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SAMPLE PAPER TEST 05 FOR BOARD EXAM 2024
(ANSWERS)

SUBJECT: MATHEMATICS | **MAX. MARKS : 80**
CLASS : X | **DURATION : 3 HRS**

General Instruction:

1. This Question Paper has 5 Sections A-E.
2. **Section A** has 20 MCQs carrying 1 mark each.
3. **Section B** has 5 questions carrying 02 marks each.
4. **Section C** has 6 questions carrying 03 marks each.
5. **Section D** has 4 questions carrying 05 marks each.
6. **Section E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

SECTION – A
Questions 1 to 20 carry 1 mark each.

1. Three cubes each of side 15 cm are joined end to end. The total surface area of the cuboid is:
(a) 3150 cm^2 (b) 1575 cm^2 (c) 1012.5 cm^2 (d) 576.4 cm^2
Ans. (a) 3150 cm^2

2. The midpoint of a line segment joining two points A(2, 4) and B(-2, -4) is
(a) (-2, 4) (b) (2, -4) (c) (0, 0) (d) (-2, -4)
Ans: (c) (0, 0)
As per midpoint formula, we know;
x-coordinate of the midpoint = [2 + (-2)]/2 = 0/2 = 0
y-coordinate of the midpoint = [4 + (-4)]/2=0/2=0
Hence, (0, 0) is the midpoint of AB.

3. If the distance between the points A(2, -2) and B(-1, x) is equal to 5, then the value of x is:
(a) 2 (b) -2 (c) 1 (d) -1
Ans: (a) 2
$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=5 \Rightarrow \sqrt{(-1-2)^2+(x+2)^2}=5 \Rightarrow \sqrt{9+(x+2)^2}=5$$
$\Rightarrow 9 + (x + 2)^2 = 25$
$\Rightarrow (2 + x)^2 = 16 \Rightarrow 2 + x = 4 \Rightarrow x = 2$

4. If cos A = 4/5, then the value of tan A is
(a) 3/5 (b) 3/4 (c) 4/3 (d) 5/3
Ans: (b) 3/4

5. If $\cos\theta + \cos^2\theta = 1$, the value of $\sin^2\theta + \sin^4\theta$ is :
(a) –1 (b) 0 (c) 1 (d) 2
Ans: (c) 1

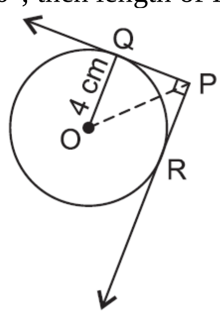
6. The HCF and the LCM of 12, 21, 15 respectively are
(a) 3, 140 (b) 12, 420 (c) 3, 420 (d) 420, 3
Ans: (c) 3, 420
Here, 12 = 22 × 3

21 = 3 × 7
15 = 3 × 5
So, HCF = 3; LCM = 22 × 3 × 7 i.e., 420

7. If the sum of LCM and HCF of two numbers is 1260 and their LCM is 900 more than their HCF, then the product of two numbers is
(a) 205400 (b) 203400 (c) 194400 (d) 198400
Ans: (c) 194400
Let the HCF of the numbers be x and their LCM be y.
It is given that the sum of the HCF and LCM is 1260, therefore
x + y = 1260 ....(i)
And, LCM is 900 more than HCF.
y = x + 900..... (ii)
Substituting (ii) in (i), we get x + x + 900 =1260
⇒ 2x + 900 = 1260
⇒ 2x = 1260 - 900 ⇒ 2x = 360 ⇒ x = 180
Substituting x = 180 in (1), we get:
y = 180 + 900 ⇒ y = 1080
We also know that the product the two numbers is equal to the product of their LCM and HCF
Thus, product of the numbers = 1080(180) = 194400

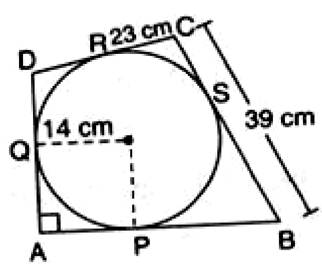
8. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and -3, then
(a) a = -7, b = -1 (b) a = 5, b = -1 (c) a = 2, b = –6 (d) a = 0, b = –6
Ans. (d) a = 0, b = –6

9. In the given figure, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If ∠QPR = 90°, then length of PQ is



(a) 3 cm (b) 4 cm (c) 2 cm (d) 2.2 cm
Ans: (b) 4 cm

10. In the given figure, quadrilateral ABCD is circumscribed, touching the circle at P, Q, R and S such that ∠DAB = 90°, If CR = 23 cm and CB = 39 cm and the radius of the circle is 14 cm, then the measure of AB is

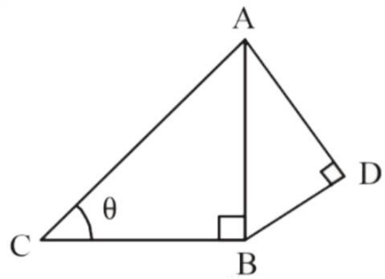


(a) 37 cm (b) 16cm (c) 30 cm (d) 39 cm
Ans: (c) 30 cm
∵ Tangent is perpendicular to the radius through the point of contact.
∠OQA = ∠OPA = 90° and OQ = OP [Radii]
∴ OQAP is a square.

