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**SAMPLE PAPER TEST 03 FOR BOARD EXAM 2024**  
**(ANSWERS)**

**SUBJECT: MATHEMATICS**  
**CLASS : X**

**MAX. MARKS : 80**  
**DURATION : 3 HRS**

**General Instruction:**

1. This Question Paper has 5 Sections A-E.
2. **Section A** has 20 MCQs carrying 1 mark each.
3. **Section B** has 5 questions carrying 02 marks each.
4. **Section C** has 6 questions carrying 03 marks each.
5. **Section D** has 4 questions carrying 05 marks each.
6. **Section E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take  $\pi = 22/7$  wherever required if not stated.

**SECTION – A**

**Questions 1 to 20 carry 1 mark each.**

1. A card is selected from a deck of 52 cards. The probability of being a red face card is  
(a)  $3/26$                       (b)  $3/13$                       (c)  $2/13$                       (d)  $1/2$   
Ans: (a)  $3/26$   
Total number of red face cards = 6  
 $\therefore$  Probability of being a red face card =  $6/52 = 3/26$
2. If two tangents inclined at an angle of  $60^\circ$  are drawn to a circle of radius 3cm, then the length of each tangent is equal to  
(a)  $\frac{3\sqrt{3}}{2}$  cm                      (b) 3 cm                      (c) 6 cm                      (d)  $3\sqrt{3}$   
Ans: (d)  $3\sqrt{3}$
3. If the mean of a frequency distribution is 8.1 and  $\sum f_i = 20$ ,  $\sum f_i x_i = 132 + 5k$ , then k =  
(a) 3                                      (b) 4                                      (c) 5                                      (d) 6  
Ans: (d) 6
4. If the radii of two circles are in the ratio of 4 : 3, then their areas are in the ratio of :  
(a) 4 : 3                                      (b) 8 : 3                                      (c) 16 : 9                                      (d) 9 : 16  
Ans: (c) 16 : 9
5. If one zero of the quadratic polynomial  $x^2 + 3x + k$  is 2, then the value of k is  
(a) 10                                      (b) -10                                      (c) 5                                      (d) -5  
Ans: (b) -10
6. If two positive integers a and b are written as  $a = x^3 y^2$  and  $b = x y^3$ ; x, y are prime numbers, then HCF (a, b) is  
(a) xy                                      (b)  $xy^2$                                       (c)  $x^3 y^3$                                       (d)  $x^2 y^2$   
Ans: (b)  $xy^2$
7. When 2120 is expressed as the product of its prime factors we get  
(a)  $2 \times 5^3 \times 53$                       (b)  $2^3 \times 5 \times 53$                       (c)  $5 \times 7^2 \times 31$                       (d)  $5^2 \times 7 \times 33$   
Ans: (b)  $2^3 \times 5 \times 53$

8. In the  $\triangle ABC$ , D and E are points on side AB and AC respectively such that  $DE \parallel BC$ .  
If  $AE = 2$  cm,  $AD = 3$  cm and  $BD = 4.5$  cm, then CE equals  
(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm  
Ans: (c) 3 cm

9. If the distance between the points  $(2, -2)$  and  $(-1, x)$  is 5, one of the values of x is  
(a) -2 (b) 2 (c) -1 (d) 1  
Ans: (b) 2

Let us consider the points as  $A = (2, -2)$  and  $B = (-1, x)$

$AB = 5$  units

$$\Rightarrow 5^2 = (-1 - 2)^2 + (x + 2)^2 \Rightarrow 25 = (-3)^2 + (x + 2)^2$$

$$\Rightarrow 25 = 9 + x^2 + 4 + 4x \Rightarrow 25 = x^2 + 4x + 13$$

$$\Rightarrow x^2 + 4x + 13 - 25 = 0 \Rightarrow x^2 + 4x - 12 = 0$$

$$\Rightarrow x^2 + 6x - 2x - 12 = 0 \Rightarrow x(x + 6) - 2(x + 6) = 0$$

$$\Rightarrow (x + 6)(x - 2) = 0 \Rightarrow x + 6 = 0 \Rightarrow x = -6$$

$$\text{And } x - 2 = 0 \Rightarrow x = 2$$

Therefore, one of the values of x is 2.

10. The value of k for which the pair of equation  $kx - y = 2$  and  $6x - 2y = 3$  has unique solution  
(a)  $k = 3$  (b)  $k \neq 3$  (c)  $k \neq 0$  (d)  $k = 0$

Ans: (b)  $k \neq 3$

For unique solution, we have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{k}{6} \neq \frac{-1}{-2} \Rightarrow \frac{k}{6} \neq \frac{1}{2} \Rightarrow k \neq 3$$

11. The median class of the following data is:

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
No. of students	8	10	12	22	30	18

