\circ}$
$\Rightarrow \mathrm{A}+\mathrm{B}=60^{\circ}$
$\sin (\mathrm{A}-\mathrm{B})=1 / 2=\sin 30^{\circ}$
$\Rightarrow \mathrm{A}-\mathrm{B}=30^{\circ}$
Solving eq. (i) and (ii), $\mathrm{A}=45^{\circ}$ and $\mathrm{B}=15^{\circ}$
OR

If $\tan \theta=3 / 4$, evaluate $\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$
Ans: $\tan \theta=\frac{3}{4} \Rightarrow \cot \theta=\frac{4}{3}$
$\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}=\frac{1-\sin ^{2} \theta}{1-\cos ^{2} \theta}=\frac{\cos ^{2} \theta}{\sin ^{2} \theta}=\cot ^{2} \theta=\left(\frac{4}{3}\right)^{2}=\frac{16}{9}$
25. Find the area of the sector of a circle with radius 4 cm and of angle $30^{\circ}$. Also, find the area of the corresponding major sector. (Use $\pi=3.14$ )
Ans: Area of sector $A O B=\frac{\pi r^{2} \theta}{360^{0}}=\frac{3.14 \times 4^{2} \times 30^{0}}{360^{0}}=4.19 \mathrm{~cm}^{2}$
Area of major sector $=$ Area of circle - Area of sector AOB
$=\pi \mathrm{r}^{2}-4.19=3.14 \times 16-4.19=46.1 \mathrm{~cm}^{2}$

## OR

What is the angle subtended at the centre of a circle of radius 10 cm by an arc of length $5 \pi \mathrm{~cm}$ ?
Ans: Arc length of a circle of radius $r=\frac{\theta}{360^{\circ}} \times 2 \pi r$
$\Rightarrow 5 \pi=\frac{\theta}{360^{0}} \times 2 \pi \times 10 \Rightarrow \frac{\theta}{360^{\circ}}=\frac{5 \pi}{20 \pi}=\frac{1}{4}$
$\Rightarrow \theta=90^{\circ}$

## SECTION-C

## Questions 26 to 31 carry 3 marks each

26. Four bells toll at an interval of $8,12,15$ and 18 seconds respectively. All the four begin to toll together. Find the number of times they toll together in one hour excluding the one at the start.
Ans: Prime factorisation of the given numbers are:
$8=2 \times 2 \times 2=2^{3}$
$12=2 \times 2 \times 3=2^{2} \times 3^{1}$
$15=3 \times 5=3^{1} \times 5^{1}$
$18=2 \times 3 \times 3=2^{1} \times 3^{2}$
$\operatorname{LCM}(8,12,15$ and 18$)=2^{3} \times 3^{2} \times 5^{1}=8 \times 9 \times 5=360 \mathrm{sec}=6 \mathrm{~min}$
$\therefore$ Four bells toll together in one hour $=60 \div 6=10$ times.
27. Find the zeroes of the quadratic polynomial $6 x^{2}-3-7 x$ and verify the relationship between the zeroes and the coefficients of the polynomial.
Ans: $6 x^{2}-7 x-3=0$
$\Rightarrow 6 \mathrm{x}^{2}-9 \mathrm{x}+2 \mathrm{x}-3=0$
$\Rightarrow 3 \mathrm{x}(2 \mathrm{x}-3)+1(2 \mathrm{x}-3)=0$
$\Rightarrow(3 \mathrm{x}+1)(2 \mathrm{x}-3)=0$
$\Rightarrow \mathrm{x}=\frac{-1}{3}, \frac{3}{2}$
Now, $\alpha+\beta=\frac{-1}{3}+\frac{3}{2}=\frac{-2+9}{6}=\frac{7}{6}$ and $\frac{-b}{a}=\frac{7}{6} \Rightarrow \alpha+\beta=\frac{-b}{a}$
$\alpha \beta=\frac{-1}{3} \times \frac{3}{2}=\frac{-1}{2}$ and $\frac{c}{a}=\frac{-1}{2} \Rightarrow \alpha \beta=\frac{c}{a}$