⇒ AP = 14cm
Now, CR = CS = 23 cm [Tangents from an external point to a circle are equal]
∴ BS = 39 - 23 = 16cm
And BS = BP = 16 cm [Tangents from an external point to a circle are equal]
Now, AB = AP + BP = 14 + 16 = 30cm

11. If the circumference of a circle increases from 2π to 4π then its area ......the original area :
(a) Half (b) Double (c) Three times (d) Four times
Ans: (d) Four times

12. In the figure given below, AD = 4 cm, BD = 3 cm and CB = 12 cm, then cot θ equals :



(a) 3/4 (b) 5/12 (c) 4/3 (d) 12/5
Ans: (d) 12/5

13. The perimeters of two similar triangles are 26 cm and 39 cm. The ratio of their areas will be :
(a) 2 : 3 (b) 6 : 9 (c) 4 : 6 (d) 4 : 9
Ans: (d) 4 : 9

14. If ΔABC ~ ΔEDF and ΔABC is not similar to ΔDEF, then which of the following is not true?
(a) BC.EF = AC.FD (b) AB.EF = AC.DE (c) BC.DE = AB.EF (d) BC.DE = AB.FD
Ans. (c) BC.DE = AB.EF

15. The radii of 2 cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. Then, the ratio of their volumes is:
(a) 19 : 20 (b) 20 : 27 (c) 18:25 (d) 17:23
Ans: (b) 20 : 27

16. Consider the following frequency distribution

Class	0 – 5	6 – 11	12 – 17	18 – 23	24 – 29
Frequency	13	10	15	8	11

The upper limit of the median class is
(a) 7 (b) 17.5 (c) 18 (d) 18.5
Ans: (b) 17.5

17. Consider the following distribution:

Marks obtained	**Number of students**
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

the frequency of the class 30-40 is
(a) 4 (b) 48 (c) 51 (d) 3
Ans: (d) 3

18. Two dice are thrown simultaneously. The probability that the product of the numbers appearing on the dice is 7 is

(a) 7/36 (b) 2/36 (c) 0 (d) 1/36

Ans: (c) 0

Direction : In the question number 19 & 20 , A statement of Assertion (A) is followed by a statement of Reason(R) . Choose the correct option

19. Assertion (A): The mid-point of the line segment joining the points A (3, 4) and B (k, 6) is P (x, y) and x + y – 10 = 0, the value of k is 7

Reason (R): Midpoint of line segment is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true and R is false

(d) A is false and R is true

Ans: (b) Both A and R are true but R is not the correct explanation of A

20. Assertion (A): For any two positive integers a and b, HCF(a, b) x LCM(a, b) = a x b

Reason (R): The HCF of two numbers is 5 and their product is 150. Then their LCM is 40.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of Assertion (A)

(c) Assertion (A) is true but reason(R) is false.

(d) Assertion (A) is false but reason(R) is true.

Ans: (c) Assertion (A) is true but reason(R) is false.

LCM(a, b) x HCF(a, b) =a x b

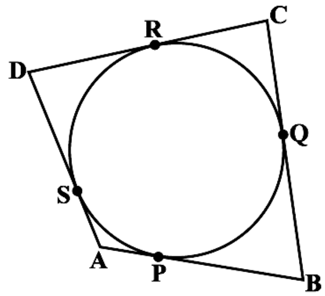
⇒ LCM x 5 = 150

⇒ LCM = 150/5 = 30 ⇒ LCM = 30

SECTION-B

Questions 21 to 25 carry 2M each

21. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.



Ans: We know that the lengths of the tangents drawn from an external point to the circle are equal.

DR = DS ...... (i)

BP = BQ ...... (ii)

AP = AS ...... (iii)

CR = CQ ...... (iv)

Adding (i), (ii), (iii), (iv), we get DR + BP + AP + CR = DS + BQ + AS + CQ

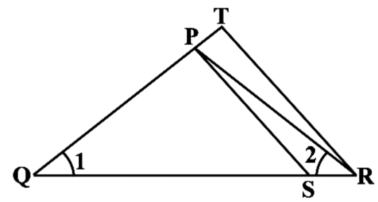
By rearranging the terms we get,

(DR + CR) + (BP + AP) = (CQ + BQ) + (DS + AS)

⇒ CD + AB = BC + AD

Hence it is proved AB + CD = AD + BC.