- (a) 20 - 30 (b) 30 - 40 (c) 40 - 50 (d) 50 - 60

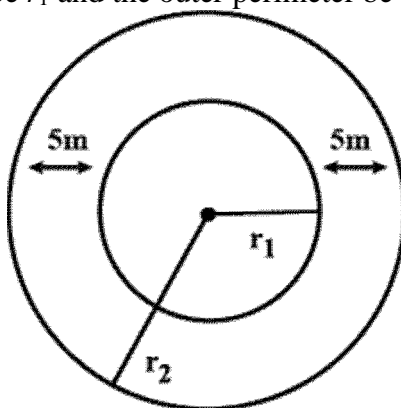
Ans: (b) 30 - 40

12. The ratio of outer and inner perimeters of circular path is 23:22. If the path is 5 m wide, the diameter of the inner circle is

- (a) 55 m (b) 110 m (c) 220 m (d) 230 m

Ans: (c) 220 m

Let the radius of inner perimeter be  $r_1$  and the outer perimeter be  $r_2$



$$\text{Now, } \frac{2r_2\pi}{2r_1\pi} = \frac{23}{22} \Rightarrow r_2 = \frac{23}{22}r_1$$

According to the question,  $r_2 - r_1 = 5$

$$\Rightarrow \frac{23}{22}r_1 - r_1 = 5 \Rightarrow r_1 = 110 \text{ \& } r_2 = 115$$

$\therefore$  diameter of the inner circle =  $2 \times 110 = 220$  m

13. In  $\triangle ABC$ , right angled at B,  $AB = 5$  cm and  $\sin C = 1/2$ . Determine the length of side AC.  
(a) 10 cm (b) 15 cm (c) 20 cm (d) none of these

Ans: (a) 10 cm

14. If  $x^2 + k(4x + k - 1) + 2 = 0$  has equal roots, then  $k = \dots\dots\dots$

- (a)  $-\frac{2}{3}, 1$  (b)  $\frac{2}{3}, -1$  (c)  $\frac{3}{2}, \frac{1}{3}$  (d)  $\frac{3}{2}, -\frac{1}{3}$

Ans: (b)  $\frac{2}{3}, -1$

15. If  $x = a \cos \theta$  and  $y = b \sin \theta$ , then the value of  $b^2x^2 + a^2y^2$  is

- (a)  $a^2 + b^2$  (b)  $a^2/b^2$  (c)  $a^2b^2$  (d) None of these

Ans: (c)  $a^2b^2$

We have,  $b^2x^2 + a^2y^2 = b^2 \times a^2 \cos^2 \theta + a^2 \times b^2 \sin^2 \theta$   
 $= a^2b^2 (\cos^2 \theta + \sin^2 \theta) = a^2b^2 \times 1 = a^2b^2$

16. ABCD is a trapezium with  $AD \parallel BC$  and  $AD = 4$ cm. If the diagonals AC and BD intersect each other at O such that  $AO/OC = DO/OB = 1/2$ , then  $BC =$

- (a) 6cm (b) 7cm (c) 8cm (d) 9cm

Ans: (c) 8cm

17. The value of  $(\sin 45^\circ + \cos 45^\circ)$  is

- (a)  $1\sqrt{2}$  (b)  $\sqrt{2}$  (c)  $\sqrt{3}/2$  (d) 1

Ans: (b)  $\sqrt{2}$

18. Volumes of two spheres are in the ratio 64:27. The ratio of their surface areas is

- (a) 3:4 (b) 4:3 (c) 9:16 (d) 16:9

Ans. (d) 16:9

**Direction : In the question number 19 & 20 , A statement of Assertion (A) is followed by a statement of Reason(R) . Choose the correct option**

19. **Statement A (Assertion):** If product of two numbers is 5780 and their HCF is 17, then their LCM is 340

**Statement R( Reason) :** HCF is always a factor of LCM

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Ans: (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

20. **Assertion (A):** The point (0, 4) lies on y-axis.

**Reason (R):** The y co-ordinate of the point on x-axis is zero.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of Assertion (A)

(c) Assertion (A) is true but reason(R) is false.

(d) Assertion (A) is false but reason(R) is true.

Ans: (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of Assertion (A)

The x co-ordinate of the point (0, 4) is zero and y co-ordinate of the point on x-axis is zero.  
 $\therefore$  Point (0, 4) lies on y-axis.

**SECTION-B**  
**Questions 21 to 25 carry 2M each**

- 21.** If the system of equations  $2x + 3y = 7$  and  $(a + b)x + (2a - b)y = 21$  has infinitely many solutions, then find a and b.

Ans: Given system of equations

$$2x + 3y = 7 \dots(i)$$

$$(a + b)x + (2a - b)y = 21 \dots(ii)$$

Equations have infinitely many solutions, if  $\frac{2}{a+b} = \frac{3}{2a-b} = \frac{7}{21} = \frac{1}{3} \Rightarrow \frac{2}{a+b} = \frac{1}{3}$

$$\Rightarrow 6 = a + b \Rightarrow a + b = 6 \dots(i)$$

$$\text{and } \frac{3}{2a-b} = \frac{1}{3} \Rightarrow 2a - b = 9 \dots(ii)$$

On solving equation (i) and (ii), we get  $a = 5, b = 1$

- 22.** Simplify:  $\frac{\tan^2 \theta}{1 + \tan^2 \theta} + \frac{\cot^2 \theta}{1 + \cot^2 \theta}$

$$\begin{aligned} \text{Ans: } \frac{\tan^2 \theta}{1 + \tan^2 \theta} + \frac{\cot^2 \theta}{1 + \cot^2 \theta} &= \frac{\tan^2 \theta}{\sec^2 \theta} + \frac{\cot^2 \theta}{\operatorname{cosec}^2 \theta} \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} \times \frac{\cos^2 \theta}{1} + \frac{\cos^2 \theta}{\sin^2 \theta} \times \frac{\sin^2 \theta}{1} = \sin^2 \theta + \cos^2 \theta = 1 \end{aligned}$$