## OR

Find the quadratic polynomial sum and product of whose zeros are -1 and -20 respectively. Also find the zeroes of the polynomial so obtained.
Ans: Let $\alpha$ and $\beta$ be the zeros of the quadratic polynomial.
$\therefore$ Sum of zeros, $\alpha+\beta=-1$
and product of zeros, $\alpha . \beta=-20$
Now, quadratic polynomial be
$x^{2}-(\alpha+\beta) \cdot x+\alpha \beta=x^{2}-(-1) x-20=x^{2}+x-20$

Now, for zeroes of this polynomial

$$
\begin{aligned}
& \mathrm{x}^{2}+\mathrm{x}-20=0 \Rightarrow \mathrm{x}^{2}+5 \mathrm{x}-4 \mathrm{x}-20=0 \\
& \Rightarrow \mathrm{x}(\mathrm{x}+5)-4(\mathrm{x}+5)=0 \Rightarrow(\mathrm{x}+5)(\mathrm{x}-4)=0 \\
& \Rightarrow \mathrm{x}=-5,4 \\
& \therefore \text { zeroes are }-5 \text { and } 4
\end{aligned}
$$

28. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.
Ans: Let length and breadth be x and y , Area $=\mathrm{xy}$
1st condition: $(x-5)(y+3)=x y-9$
$\Rightarrow 3 \mathrm{x}-5 \mathrm{y}=6$
2nd condition: $(x+3)(y+2)=x y+67$
$\Rightarrow 2 \mathrm{x}+3 \mathrm{y}=61$
Solve 1st and 2nd equations, we get $\mathrm{x}=17$ and $\mathrm{y}=9$
Hence, Length of rectangle $=17$ units and breadth of rectangle $=9$ units
29. In the below figure, $X Y$ and $X^{\prime} Y^{\prime}$ are two parallel tangents to a circle with centre $O$ and another tangent AB with point of contact C intersecting XY at A and $\mathrm{X}^{\prime} \mathrm{Y}^{\prime}$ at B . Prove that $\angle \mathrm{AOB}=90^{\circ}$.


Ans: Join OC. Since, the tangents drawn to a circle from an external point are equal. $\therefore \mathrm{AP}=\mathrm{AC}$


In $\triangle \mathrm{PAO}$ and $\triangle \mathrm{AOC}$, we have:
$\mathrm{AO}=\mathrm{AO}$ [Common]
$\mathrm{OP}=\mathrm{OC}$ [Radii of the same circle]
$\mathrm{AP}=\mathrm{AC}$
$\Rightarrow \triangle \mathrm{PAO} \cong \triangle \mathrm{AOC}$ [SSS Congruency]
$\therefore \angle \mathrm{PAO}=\angle \mathrm{CAO}=\angle 1$
$\angle \mathrm{PAC}=2 \angle 1$
Similarly $\angle \mathrm{CBQ}=2 \angle 2$
Again, we know that sum of internal angles on the same side of a transversal is $180^{\circ}$.
$\therefore \angle \mathrm{PAC}+\angle \mathrm{CBQ}=180^{\circ}$
$\Rightarrow 2 \angle 1+2 \angle 2=180^{\circ}$ [From (1) and (2)]
$\Rightarrow \angle 1+\angle 2=180^{\circ} / 2=90^{\circ}$

Also $\angle 1+\angle 2+\angle \mathrm{AOB}=180^{\circ}$ [Sum of angles of a triangle]
$\Rightarrow 90^{\circ}+\angle \mathrm{AOB}=180^{\circ}$
$\Rightarrow \angle \mathrm{AOB}=180^{\circ}-90^{\circ} \Rightarrow \angle \mathrm{AOB}=90^{\circ}$.
OR
In the below figure, two equal circles, with centres O and $\mathrm{O}^{\prime}$, touch each other at X . OO' produced meets the circle with centre $\mathrm{O}^{\prime}$ at A . AC is tangent to the circle with centre O , at the point C . $\mathrm{O}^{\prime} \mathrm{D}$ is perpendicular to AC. Find the value of $\frac{D O^{\prime}}{C O}$.