22. In the figure, $\frac{QR}{QS}=\frac{QT}{PR}$ and ∠1 = ∠2, Show that ΔPQS ~ ΔTQR.



Ans: In △PQR,

Since, ∠1=∠2

∴ PR=PQ (Opposite sides of equal angles are equal) .....(1)

In △PQS and △TQR, $\frac{QR}{QS}=\frac{QT}{PR}$.... $(Given)$

i.e., $\frac{QR}{QS}=\frac{QT}{PQ}$....($From$ 1)

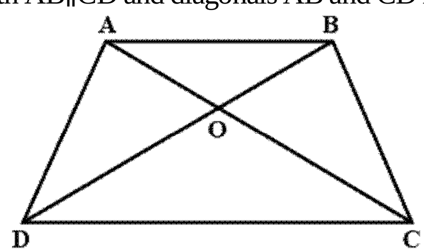
Also, ∠Q is common

∴ By SAS criterion of similarity, △PQS~△TQR.

OR

ABCD is a trapezium in which AB || CD and its diagonals intersect each other at the point O. Using a similarity criterion of two triangles, show that $\frac{OA}{OC}=\frac{OB}{OD}$

Ans: ABCD is a trapezium with AB||CD and diagonals AB and CD intersecting at O.



In △OAB and △OCD

∠AOB =∠DOC [Vertically opposite angles]

∠ABO = ∠CDO [Alternate angles]

∠BAO = ∠OCD [Alternate angles]

∴ △OAB ~ △OCD [AAA similarity]

We know that if triangles are similar, their corresponding sides are in proportional

$\therefore \frac{OA}{OC}=\frac{OB}{OD}$

23. If sin(A – B) = $\frac{1}{2}$, cos(A + B) = $\frac{1}{2}$, $0^0 < A + B \le 90^0$, A > B. Find A and B.

Ans: $\sin(A-B)=\frac{1}{2}\Rightarrow \sin(A-B)=30^\circ\left[\because \sin 30^\circ=\frac{1}{2}\right]$

On equating both sides

$\text{A}-\text{B}=30^\circ\ldots(1)$

$\cos(A+B)=\frac{1}{2}\Rightarrow \cos(A-B)=\cos(60^\circ)\left[\because \cos(60^\circ)=\frac{1}{2}\right]$

On equating both sides

$A+B=60^\circ..(2)$

Adding (1) and (2),we get $2A = 90^0 \Rightarrow A = 45^0$
Putting value of A in (i)
$45^0 + B = 60^0 \Rightarrow B = 15^0$

24. Find the value of p if the pair of equations $2x + 3y – 5 = 0$ and $px – 6y – 8 = 0$ has a unique solution.
Ans: Given, pair of linear equations is $2x + 3y – 5 = 0$ and $px – 6y – 8 = 0$
On comparing with $ax + by + c = 0$ we get
Here, $a_1 = 2$, $b_1 = 3$, $c_1 = – 5$;
And $a_2 = p$, $b_2 = – 6$, $c_2 = – 8$;
$a_1 /a_2 = 2/p$, $b_1 /b_2 = – 3/6 = – ½$, $c_1 /c_2 = 5/8$
Since, the pair of linear equations has a unique solution,
$\therefore a_1/a_2 \neq b_1/b_2$
$\Rightarrow 2/p \neq – ½$
$\Rightarrow p \neq – 4$
Hence, the pair of linear equations has a unique solution for all values of p except – 4.

25. The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 2 days
Ans: In 2 days, the short hand will complete 4 rounds.
∴ Distance moved by its tip = 4(circumference of a circle of radius 4 cm)
$$= 4 \times \left(2 \times \frac{22}{7} \times 4 \right) cm = \frac{704}{7} cm$$
In 2 days, the long hand will complete 48 rounds.
∴ Distance moved by its tip = 48(circumference of a circle of radius 6 cm)
$$= 48 \times \left(2 \times \frac{22}{7} \times 6 \right) cm = \frac{12.672}{7} cm$$
Hence, sum of distances moved by the tips of two hands of the clock $= \left(\frac{704}{7} + \frac{12672}{7} \right) cm$
= 1910.85 cm

OR

A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle of 120°. Find the total area cleaned at each sweep of the blades
Ans: Here, r = 21 cm, θ = 120°
Area of a sector = $\frac{\theta}{360^0} \times \pi r^2 = \frac{120^0}{360^0} \times \frac{22}{7} \times 21 \times 21$
= 462 cm^2
∴ Total area cleaned by two wipers
= 2 × 462 = 924 cm^2

SECTION-C
Questions 26 to 31 carry 3 marks each

26. 4 Bells toll together at 9.00 am. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours?
Ans: 7 = 7 × 1
8 = 2 × 2 × 2
11 = 11 × 1
12 = 2 × 2 × 3
∴ LCM of 7, 8, 11, 12 = 2 × 2 × 2 × 3 × 7 × 11 = 1848
∴ Bells will toll together after every 1848 sec.