**OR**

If  $7 \sin^2 A + 3 \cos^2 A = 4$ , then find  $\tan A$

Ans: Given,  $7 \sin^2 A + 3 \cos^2 A = 4$

Dividing both sides by  $\cos^2 A$ , we get

$$7 \tan^2 A + 3 = 4 \sec^2 A \quad [\because \sec^2 \theta = 1 + \tan^2 \theta]$$

$$\Rightarrow 7 \tan^2 A + 3 = 4(1 + \tan^2 A)$$

$$\Rightarrow 7 \tan^2 A + 3 = 4 + 4 \tan^2 A$$

$$\Rightarrow 3 \tan^2 A = 1 \Rightarrow \tan^2 A = 1/3 \Rightarrow \tan A = 1/\sqrt{3}$$

- 23.** If the perimeter of a protractor is 72 cm, calculate its area. (Use  $\pi = \frac{22}{7}$ )

$$\text{Ans: } (\pi r + 2r) = \frac{36r}{7} \Rightarrow \frac{36r}{7} = 72 \Rightarrow r = 14$$

$$\text{Now area of the protractor is } \frac{\pi r^2}{2} = \frac{11}{7} \times 14 \times 14 = 22 \times 14 = 308 \text{ cm}^2$$

**OR**

Two circular pieces of equal radii and maximum area, touching each other are cut out from a Rectangular card board of dimensions 14 cm  $\times$  7 cm. Find the area of the remaining card board. [Use  $\pi = 22/7$ ]

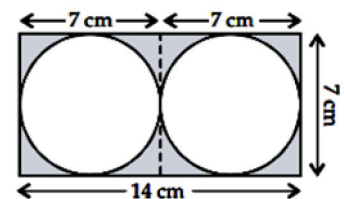
Ans: Here,  $r = 7/2$  cm,  $l = 14$  cm,  $b = 7$  cm

Area of the remaining card board

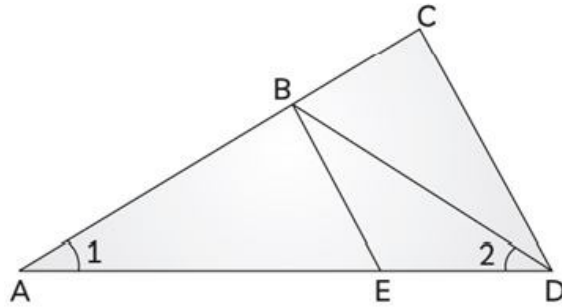
$$= \text{area of rectangle} - 2 (\text{area of circle})$$

$$= l \times b - 2 \pi r^2$$

$$= 14 \times 7 - 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 98 - 77 = 21 \text{ cm}^2$$



- 24.** In the given figure below,  $AD/AE = AC/BD$  and  $\angle 1 = \angle 2$ . Show that  $\Delta BAE \sim \Delta CAD$ .



Ans: In  $\triangle ABC$ ,  $\angle 1 = \angle 2$

$\therefore AB = BD$  .....(i)

Given,  $\frac{AD}{AE} = \frac{AC}{BD}$

Using equation (i), we get  $\frac{AD}{AE} = \frac{AC}{AB}$  .....(ii)

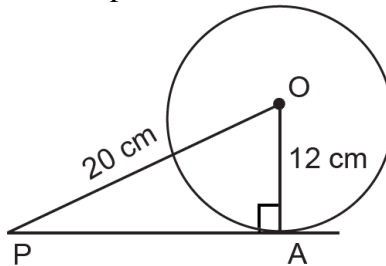
In  $\triangle BAE$  and  $\triangle CAD$ , by equation (ii),  $\frac{AC}{AB} = \frac{AD}{AE}$

and  $\angle A = \angle A$  (common)

$\therefore \triangle BAE \sim \triangle CAD$  [By SAS similarity criterion]

25. Find the length of the tangent from an external point P at a distance of 20 cm from the centre of a circle of radius 12 cm.

Ans: Let PA be the tangent to the circle of point A.



Here,  $OP = 20$  cm and  $OA = 12$  cm

Since PA is tangent of A, therefore  $OA \perp PA$ .

In right angled triangle  $\triangle OAB$ ,  $OP^2 = PA^2 + OA^2$  (Using Pythagoras Theorem)

$$\Rightarrow (20)^2 = PA^2 + (12)^2 \Rightarrow PA^2 = 400 - 144 = 256$$

$$\Rightarrow PA = 16 \text{ cm}$$

Hence, Length of tangent be 16 cm.

### SECTION-C

#### Questions 26 to 31 carry 3 marks each

26. Two numbers are in the ratio of 1 : 3. If 5 is added to both the numbers, the ratio becomes 1 : 2. Find the numbers.

Ans: Let two numbers are x and y respectively such that its fraction =  $x/y$ .