Ans: AC is tangent to circle with centre O .
Thus $\angle \mathrm{ACO}=90^{\circ}$
In $\triangle A^{\prime}$ D and $\triangle \mathrm{AOC}$
$\angle \mathrm{ADO}^{\prime}=\angle \mathrm{ACO}=90^{\circ}$
$\angle \mathrm{A}=\angle \mathrm{A}$ (Common)
$\therefore \triangle \mathrm{AO}{ }^{\prime} \mathrm{D} \sim \triangle \mathrm{AOC}$ (By AA similarity)
$\Rightarrow \frac{A O^{\prime}}{A O}=\frac{D O^{\prime}}{C O}$
Now, $\mathrm{AO}=\mathrm{AO}^{\prime}+\mathrm{O}^{\prime} \mathrm{X}+\mathrm{XO}=3 r$
$\therefore \frac{D O^{\prime}}{C O}=\frac{r}{3 r}=\frac{1}{3}$
30. Prove that $\frac{\sin \theta-\cos \theta+1}{\sin \theta+\cos \theta-1}=\sec \theta+\tan \theta$

$$
\frac{\tan \theta-1+\sec \theta}{\tan \theta+1-\sec \theta} \text { (Dividing numerator and denominator by } \cos \square \text { ) }
$$

$=\frac{\tan \theta+\sec \theta-1}{\tan \theta+1-\sec \theta}$
$=\frac{\tan \theta+\sec \theta-\left(\sec ^{2} \theta-\tan ^{2} \theta\right)}{\tan \theta+1-\sec \theta}$
$=\frac{(\sec \theta+\tan \theta)(1-\sec \theta+\tan \theta)}{\tan \theta+1-\sec \theta}$
$=\sec \theta+\tan \theta=$ RHS
31. Two dice are thrown at the same time. What is the probability that the sum of the two numbers appearing on the top of the dice is (i) 5 ? (ii) 10 ? (iii) at least 9 ?
Ans: Total number of outcomes $=36$
(i) Number of outcomes in which the sum of the two numbers is $5=4$
$\therefore$ Required Probability $=4 / 36=1 / 9$
(ii) Number of outcomes in which the sum of the two numbers is $10=3$
$\therefore$ Required Probability $=3 / 36=1 / 12$
(i) Number of outcomes in which the sum of the two numbers is at least $9=10$
$\therefore$ Required Probability $=10 / 36=5 / 18$

## SECTION-D

## Questions 32 to 35 carry 5M each

32. A motor boat whose speed is $18 \mathrm{~km} / \mathrm{h}$ in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.
Ans: Let the speed of the stream be $\mathrm{xm} / \mathrm{h}$.
Therefore, the speed of the boat upstream $=(18-x) \mathrm{km} / \mathrm{h}$ and the speed of the boat downstream $=$ $(18+\mathrm{x}) \mathrm{km} / \mathrm{h}$.
The time taken to go upstream $=$ distance $/$ speed $=\frac{24}{18+x}$
Similarly, the time taken to go downstream $=\frac{24}{18-x}$
According to the question, $\frac{24}{18-x}-\frac{24}{18+x}=1$
$\Rightarrow 24(18+\mathrm{x})-24(18-\mathrm{x})=(18-\mathrm{x})(18+\mathrm{x})$
$\Rightarrow \mathrm{x}^{2}+48 \mathrm{x}-324=0$
$\Rightarrow x=6$ or -54
Since $x$ is the speed of the stream, it cannot be negative. So, we ignore the root $x=-54$. Therefore, $x=6$ gives the speed of the stream as $6 \mathrm{~km} / \mathrm{h}$.

## OR

An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is $11 \mathrm{~km} / \mathrm{h}$ more than that of the passenger train, find the average speed of the two trains.
Ans: Ans: Let the average speed of the passenger train $=x \mathrm{~km} / \mathrm{hour}$
And the average speed of the express train $=(x+11) \mathrm{km} /$ hour.
The time taken by the passenger train $=\frac{132}{x}$ hour
and the time taken by the express train $=\frac{132}{x+11}$ hour
According to the question, $\frac{132}{x+11}=\frac{132}{x}+1$

$$
\begin{gathered}
\Rightarrow \quad x^{2}+11 x-1452=0 \\
\Rightarrow \quad(x-33)(x+44)=0 \Rightarrow x=33, x=-44
\end{gathered}
$$

The speed cannot be negative, so the speed of the passenger train is $33 \mathrm{~km} /$ hour and the speed of the express train is $33+11=44 \mathrm{~km} /$ hour.
33. State and prove Basic Proportional Theorem.