∴ In next 3 hrs, number of times the bells will toll together = $\frac{3\times3600}{1848}$ = 5.84 = 5 times.

OR

Given that √3 is irrational, prove that (2 + 5√3) is an irrational number.

Ans: Let 2 + 5√3 be a rational number such that

2 + 5√3 = *a*, where *a* is a non-zero rational number.

$\Rightarrow 5\sqrt{3} = a - 2 \Rightarrow \sqrt{3} = \frac{a-2}{5}$

Since 5 and 2 are integers and a is a rational number, therefore $\frac{a-2}{5}$ is a rational number

⇒ √3 is a rational number which contradicts the fact that √3 is an irrational number.

Therefore, our assumption is wrong.

Hence 2 + 5√3 is an irrational number

27. Find the ratio in which the line 2x + y – 4 = 0 divides the line segment joining the points A (2, –2) and B (3, 7)

Ans: Let P(x, y) be the point on the line 2x + y – 4 = 0 dividing the line segment joining the points A(2, –2) and B(3, 7) in the ratio k : 1.

∴ The coordinate of P are $\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

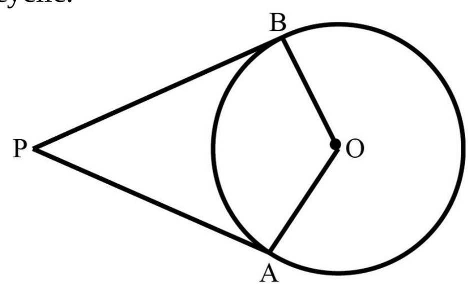
Since, point (x, y) lies on the line 2x + y = 4.

$\Rightarrow 2\left(\frac{3k+2}{k+1}\right) + \left(\frac{7k-2}{k+1}\right) = 4 \Rightarrow \frac{6k+4+7k-2}{k+1} = 4$

⇒ 13k + 2 = 4k + 4 ⇒ 9k = 2 ⇒ k = 2/9

Thus, required ratio is 2 : 9.

28. In the given figure, PA and PB are the tangent segments to a circle with centre O. Show that the points A, O, B and P are concyclic.



Ans: Here, OA = OB

And OA ⊥ AP, OB ⊥ BP (tangent ⊥ radius)

∴ ∠OAP = 90°,

∠OBP = 90°

∴ ∠OAP + ∠OBP = 90° + 90° = 180°

∴ ∠AOB + ∠APB = 180°

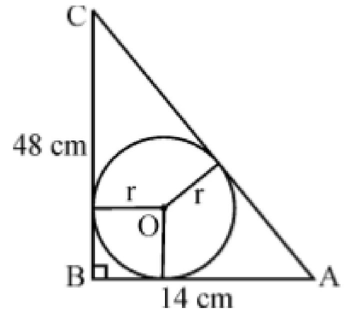
(Since, ∠AOB + ∠OAP + ∠OBP + ∠APB = 360°)

Thus, sum of opposite angle of a quadrilateral is 180°.

Hence, A, O, B and P are concyclic.

OR

In the given figure, ABC is a triangle in which ∠B = 90°, BC = 48 cm and AB = 14 cm. A circle is inscribed in the triangle, whose centre is O. Find radius r of in-circle.



Ans: In ΔABC, $AC^2 = AB^2 + BC^2 = \sqrt{(14)^2 + (48)^2} = 50\,cm$
∠OQB = 90°⇒ OPBQ is a square ⇒ BQ = r
⇒ QA = 14 - r = AR (tangents from a external point are equal in length)
Again PB = r ⇒ PC = 48 - r
⇒ RC = 48 - r (tangents from a external point are equal in length)
⇒ AR + RC = AC
⇒ 14 - r + 48 - r = 50
⇒ r = 6cm.

29. From a pack of 52 playing cards, jacks, queens, kings and aces of red colour are removed. From the remaining a card is drawn at random. Find the probability that the card drawn is (i) a black queen (ii) a red card (iii) a face card.
Ans: From the total playing 52 cards, red coloured jacks, queen, kings and aces are removed(i.e., 2 jacks, 2 queens, 2 kings, 2 aces) ∴ Remaining cards = 52 – 8 = 44
(i) Favourable cases for a black queen are 2 (i.e., queen of club or spade)
∴ Probability of drawing a black queen = 2/44 = 1/22
(ii) Favourable cases for red cards are 26 – 8 = 18 (as 8 cards have been removed) (i.e.9 diamonds + 9 hearts)
∴ Probability of drawing a red card = 18/44 = 9/22
(iii) Favourable cases for a face card are 6 (i.e. 2 black jacks, queens and kings each)
∴ Probability of drawing a face card = 6/44 = 3/22