According to the question,  $\frac{x}{y} = \frac{1}{3}$

$$\Rightarrow 3x = y \Rightarrow y = 3x \dots(i)$$

Also,  $\frac{x+5}{y+5} = \frac{1}{2}$

$$\Rightarrow 2x + 10 = y + 5 \Rightarrow 2x - y = -5 \dots(ii)$$

Putting the value of  $y=3x$  in (ii), we get

$$2x - 3x = -5 \Rightarrow -x = -5 \Rightarrow x = 5$$

Putting the value of  $x = 5$  in (i), we get  $y = 3 \times 5 = 15$   
Hence, Numbers are 5 and 15.

**OR**

A train covered a certain distance at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr; it would have taken 6 hours more than the scheduled time. Find the length of the journey.

Ans: Let the actual speed of the train be  $x$  km/hr and let the actual time taken be  $y$  hours.

Distance covered is  $xy$  km If the speed is increased by 6 km/hr, then time of journey is reduced by 4 hours i.e., when speed is  $(x+6)$ km/hr, time of journey is  $(y-4)$  hours.

$$\therefore \text{Distance covered} = (x + 6)(y - 4)$$

$$\Rightarrow xy = (x + 6)(y - 4) \Rightarrow -4x + 6y - 24 = 0 \Rightarrow -2x + 3y - 12 = 0 \quad \dots\dots\dots(i)$$

$$\text{Similarly } xy = (x - 6)(y + 6) \Rightarrow 6x - 6y - 36 = 0 \Rightarrow x - y - 6 = 0 \quad \dots\dots\dots(ii)$$

Solving (i) and (ii) we get  $x=30$  and  $y=24$

Putting the values of  $x$  and  $y$  in equation (i), we obtain

$$\text{Distance} = (30 \times 24)\text{km} = 720\text{km.}$$

Hence, the length of the journey is 720km.

**27.** Given that  $\sqrt{3}$  is irrational, prove that  $2 + 5\sqrt{3}$  is irrational.

Ans: Let us assume  $2 + 5\sqrt{3}$  is rational, then it must be in the form of  $p/q$  where  $p$  and  $q$  are co-prime integers and  $q \neq 0$

i.e  $2 + 5\sqrt{3} = p/q$

$$\text{So } \sqrt{3} = \frac{p - 2q}{5q} \quad \dots(i)$$

Since  $p, q, 5$  and  $2$  are integers and  $q \neq 0$ , RHS of equation (i) is rational.

But LHS of (i) is  $\sqrt{3}$  which is irrational. This is not possible. This contradiction has arisen due to our wrong assumption that  $2 + 5\sqrt{3}$  is rational

So,  $2 + 5\sqrt{3}$  is irrational.

**28.** Two dice are thrown at the same time. What is the probability that the sum of the two numbers appearing on the top of the dice is (i) 7? (ii) 14? (iii) equal to 12?

Ans: (i)  $P(\text{sum of the numbers is } 7) = 6/36 = 1/6$

(ii)  $P(\text{sum of the numbers is } 14) = 0/36 = 0$

(iii)  $P(\text{sum of the numbers is } 12) = 1/36$

**29.** If  $\tan \theta = \frac{a}{b}$ , prove that  $\frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} = \frac{a^2 - b^2}{a^2 + b^2}$

$$\text{Ans: } LHS = \frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} = \frac{a \frac{\sin \theta}{\cos \theta} - b}{a \frac{\sin \theta}{\cos \theta} + b} = \frac{a \tan \theta - b}{a \tan \theta + b}$$

$$= \frac{a \tan \theta - b}{a \tan \theta + b} = \frac{a \times \frac{a}{b} - b}{a \times \frac{a}{b} + b} = \frac{\frac{a^2 - b^2}{b}}{\frac{a^2 + b^2}{b}} = \frac{a^2 - b^2}{a^2 + b^2} = RHS$$

**30.** If the zeroes of the polynomial  $x^2 + px + q$  are double in value to the zeroes of  $2x^2 - 5x - 3$ , then find the values of  $p$  and  $q$

Ans:

Let  $\alpha$  and  $\beta$  be the zeroes of  $2x^2 - 5x - 3$ .

$$\therefore \alpha + \beta = \frac{5}{2}, \quad \alpha\beta = \frac{-3}{2}$$

As per the question,

It is given that  $2\alpha, 2\beta$  are the zeroes of  $x^2 + px + q$

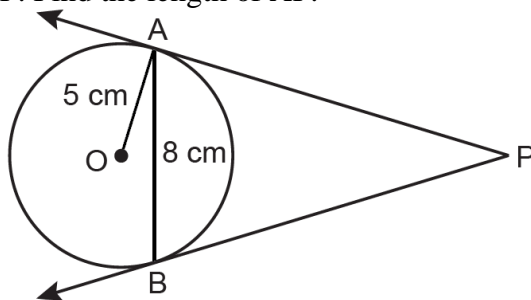
$$\therefore 2\alpha + 2\beta = -p$$

$$\Rightarrow 2(\alpha + \beta) = -p \Rightarrow 2 \times \frac{5}{2} = -p \Rightarrow p = -5$$