Ans: Statement - 1 mark
Given, To Prove, Construction and Figure - 2 marks
Correct Proof - 2 marks
34. If the median of the following distribution is 58 and sum of all the frequencies is 140 . What is the value of $x$ and $y$ ?

| Class | $15-25$ | $25-35$ | $35-45$ | $45-55$ | $55-65$ | $65-75$ | $75-85$ | $85-95$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 8 | 10 | $x$ | 25 | 40 | $y$ | 15 | 7 |

Ans:

| Variable | Frequency | c.f. |
| :---: | :---: | :---: |
| $15-25$ | 8 | 8 |
| $25-35$ | 10 | 18 |
| $35-45$ | $x$ | $18+x$ |
| $45-55$ | 25 | $43+x$ |
| $55-65$ | 40 | $83+x$ |
| $65-75$ | $y$ | $83+x+y$ |
| $75-75$ | 15 | $98+x+y$ |
| $85-95$ | 7 | $105+x+y$ |
| Total | $105+x+y$ |  |

And, $105+x+y=140$
$\Rightarrow \quad x+y=35$ (i)
Here, Median $=58$
Then, median class is 55-65, $l=55, \frac{\mathrm{~N}}{2}=\frac{140}{2}=70$
Then, c.f. $=43+x \quad f=40$
Median $=l+\left(\frac{\frac{N}{2}-\text { c.f. }}{f}\right) \times h$
$\Rightarrow 58=55+\left(\frac{70-43-x}{40}\right) \times 10$
$\Rightarrow 3=\frac{27-x}{4} \Rightarrow 12=27-x$
$\Rightarrow x=27-12=15 \Rightarrow y=35-15=20$
35. A toy is in the form of a hemisphere surmounted by a right circular cone of the same base radius as that of the hemisphere. If the radius of the base of the cone is 21 cm and its volume is $2 / 3$ of the volume of the hemisphere, calculate the height of the cone and the surface area of the toy.
Ans: We have, Radius of cone $=$ Radius of hemisphere $=21 \mathrm{~cm}$
$\Rightarrow \mathrm{r}=21 \mathrm{~cm}$


According to question, Volume of cone $=\frac{2}{3} \times$ volume of hemisphere
$\Rightarrow \frac{1}{3} \pi r^{2} h=\frac{2}{3} \times \frac{2}{3} \pi r^{3} \Rightarrow h=\frac{4}{3} \mathrm{r}=\frac{4}{3} \times 21=28 \mathrm{~cm}$
Slant height, $l=\sqrt{\mathrm{r}^{2}+\mathrm{h}^{2}}=\sqrt{21^{2}+28^{2}}=35 \mathrm{~cm}$
Total surface area $=C S A_{\text {cone }}+C S A_{\text {hemisphere }}=\pi r l+2 \pi \mathrm{r}^{2}=\pi r(l+2 \mathrm{r})$
$=\frac{22}{7} \times 21 \times(42+35)=22 \times 3 \times 77=5082 \mathrm{~cm}^{2}$

## OR

A vessel full of water is in the form of an inverted cone of height 8 cm and the radius of its top, which is open, is 5 cm . 100 spherical lead balls are dropped into the vessel. One fourth of the water flows out of the vessel. Find the radius of a spherical ball.
Ans: Height (h) of the cone $=8 \mathrm{~cm}$ and radius $(\mathrm{r})$ of the cone $=5 \mathrm{~cm}$
$\therefore$ Volume of water flows out $=\frac{1}{4} \times$ volume of cone
$=\frac{1}{4} \times \frac{1}{3} \pi r^{2} h=\frac{1}{12} \times \pi \times 25 \times 8$
$\therefore$ Volume of water flows out $=100 \times$ volume of spherical ball

$$
\begin{aligned}
& \Rightarrow \frac{1}{12} \times \pi \times 25 \times 8=100 \times \frac{4}{3} \pi R^{3} \\
& \Rightarrow R^{3}=\frac{1}{8} \Rightarrow R=\frac{1}{2} c m=0.5 \mathrm{~cm}
\end{aligned}
$$

## SECTION-E (Case Study Based Questions) <br> Questions 36 to 38 carry 4M each

36. The top of a table is shown in the figure given below:



On the basis of above information answer the following questions.
(i) Find the distance between points A and B.
(ii) Write the co-ordinates of the mid point of line segment joining points M and Q .
(iii) If G is taken as the origin, and x , y axis put along GF and GB , then find the point denoted by coordinates $(4,2)$ and $(8,4)$.