30. If a, b are the zeroes of the polynomial $2x^2 – 5x + 7$, then find a polynomial whose zeroes are 2a + 3b, 3a + 2b
Ans: Since a, b are the zeroes of $2x^2 – 5x + 7$
∴ $a + b = \frac{-(-5)}{2} = \frac{5}{2}$ and $ab = \frac{7}{2}$
The given zeroes of required polynomial are 2a + 3b and 3a + 2b
Sum of the zeroes = 2a + 3b + 3a + 2b = 5a + 5b = 5(a + b) = $5 \times \frac{5}{2} = \frac{25}{2}$
Again, product of the zeroes = (2a + 3b) (3a + 2b) = $6(a^2 + b^2) + 13ab$
= $6[(a + b)^2 – 2ab)] + 13ab = 6(a + b)^2 + ab$
= 6(5/2)2 + 7/2 = 75/2 + 7/2 = 82/2 = 41
Now, required polynomial is given by
k [x^2 – (Sum of the zeroes) x + Product of the zeroes] = k [$x^2 - \frac{25}{2}x + 41$]
= $\frac{k}{2}[2x^2 - 25x + 82]$, where k is any non-zero real number.
Hence the required polynomial is $2x^2 - 25x + 82$.

31. Prove that $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

Ans: $LHS = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$= \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)} = \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{\cos A(1+\sin A)}$

$= \frac{2+2\sin A}{\cos A(1+\sin A)} = \frac{2}{\cos A} = 2\sec A = RHS$

SECTION-D

Questions 32 to 35 carry 5M each

32. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Compute the missing frequencies f_1 and f_2.

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	f_1	10	f_2	7	8

Ans:

Class	Frequency	Class mark (x)	*u*	*fu*
0-20	5	10	-1	-5
20-40	f_1	30	0	0
40-60	10	50	1	10
60-80	f_2	70	2	$2f_2$
80-100	7	90	3	21
100-120	8	110	4	32
Total	$f_1 + f_2 + 30$			$2f_2 + 58$

$f_1 + f_2 + 30 = 50 \Rightarrow f_1 + f_2 = 20$

$\Sigma fu = 2f_2 + 58$, $h = 20$, A = 30

$Mean = A + \left(\frac{\sum fu}{\sum f} \times h\right) \Rightarrow 62.8 = 30 + \left(\frac{2f_2 + 58}{50} \times 20\right)$

$\Rightarrow 62.8 - 30 = \frac{2f_2 + 58}{5} \times 2 \Rightarrow 32.8 \times 5 = 2(2f_2 + 58)$

$\Rightarrow 4f_2 + 116 = 164 \Rightarrow 4f_2 = 164 - 116 = 48 \Rightarrow f_2 = 12$

$\Rightarrow f_1 = 20 - 12 = 8$

33. A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.

Ans: Let original speed of train = x km/h

We know, Time = distance/speed

According to the question, we have Time taken by train = 360/x hour

And, Time taken by train its speed increase 5 km/h = 360/(x + 5)

It is given that,

Time taken by train in first – time taken by train in 2nd case = 48 min = 48/60 hour

$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60} = \frac{4}{5} \Rightarrow 360\left(\frac{1}{x} - \frac{1}{x+5}\right) = \frac{4}{5}$

$\Rightarrow 360\left(\frac{x+5-x}{x(x+5)}\right) = \frac{4}{5} \Rightarrow 360\left(\frac{5}{x(x+5)}\right) = \frac{4}{5}$

$\Rightarrow 360 \times \frac{5}{4}\left(\frac{5}{x^2+5x}\right) = 1 \Rightarrow 450 \times 5 = x^2 + 5x$

$\Rightarrow x^2 + 5x – 2250 = 0 \Rightarrow x^2 + 50x – 45x – 2250 = 0$

$\Rightarrow x(x + 50) – 45(x + 50) = 0 \Rightarrow (x + 50)(x – 45) = 0$

⇒ x = -50, 45
But x ≠ -50 because speed cannot be negative
So, x = 45 km/h
Hence, original speed of train = 45 km/h

OR

Two water taps together can fill a tank in 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Ans: Let the time taken by the smaller tap to fill the tank = x hours and
time taken by larger tap = x – 9

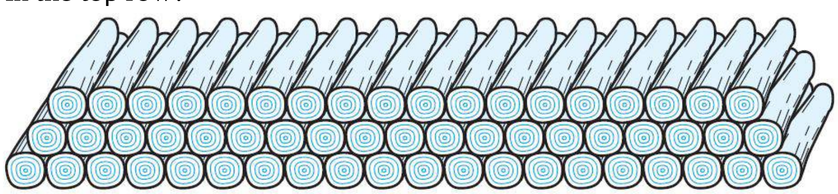
In 1 hour, the smaller tap will fill $\frac{1}{x}$ of tank and the larger tap will fill $\frac{1}{x-9}$ of tank.