$$\text{Also, } (2\alpha)(2\beta) = q \Rightarrow 4\alpha\beta = q$$

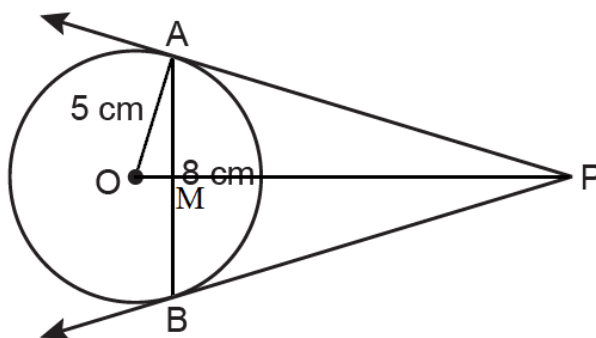
$$\Rightarrow \alpha\beta = \frac{q}{4} \Rightarrow \frac{-3}{2} = \frac{q}{4} \Rightarrow q = -6$$

31. In the given figure, AB is a chord of length 8 cm of a circle of radius 5 cm. The tangents to the circle at A and B intersect at P. Find the length of AP.



$$\text{Ans: } AB = 8 \text{ cm} \Rightarrow AM = 4 \text{ cm}$$

$$\therefore OM = \sqrt{(5^2 - 4^2)} = 3 \text{ cm}$$



$$\text{Let } AP = y \text{ cm, } PM = x \text{ cm}$$

$\therefore \triangle OPA$  is a right angle triangle

$$\therefore OP^2 = OA^2 + AP^2$$

$$(x + 3)^2 = y^2 + 25$$

$$\Rightarrow x^2 + 9 + 6x = y^2 + 25 \dots(i)$$

$$\text{Also, } x^2 + 42 = y^2 \dots(ii)$$

$$\Rightarrow x^2 + 6x + 9 = x^2 + 16 + 25$$

$$\Rightarrow 6x = 32$$

$$\Rightarrow x = \frac{32}{6} = \frac{16}{3} \text{ cm}$$

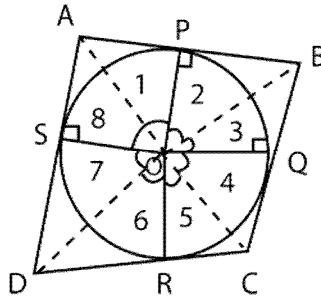
$$\therefore y^2 = x^2 + 16 = \frac{256}{9} + 16 = \frac{400}{9}$$

$$\Rightarrow y = \frac{20}{3} \text{ cm}$$

**OR**

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans: Let ABCD be the quadrilateral circumscribing a circle at the center O such that it touches the circle at the point P, Q, R, S. Let join the vertices of the quadrilateral ABCD to the center of the circle



In  $\triangle OAP$  and  $\triangle OAS$

$AP=AS$  ( Tangents from to same point A)

$PO=OS$  ( Radii of the same circle)

$OA=OA$  ( Common side)

so,  $\triangle OAP=\triangle OAS$  (SSS congruence criterion)

$\therefore \angle POA=\angle AOS$  (CPCT)

$\Rightarrow \angle 1=\angle 8$

Similarly,  $\angle 2=\angle 3$ ,  $\angle 4=\angle 5$  and  $\angle 6=\angle 7$

$\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7+\angle 8=360^\circ$

$\Rightarrow (\angle 1+\angle 8)+(\angle 2+\angle 3)+(\angle 4+\angle 5)+(\angle 6+\angle 7)=360^\circ$

$\Rightarrow 2(\angle 1)+2(\angle 2)+2(\angle 5)+2(\angle 6)=360^\circ$

$\Rightarrow (\angle 1)+(\angle 2)+(\angle 5)+(\angle 6)=180^\circ$

$\therefore \angle AOD+\angle COD=180^\circ$

Similarly,  $\angle BOC+\angle DOA=180^\circ$

### SECTION-D

#### Questions 32 to 35 carry 5M each

32. A motorboat whose speed in still water is 9 km/h, goes 15km downstream and comes back to the same spot, in a total time of 3 hours 45 minutes. Find the speed of the stream.

Ans: Let speed of stream be x km/h.

Given, Speed of boat = 9 km/h

Distance covered upstream = 15 km

Distance covered downstream = 15 km

Total time taken = 3 hours 45 minutes =  $15/4$  hours

Now, Speed of boat upstream =  $9-x$  km/h

Speed of boat downstream =  $9+x$  km/h

According to the question,  $\frac{15}{9+x} + \frac{15}{9-x} = \frac{15}{4}$

$$\Rightarrow \frac{1}{9+x} + \frac{1}{9-x} = \frac{1}{4} \Rightarrow \frac{9-x+9+x}{(9+x)(9-x)} = \frac{1}{4} \Rightarrow \frac{18}{(9+x)(9-x)} = \frac{1}{4}$$

$$\Rightarrow \frac{18}{81-x^2} = \frac{1}{4}$$

$$\Rightarrow 81-x^2 = 72$$

$$\Rightarrow x^2 = 9$$

$\Rightarrow x = 3$  (x is the speed of the stream and thus cannot have negative value)

Thus, the speed of the stream is 3 km/hr.