OR
Find the coordinates of $\mathrm{H}, \mathrm{G}$ and also find the distance between them.
Ans: (i) Distance between $\mathrm{A}(1,9)$ and $\mathrm{B}(5,13)$ is
$=\sqrt{(5-1)^{2}+(13-9)^{2}}=\sqrt{16+16}=\sqrt{32}=4 \sqrt{2}$ units
(ii) Midpoint of the line segment joining $\mathrm{M}(5,11)$ and $\mathrm{Q}(9,3)$ is given by $\left(\frac{5+9}{2}, \frac{11+3}{2}\right)=\left(\frac{14}{2}, \frac{14}{2}\right)=(7,7)$
(iii) If G is $(0,0)$ then Q is $(4,2)$ and E is $(8,4)$.

## OR

As per graph the coordinate of H is $(1,5)$ and of G is $(5,1)$.
Distance HG $=\sqrt{(5-1)^{2}+(1-5)^{2}}=\sqrt{16+16}=\sqrt{32}=4 \sqrt{2}$ units
37. Ananya saves Rs. 24 during the first month Rs. 30 in the second month and Rs. 36 in the third month. She continues to save in this manner.


On the basis of above information answer the following questions.
(i) Whether the monthly savings of Ananya form an AP or not? If yes then write the first term and common difference.
(ii) What is the amount that she will save in 15 th month?
(iii) In which month, will she save Rs. 66?

## OR

What is the common difference of an AP whose nth term is $8-5 n$ ?
Ans: (i) Savings of Ananya are Rs. 24, Rs. 30, Rs. 36, ...
Since it is uniformly increasing by Rs. 6, therefore it forms an AP.
Here, $\mathrm{a}=24, \mathrm{~d}=30-24=6$
(ii) $\mathrm{a}_{15}=\mathrm{a}+14 \mathrm{~d}=24+14 \times 6=24+84=$ Rs. 108
(iii) $\mathrm{a}_{\mathrm{n}}=66 \Rightarrow \mathrm{a}+(\mathrm{n}-1) \mathrm{d}=66$
$\Rightarrow 24+(\mathrm{n}-1) 6=66 \Rightarrow \mathrm{n}-1=42 / 6=7 \Rightarrow \mathrm{n}=8$

## OR

$\mathrm{a}_{\mathrm{n}}=8-5 \mathrm{n}$
$a_{1}=8-5=3$
$\mathrm{a}_{2}=8-10=-2 \Rightarrow \mathrm{~d}=\mathrm{a}_{2}-\mathrm{a}_{1}=-2-3=-5$
38. A person/observer on the sea coast observes two ships in the sea, both the ships are in same straight path one behind the other.
If the observer is on his building of height 20 meters (including observer) and he observes the angle of depression of two ships as $45^{\circ}$ and $60^{\circ}$ respectively.


On the basis of above information answer the following questions.
(i) If a person observes a ship whose angle of depression is $60^{\circ}$ then how much distance is the ship away from the building?
(ii) If a person observes another ship whose angle of depression is $45^{\circ}$ then how much distance that ship is away from the building?
(iii) If a person observes the ship whose angle of depression changes from $60^{\circ}$ to $30^{\circ}$ then how far be ship from the building if the observer is at 20 m of height (including him)?

## OR

At a time when a person observes two ships whose angle of depressions are $60^{\circ}$ and $45^{\circ}$ the distance between the ships is (in meter).
Ans: (i) $\tan 60^{\circ}=\frac{O C}{A C} \Rightarrow \sqrt{3}=\frac{20}{A C} \Rightarrow A C=\frac{20}{\sqrt{3}}=\frac{20 \sqrt{3}}{3} m=11.55 \mathrm{~m}$

(ii) $\tan 45^{\circ}=\frac{O C}{B C} \Rightarrow 1=\frac{20}{B C} \Rightarrow B C=20 \mathrm{~m}$

(iii) $\tan 30^{\circ}=\frac{O B}{O A} \Rightarrow \frac{1}{\sqrt{3}}=\frac{20}{O A} \Rightarrow O A=20 \sqrt{3} \mathrm{~m}$


Distance between two ships $20 \mathrm{~m}=\mathrm{BC}-\mathrm{AC}=20-11.55=8.45 \mathrm{~m}$