In 1 hour both the tank will fill the tank = $\frac{1}{x}+\frac{1}{x-9}$

According to the question, $\frac{1}{x}+\frac{1}{x-9}=\frac{1}{6} \Rightarrow \frac{x-9+x}{x(x-9)}=\frac{1}{6} \Rightarrow \frac{2x-9}{x^2-9x}=\frac{1}{6}$

Solving by cross multiplication, $6(2x - 9) = x^2 - 9x$
$\Rightarrow 12x - 54 = x^2 - 9x \Rightarrow x^2 - 9x - 12x + 54 = 0 \Rightarrow x^2 - 21x + 54 = 0$
⇒ x2−18x−3x+54=0
⇒ x(x − 18) − 3(x − 18)=0
⇒ (x − 18)(x − 3) = 0
⇒ x = 18, x = 3
Neglecting x = 3 as x – 9 can’t negative, therefore x = 18
If we take x = 18
Smaller tap = (x) = 18 hrs
Larger tap = (x – 9) = 18 – 9 = 9hrs
Hence, the time taken by the smaller tap to fill the tank = 18 hrs & the time taken by the larger tap to fill the tank = 9 hrs

34. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see below figure). In how may rows are the 200 logs placed and how many logs are in the top row?



Ans: Here, a is the first term, d is a common difference and n is the number of terms.
It can be observed that the number of logs in rows are forming an A.P. 20, 19, 18, ...

We know that sum of n terms of AP is given by the formula $S_n = \frac{n}{2}[2a + (n - 1) d]$

$\Rightarrow 200 = \frac{n}{2}[2 \times 20 + (n - 1)(- 1)] \Rightarrow 400 = n [40 - n + 1] \Rightarrow 400 = n [41 - n]$
$\Rightarrow 400 = 41n - n^2 \Rightarrow n^2 - 41n + 400 = 0 \Rightarrow n^2 - 16n - 25n + 400 = 0$
⇒ n(n - 16) - 25(n - 16) = 0 ⇒ (n - 16)(n - 25) = 0
⇒ Either (n -16) = 0 or (n - 25) = 0
∴ n = 16 or n = 25
The number of logs in nth row will be $a_n = a + (n - 1) d$
$\Rightarrow a_{16} = a + 15d \Rightarrow a_{16} = 20 + 15 \times (- 1) \Rightarrow a_{16} = 20 – 15 \Rightarrow a_{16} = 5$
Similarly, $a_{25} = 20 + 24 \times (- 1)$
$\Rightarrow a_{25} = 20 – 24 \Rightarrow a_{25} = - 4$

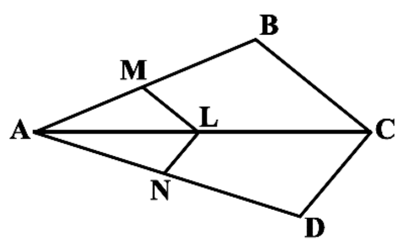
Clearly, the number of logs in the 16th row is 5. However, the number of logs in the 25th row is negative 4, which is not possible.
Therefore, 200 logs can be placed in 16 rows. The number of logs in the top (16th) row is 5.

OR

The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Ans: Here, a is the first term, d is the common difference and n is the number of terms.
Given: $a_3 + a_7 = 6$ ----- (1)
$a_3 \times a_7 = 8$ ----- (2)
We know that nth term of AP is $a_n = a + (n - 1)d$
Third term, $a_3 = a + 2d$ ----- (3)
Seventh term, $a_7 = a + 6d$ ----- (4)
Using equation (3) and equation (4) in equation (1) to find the sum of the terms,
$(a + 2d) + (a + 6d) = 6$
$\Rightarrow 2a + 8d = 6 \Rightarrow a + 4d = 3 \Rightarrow a = 3 - 4d$ ----- (5)
Using equation (3) and equation (4) in equation (2) to find the product of the terms,
$(a + 2d) \times (a + 6d) = 8$
Substituting the value of a from equation (5) above,
$(3 - 4d + 2d) \times (3 - 4d + 6d) = 8$
$\Rightarrow (3 - 2d) \times (3 + 2d) = 8$
$\Rightarrow (3)^2 - (2d)^2 = 8$ [Since $(a + b)(a - b) = a^2 - b^2$]
$\Rightarrow 9 - 4d^2 = 8 \Rightarrow 4d^2 = 1 \Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \frac{1}{2}, -\frac{1}{2}$
Case 1: When $d = \frac{1}{2}$
$a = 3 - 4d = 3 - 4 \times \frac{1}{2} = 3 - 2 = 1$
$S_n = \frac{n}{2}[2a + (n - 1)d] \Rightarrow S_{16} = \frac{16}{2}[2 \times 1 + (16 - 1) \times \frac{1}{2}] = 8 \times 19/2 = 76$
Case 2: When $d = -\frac{1}{2}$
$a = 3 - 4d = 3 - 4 \times (-\frac{1}{2}) = 3 + 2 = 5$
$S_n = \frac{n}{2}[2a + (n - 1)d] \Rightarrow S_{16} = \frac{16}{2}[2 \times 5 + (16 - 1) \times (-\frac{1}{2})] = 8[10 - 15/2] = 8 \times 5/2 = 20$