OR

A takes 6 days less than the time taken by B to finish a piece of work. If both A And B together can finish it in 4 days, find the time taken by B to finish the work.

Ans: Let B takes a total of x days to complete the work alone.

So as know that A takes 6 days less than B we can write that A takes

x - 6 days to complete the work alone.



$$\text{Work done by B in a day} = \frac{1}{x}$$

$$\text{Work done by A in a day} = \frac{1}{x-6}$$

$$\text{According to the question, } \frac{1}{x} + \frac{1}{x-6} = \frac{1}{4} \Rightarrow \frac{x-6+x}{x(x-6)} = \frac{1}{4} \Rightarrow \frac{2x-6}{x(x-6)} = \frac{1}{4}$$

$$\Rightarrow 8x - 24 = x^2 - 6x$$

$$\Rightarrow x^2 - 14x + 24 = 0$$

$$\Rightarrow x^2 - 12x - 2x + 24 = 0$$

$$\Rightarrow x(x-12) - 2(x-12) = 0$$

$$\Rightarrow x = 2, 12$$

We will reject  $x = 2$  as  $x - 6$  will become negative.

Hence, B takes 12 days to complete the work alone.

33. Prove that if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio.

Using the above theorem prove that a line through the point of intersection of the diagonals and parallel to the base of the trapezium divides the non parallel sides in the same ratio.

**Ans:** For the Theorem :

Given, To prove, Construction and figure of 1½ marks

Proof of 1½ marks

Let ABCD be a trapezium  $DC \parallel AB$  and EF is a line parallel to AB and hence to DC.

Join AC, meeting EF in G.

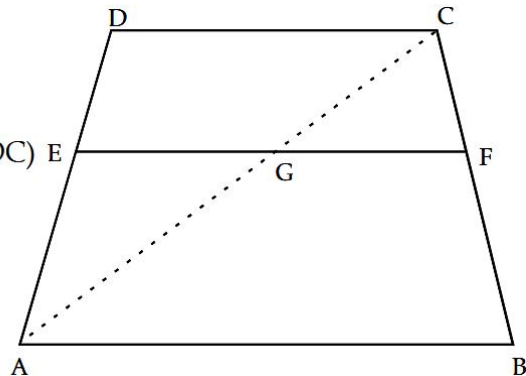
In  $\triangle ABC$ , we have  $GF \parallel AB$

$$\frac{CG}{GA} = \frac{CF}{FB} \quad [\text{By BPT}] \dots(1)$$

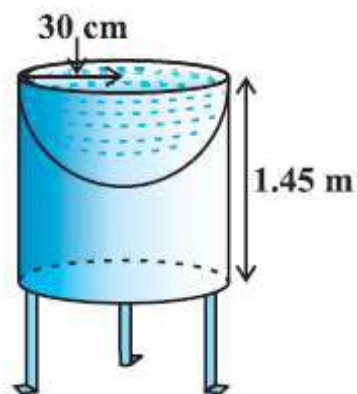
In  $\triangle ADC$ , we have  $EG \parallel DC$  ( $EF \parallel AB$  &  $AB \parallel DC$ )

$$\frac{DE}{EA} = \frac{CG}{GA} \quad [\text{By BPT}] \dots(2)$$

From (1) & (2), we get,  $\frac{DE}{EA} = \frac{CF}{FB}$



34. Ramesh made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath.



Ans: Let  $h$  be height of the cylinder, and  $r$  the common radius of the cylinder and hemisphere.

Then, the total surface area = CSA of cylinder + CSA of hemisphere

$$= 2\pi rh + 2\pi r^2 = 2\pi r (h + r)$$

$$= 2 \times \frac{22}{7} \times 30 (145 + 30) \text{ cm}^2$$

$$= 2 \times \frac{22}{7} \times 30 \times 175 \text{ cm}^2$$

$$= 33000 \text{ cm}^2 = 3.3 \text{ m}^2$$

**OR**

A tent is in shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1m and 4m respectively and the slant height of the top is 2.8m. Find the area of canvas used for making the tent. Also find the cost of canvas of the tent at the rate of 500 per m<sup>2</sup>.

Ans: Radius = 2m, Slant height l = 2.8m, height h = 2.1m

Cost of canvas per m<sup>2</sup> = Rs.500

Area of canvas used = CSA of cone + CSA of cylinder

$$= \pi r l + 2\pi r h$$

$$= 22/7 \times 2 \times 2.8 + 2 \times 22/7 \times 2 \times 2.1$$

$$= 17.6 + 26.4$$

$$= 44 \text{ m}^2$$

Cost of the canvas of tent = 44 x 500

$$= \text{Rs.}22,000$$

35. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	Number of policy holders
Below 20	2
20 – 25	4
25 – 30	18
30 – 35	21
35 – 40	33
40 – 45	11
45 – 50	3
50 – 55	6
55 – 60	2

Ans:

