$\mathcal{P M} S \mathcal{H R I}$ KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPCLS, $\mathcal{H} \mathscr{H} \mathcal{D}-32$ $S \mathcal{A M P L E} \mathcal{P A P E R} \mathcal{T E S T} 07 \mathcal{F O R} \mathcal{B O} \mathcal{A R D}$ EX $\mathcal{A M} 2024$
( $\mathcal{A N S}$ WERS)
$\mathcal{S T B I E C T}: ~ M A T \mathcal{H E M A T} I C S$
$\mathcal{M A X}$. $\operatorname{MARKS}: 80$
CLASS : $X$
$\mathcal{D L R A T}$ I O $\mathcal{N}: 3 \mathcal{H R S}$

## General Instruction:

1. This Question Paper has 5 Sections A-E.
2. Section $\mathbf{A}$ has 20 MCQs carrying 1 mark each.
3. Section $\mathbf{B}$ has 5 questions carrying 02 marks each.
4. Section $\mathbf{C}$ has 6 questions carrying 03 marks each.
5. Section $\mathbf{D}$ has 4 questions carrying 05 marks each.
6. Section $\mathbf{E}$ has 3 case based integrated units of assessment ( 04 marks each) with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi=22 / 7$ wherever required if not stated.

## SECTION - A

## Questions 1 to 20 carry 1 mark each.

1. The pair of equations $x+2 y+5=0$ and $-3 x-6 y+1=0$ have
(a) a unique solution
(b) exactly two solutions
(c) Infinitely many solutions
(d) no solution

## Ans: (d) no solution

The given equations are:
$x+2 y+5=0$
$-3 x-6 y+1=0$
From the given equations we have:
$\frac{a_{1}}{a_{2}}=\frac{1}{-3} ; \frac{b_{1}}{b_{2}}=\frac{2}{-6}=\frac{1}{-3} ; \frac{c_{1}}{c_{2}}=\frac{5}{1}$
$\Rightarrow \frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}}$
Hence the given pair of equations have no solution.
2. If $p$ and $q$ are the zeroes of the quadratic polynomial $f(x)=2 x^{2}-7 x+3$, find the value of $p+q-$ $p q$ is
(a) 1
(b) 2
(c) 3
(d) None of these

Ans: (b) 2
Here, $a=2, b=-7$ and $c=3$
$\mathrm{p}+\mathrm{q}=-\mathrm{b} / \mathrm{a}=7 / 2$ and $\mathrm{pq}=\mathrm{c} / \mathrm{a}=3 / 2$
$p+q-p q=\frac{7}{2}-\frac{3}{2}=\frac{7-3}{2}=\frac{4}{2}=2$
3. If $\sin 2 \mathrm{~A}=\frac{1}{2} \tan ^{2} 45^{\circ}$ where A is an acute angle, then the value of A is
(a) $60^{\circ}$
(b) $45^{\circ}$
(c) $30^{\circ}$
(d) $15^{\circ}$

Ans: (d) $\mathbf{1 5}^{\circ}$
$\sin 2 A=\frac{1}{2} \tan ^{2} 45^{\circ}=\frac{1}{2} \times 1^{2}=\frac{1}{2}=\sin 30^{\circ} \Rightarrow 2 A=30^{\circ} \Rightarrow A=15^{\circ}$
4. A girl walks 200 m towards East and then 150 m towards North. The distance of the girl from the starting point is
(a) 350 m
(b) 250 m
(c) 300 m
(d) 325 m

Ans: (b) 250m
Let girl starting point is A and she goes to B towards east covering 200 m distance .
Now from B she moves 150 m towards north let at point C.


Then, using Pythagoras theorem we get,
$\Rightarrow \mathrm{AC}=\sqrt{ }\left(\mathrm{AB}^{2}+\mathrm{BC}^{2}\right) \Rightarrow \mathrm{AC}=\sqrt{ }\left(200^{2}+150^{2}\right) \Rightarrow \mathrm{AC}=\sqrt{ }(40000+22500)$
$\Rightarrow A C=\sqrt{ } 62500 \Rightarrow A C=250 \mathrm{~m}$
5. In $\triangle A B C$ right angled at $B$, if $\cot C=\sqrt{ } 3$, then then $\cos A \sin C+\sin A \cos C=$
(a) -1
(b) 0
(c) 1
(d) $\sqrt{ } 3 / 2$

Ans: (c) 1
$\cot \mathrm{C}=\sqrt{3}=\cot 30^{\circ}$, so, $\angle \mathrm{C}=30^{\circ}$,
Hence, $\angle \mathrm{A}=60^{\circ}$.
So, $\cos \mathrm{A} \sin \mathrm{C}+\sin \mathrm{A} \cos \mathrm{C}=\frac{1}{2} \times \frac{1}{2}+\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}=\frac{1}{4}+\frac{3}{4}=\frac{1+3}{4}=\frac{4}{4}=1$
6. If $\theta$ is an acute angle and $\tan \theta+\cot \theta=2$, then the value of $\sin ^{3} \theta+\cos ^{3} \theta$ is
(a) 1
(b) $\frac{1}{2}$
(c) $\frac{\sqrt{2}}{2}$
(d) $\sqrt{2}$