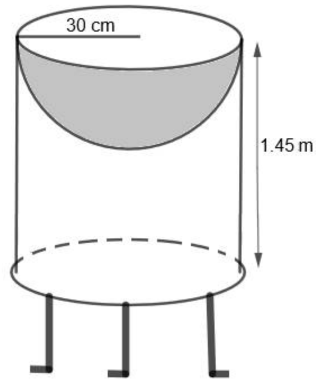
35. Prove that if a line is a drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio. Using the above theorem. Prove that $\frac{AM}{MB} = \frac{AN}{ND}$ if LM || CB and LN || CD as shown in the figure.



Ans: Given, To Prove, Figure and Construction – 1 ½ marks
Proof - 1 ½ mark
In △ABC, LM ∥ BC
∴ By Basic proportionality theorem, $\frac{AM}{AB} = \frac{AL}{AC}$………….(1)
Similarly, In △ADC, LN ∥ CD
∴ By Basic proportionality theorem, $\frac{AN}{AD} = \frac{AL}{AC}$…………(2)
∴ from (1) and (2), $\frac{AM}{AB} = \frac{AN}{AD}$

SECTION-E (Case Study Based Questions)
Questions 36 to 38 carry 4M each

36. Mayank a student of class 7th loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10th helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is Rs. 40 per square meter.



(i) Find the curved surface area of the hemisphere. (Take π = 3.14)
(ii) Find the total surface area of the bird-bath. (Take π = 22/7)
(iii) What is total cost for making the bird bath?

OR

(iii) Mayank and his brother thought of increasing the radius of hemisphere to 35 cm with same material so that birds get more space, then what is the new height of cylinder?

Ans: (i) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.
Then, r = 30 cm and h = 1.45 m = 145 cm.
Curved surface area of the hemisphere = $2\pi r^2$
$=2 \times 3.14 \times 30^2 = 0.56 \text{ m}^2$
(ii) Let S be the total surface area of the birdbath.
S = Curved surface area of the cylinder + Curved surface area of the hemisphere
$\Rightarrow S = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$
$\Rightarrow S = 2 \times \frac{22}{7} \times 30(145 + 30) = 33000 \text{ cm}^2 = 3.3 \text{ m}^2$
(iii) Total Cost of material = Total surface area x cost per sq m^2
= 3.3 x 40 = Rs. 132

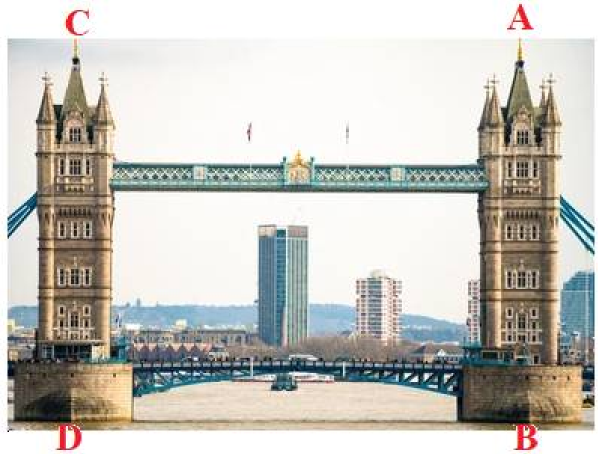
OR

We know that S = 3.3 m^2

$$S = 2\pi r(r+h)$$
$$\Rightarrow 3.3 = 2\times\frac{22}{7}\times\frac{35}{100}\left(\frac{35}{100}+h\right)$$
$$\Rightarrow 3.3 = \frac{22}{10}\left(\frac{35}{100}+h\right) \Rightarrow \frac{33}{22} = \frac{35}{100}+h$$
$$\Rightarrow h = \frac{3}{2}-\frac{7}{20} = \frac{23}{20} = 1.15m$$

37. Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The

bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.
In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was 60° and 30° respectively.