Age (in years)	Number of policy holders	cf
Below 20	2	2
20 – 25	4	6
25 – 30	18	24
30 – 35	21	45
35 – 40	33	78
40 – 45	11	89
45 – 50	3	92
50 – 55	6	98
55 – 60	2	100

Here, n = 100 ⇒ n/2 = 50, therefore median class is 35 – 40

So, l = 35, cf = 45, f = 33, h = 5

$$\text{Now, Median} = l + \left( \frac{\frac{n}{2} - cf}{f} \times h \right) = 35 + \left( \frac{50 - 45}{33} \times 5 \right) = 35 + \left( \frac{5}{33} \times 5 \right)$$

$$= 35 + \frac{25}{33} = 35 + 0.76 = 35.76$$

Hence, median age is 35.76 years

**SECTION-E (Case Study Based Questions)**  
**Questions 36 to 38 carry 4M each**

36. Aditya is a fitness freak and great athlete. He always wants to make his nation proud by winning medals and prizes in the athletic activities.



An upcoming activity for athletes was going to be organised by Railways. Aditya wants to participate in 200 m race. He can currently run that distance in 51 seconds. But he wants to increase his speed, so to do it in 31 seconds. With each day of practice, it takes him 2 seconds less.

- (i) He wants to make his best time as 31 sec. In how many days will he be able to achieve his target?  
(ii) What will be the difference between the time taken on 5th day and 7th day.

**OR**

- (ii) Which term of the arithmetic progression 3, 15, 27, 39 .... will be 120 more than its 21st term?

Ans: Ans:

- (i) Let, the number of days taken to achieve the target be  $n$ .

In the given A.P.,  $a = 51$ ,  $d = -2$

$$\text{Since } a_n = a + (n - 1)d \Rightarrow 31 = 51 + (n - 1)(-2)$$

$$\Rightarrow 31 - 51 = (n - 1)(-2) \Rightarrow -20 = (n - 1)(-2)$$

$$\Rightarrow (n - 1) = 10 \Rightarrow n = 11$$

Hence, 11 days are needed to achieve the target.

$$(ii) a_5 = a + 4d = 51 + 4(-2) = 51 - 8 = 43 \text{ sec}$$

$$a_7 = a + 6d = 51 + 6(-2) = 51 - 12 = 39 \text{ sec}$$

$$\text{Now, time difference} = 43 - 39 = 4 \text{ sec.}$$

**OR**

- (ii) We have,  $a = 3$  and  $d = 12$

$$\therefore a_{21} = a + 20d = 3 + 20 \times 12 = 243$$

Let  $n$ th term of the given AP be 120 more than its 21st term. Then,  $a_n = 120 + a_{21}$

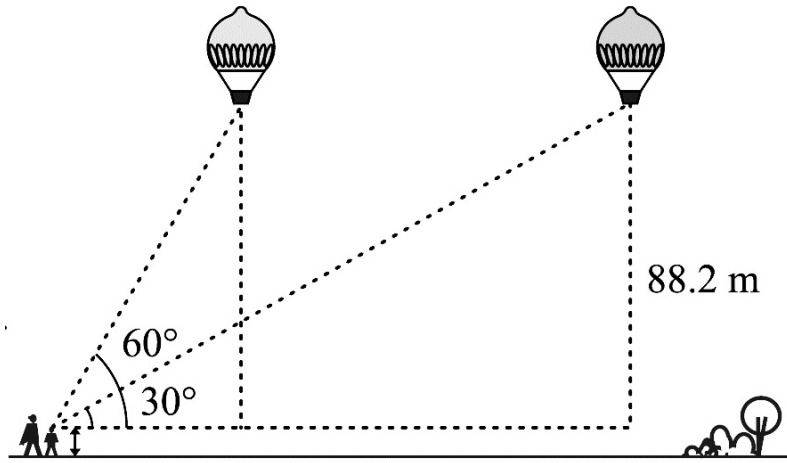
$$\Rightarrow 3 + (n - 1)d = 120 + 243$$

$$\Rightarrow 3 + 12(n - 1) = 363 \Rightarrow 12(n - 1) = 360$$

$$\Rightarrow n - 1 = 30 \Rightarrow n = 31$$

Hence, 31st term of the given AP is 120 more than its 21st term.

37. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is  $60^\circ$ . After 30 seconds, the angle of elevation reduces to  $30^\circ$  (see the below figure).



Based on the above information, answer the following questions. (Take  $\sqrt{3} = 1.732$ )

- (i) Find the distance travelled by the balloon during the interval. (2)  
 (ii) Find the speed of the balloon. (2)

**OR**

- (ii) If the elevation of the sun at a given time is  $30^\circ$ , then find the length of the shadow cast by a tower of 150 feet height at that time. (2)

Ans: (i) In the figure, let C be the position of the observer (the girl).

A and P are two positions of the balloon.

CD is the horizontal line from the eyes of the (observer) girl.

Here  $PD = AB = 88.2 \text{ m} - 1.2 \text{ m} = 87 \text{ m}$

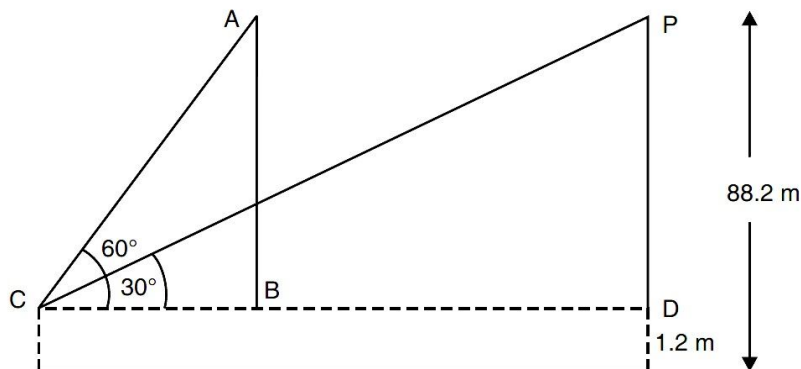
In right  $\triangle ABC$ , we have  $\frac{AB}{BC} = \tan 60^\circ$

$$\Rightarrow \frac{87}{BC} = \sqrt{3} \Rightarrow BC = \frac{87}{\sqrt{3}} \text{ m}$$

In right  $\triangle PDC$ , we have  $\frac{PD}{CD} = \tan 30^\circ$

$$\Rightarrow \frac{87}{CD} = \frac{1}{\sqrt{3}} \Rightarrow CD = 87\sqrt{3}$$

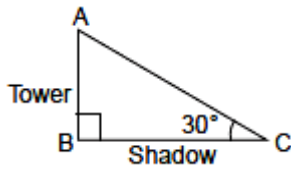
$$\text{Now, } BD = CD - BC = 87\sqrt{3} - \frac{87}{\sqrt{3}} = 58\sqrt{3} \text{ m}$$



Thus, the required distance between the two positions of the balloon =  $58\sqrt{3} \text{ m}$   
 $= 58 \times 1.732 = 100.46 \text{ m}$  (approx.)

(ii) Speed of the balloon = Distance/time =  $100.46/30 = 3.35 \text{ m/s}$  (approx.)

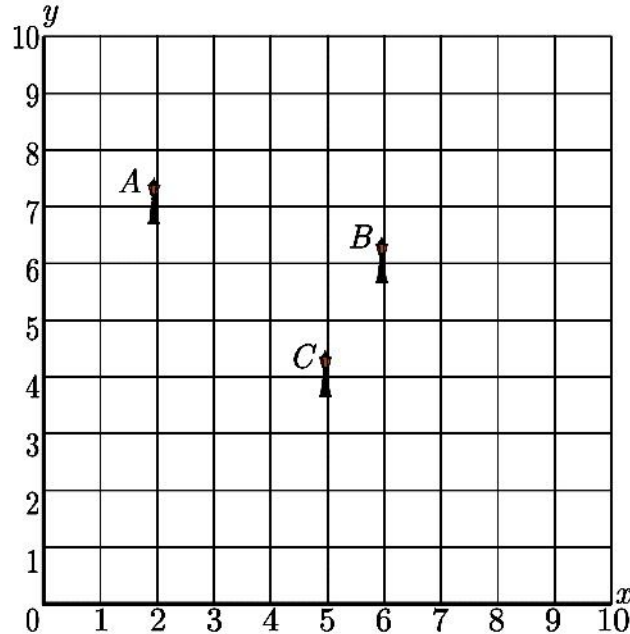
**OR**



In right  $\triangle ABC$

$$\frac{AB}{BC} = \tan 30^\circ \Rightarrow \frac{150}{BC} = \frac{1}{\sqrt{3}} \Rightarrow BC = 150\sqrt{3} \text{ feet}$$

38. Resident Welfare Association (RWA) of a Gulmohar Society in Delhi have installed three electric poles A, B and C in a society's common park. Despite these three poles, some parts of the park are still in dark. So, RWA decides to have one more electric pole D in the park. The park can be modelled as a coordinate systems given below.



On the basis of the above information, answer any four of the following questions:

- (i) What is the position of the pole C? (1)
- (ii) What is the distance of the pole B from the corner O of the park? (1)
- (iii) Find the position of the fourth pole D so that four points A, B, C and D form a parallelogram. (2)

**OR**

- (iii) What is the distance between poles A and C? (2)

**Ans:** (i) From the given diagram we can easily get that position of the pole C (5, 4).

(ii) Coordinates of B is (6, 6).

$$\text{Distance from origin} = \sqrt{(6-0)^2 + (6-0)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

(iii) If ABCD is a parallelogram, the diagonals bisect each other. Here AC and BD are diagonals.

$$\text{Mid-point of AC} = \left( \frac{2+5}{2}, \frac{7+4}{2} \right) = (3.5, 5.5)$$

Now, mid-point of diagonal, BD will be (3.5, 5.5) also.

Let, the coordinates of D be (x, y)

$$\text{Now, } \frac{6+x}{2} = 3.5, \frac{6+y}{2} = 5.5 \Rightarrow x = 1 \text{ and } y = 5$$

**OR**

(iii) Coordinates of A are (2, 7) and coordinates of C are (5, 4).

$$\text{Distance between pole A and C, } AC = \sqrt{(5-2)^2 + (4-7)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$