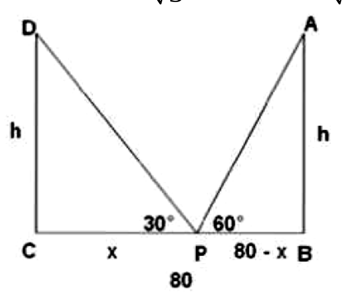


(i) Find the distances of the point from the base of the towers where Neeta was standing while measuring the height. [2]
(ii) Neeta used some applications of trigonometry she learned in her class to find the height of the towers without actually measuring them. What would be the height of the towers she would have calculated? [2]

OR

(ii) Find the distance between Neeta and top of tower AB? Also, Find the distance between Neeta and top tower CD? [2]
Ans: (i) Suppose AB and CD are the two towers of equal height h m. BC be the 80 m wide road. P is any point on the road. Let CP be x m, therefore BP = (80 - x).
Also, ∠APB = 60° and ∠DPC = 30°

In right angled triangle DCP, $\tan 30^0 = \frac{CD}{CP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow h = \frac{x}{\sqrt{3}}$ …… (1)



In right angled triangle ABP, $\tan 60^0 = \frac{AB}{BP} \Rightarrow \frac{h}{80-x} = \sqrt{3} \Rightarrow h = \sqrt{3}(80-x)$

$\Rightarrow \frac{x}{\sqrt{3}} = \sqrt{3}(80-x)$ ⇒ x = 3(80 – x) ⇒ x = 240 -3x

⇒ x + 3x = 240 ⇒ 4x = 240 ⇒ x=60
Thus, the position of the point P is 60 m from C.

(ii) Height of the tower, h = $\frac{x}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$

The height of each tower is $20\sqrt{3}$ m.

OR

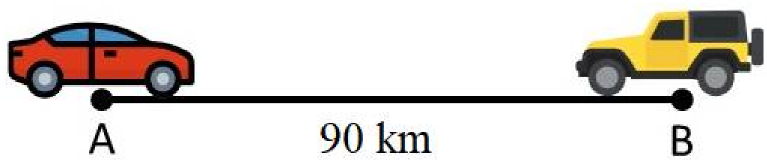
(ii) The distance between Neeta and top of tower AB.

In ΔABP, $\sin 60^0 = \frac{AB}{AP} \Rightarrow \frac{\sqrt{3}}{2} = \frac{20\sqrt{3}}{AP} \Rightarrow AP = 40m$

Similarly, the distance between Neeta and top of tower CD.

In ΔCDP, $\sin 30^0 = \frac{CD}{PD} \Rightarrow \frac{1}{2} = \frac{20\sqrt{3}}{PD} \Rightarrow PD = 40\sqrt{3}m$

38. On the roadway, Points A and B, which stand in for Chandigarh and Kurukshetra, respectively, are located nearly 90 kilometres apart. At the same time, a car departs from Kurukshetra and one from Chandigarh. These cars will collide in 9 hours if they are travelling in the same direction, and in 9/7 hours if they are travelling in the other direction. Let X and Y be two cars that are travelling at x and y kilometres per hour from places A and B, respectively. On the basis of the above information, answer the following questions:



(a) When both cars move in the same direction, then find the situation can be represented algebraically. [2]

OR

(a) When both cars move in the opposite direction, then find the situation can be represented algebraically. [2]

(b) Find the speed of car x. [1]

(c) Find the speed of car y. [1]

Ans: (a) Suppose two cars meet at point Q. Then, Distance travelled by car X = AQ, Distance travelled by car Y = BQ. It is given that two cars meet in 9 hours.

∴ Distance travelled by car X in 9 hours = 9x km = AQ = 9x

Distance travelled by car Y in 9 hours = 9y km = BQ = 9y

Clearly, AQ - BQ = AB = 9x - 9y = 90 = x - y = 10

OR

Suppose two cars meet at point P. Then Distance travelled by car X = AP and Distance travelled by car Y = BP.

In this case, two cars meet in $\frac{9}{7}$ hours. Distance travelled by car X in $\frac{9}{7}$ hours

= $\frac{9}{7}$ x km ⇒ AP = $\frac{9}{7}$x

Distance travelled by car Y in $\frac{9}{7}$ hours

= $\frac{9}{7}$y km ⇒ BP = $\frac{9}{7}$y

Clearly, AP + BP = AB

⇒ $\frac{9}{7}$x + $\frac{9}{7}$ y = 90 ⇒ $\frac{9}{7}$ (x + y) = 90 ⇒ x + y = 70

(b) We have x - y = 10 and x + y = 70

Adding equations (i) and (ii), we get 2x = 80 ⇒ x = 40

Hence, speed of car X is 40 km/hr.

(c) We have x - y = 10 ⇒ 40 - y = 10 ⇒ y = 30

Hence, speed of car y is 30 km/hr