Ans: (c) $\frac{\sqrt{2}}{2}$
$\tan \theta+\cot \theta=2 \Rightarrow \tan \theta+\frac{1}{\tan \theta}=2 \Rightarrow \tan ^{2} \theta-2 \tan \theta+1=0$
$\Rightarrow(\tan \theta-1)^{2}=0 \Rightarrow \tan \theta=1=\tan 45^{\circ} \Rightarrow \theta=45^{\circ}$
Now, $\sin ^{3} \theta+\cos ^{3} \theta=\sin ^{3} 45^{\circ}+\cos ^{3} 45^{0}=\left(\frac{1}{\sqrt{2}}\right)^{3}+\left(\frac{1}{\sqrt{2}}\right)^{3}=\frac{1}{2 \sqrt{2}}+\frac{1}{2 \sqrt{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$
7. If the distance between the points $(4, p)$ and $(1,0)$ is 5 , then value of $p$ is
(a) 4 only
(b) $\pm 4$
(c) -4 only
(d) 0

Ans: (b) $\pm 4$
The points given are $(4, p)$ and $(1,0)$
By distance formula, $5=\sqrt{ }(4-1)^{2}+\mathrm{p}^{2}$
$\Rightarrow 25=(4-1)^{2}+\mathrm{p}^{2} \Rightarrow 25=3^{2}+\mathrm{p}^{2} \Rightarrow 25=9+\mathrm{p}^{2}$
$\Rightarrow \mathrm{p}^{2}=25-9=16 \Rightarrow \mathrm{p}= \pm 4$
8. If $p$ and $q$ are positive integers such that $p=a^{3} b^{2}$ and $q=a^{2} b$, where ' $a$ ' and ' $b$ ' are prime numbers, then the $\operatorname{LCM}(p, q)$ is .....
(a) ab
(b) $a^{2} b^{2}$
(c) $a^{3} b^{2}$
(d) $a^{3} b^{3}$

Ans: (c) $\mathbf{a}^{\mathbf{3}} \mathbf{b}^{\mathbf{2}}$
9. 108 can be expressed as a product of its primes as
(a) $2^{3} \times 3^{2}$
(b) $2^{3} \times 3^{3}$
(c) $2^{2} \times 3^{2}$
(d) $2^{2} \times 3^{3}$

Ans: (d) $\mathbf{2}^{\mathbf{2}} \times \mathbf{3}^{\mathbf{3}}$
$108=2 \times 2 \times 3 \times 3 \times 3=2^{2} \times 3^{3}$
10. If angle between two radii of a circle is $130^{\circ}$, the angle between the tangents at the ends of the radii is :
(a) $90^{\circ}$
(b) $50^{\circ}$
(c) $70^{\circ}$
(d) $40^{\circ}$

Ans: (b) $\mathbf{5 0}^{\boldsymbol{0}}$
11. The relationship between mean, median and mode for a moderately skewed distribution is
(a) mode $=$ median -2 mean
(b) mode $=3$ median -2 mean
(c) mode $=2$ median -3 mean
(d) mode $=$ median - mean

Ans: (b) mode $=\mathbf{3}$ median $\mathbf{- 2}$ mean
12. For the following distribution:

| Marks | Below | Below | Below | Below | Below | Below |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 10 | 20 | 30 | 40 | 50 | 60 |
| No. of Students | 3 | 12 | 27 | 57 | 75 | 80 |

the modal class is
(a) $10-20$
(b) $20-30$
(c) $30-40$
(d) $50-60$

Ans: (c) 30-40

| Marks | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of Students | 3 | 9 | 15 | 30 | 18 | 5 |

Highest frequency is 30 which belong to $30-40$. Hence, Modal class is $30-40$
13. The area of a quadrant of a circle, whose circumference is 22 cm , is
(a) $\frac{11}{8} \mathrm{~cm}^{2}$
(b) $\frac{77}{8} \mathrm{~cm}^{2}$
(c) $\frac{77}{2} \mathrm{~cm}^{2}$
(d) $\frac{77}{4} \mathrm{~cm}^{2}$

Ans: (b) $\frac{77}{8} \mathbf{c m}^{2}$
Let the radius of the circle be ' r '
Circumference (C) $=22 \mathrm{~cm}$
$\Rightarrow$ radius $(\mathrm{r})=\mathrm{C} / 2 \pi=22 /(2 \times 22 / 7)=(22 \times 7) /(2 \times 22)=7 / 2 \mathrm{~cm}$
Therefore, the area of a quadrant $=1 / 4 \times \pi r^{2}$
$=1 / 4 \times 22 / 7 \times 7 / 2 \times 7 / 2$
$=77 / 8 \mathrm{~cm}^{2}$
14. If the quadratic equation $x^{2}+4 x+k=0$ has real and equal roots, then
(a) $\mathrm{k}<4$
(b) $\mathrm{k}>4$
(c) $\mathrm{k}=4$
(d) $\mathrm{k} \geq 4$

Ans: (c) $k=4$
For a quadratic equation to have equal and real roots the discriminant should be equal to zero.
$\mathrm{D}=0$. Now, $\mathrm{D}=\mathrm{b}^{2}-4 \mathrm{ac}$
$\Rightarrow 0=(4)^{2}-4(1)(\mathrm{k}) \Rightarrow 0=16-4 \mathrm{k} \Rightarrow 4 \mathrm{k}=16 \Rightarrow \mathrm{k}=16 / 4 \Rightarrow \mathrm{k}=4$
15. Volumes of two spheres are in the ratio $64: 27$. The ratio of their surface areas is
(a) $3: 4$
(b) $4: 3$
(c) $9: 16$
(d) $16: 9$

Ans: (d) $16: 9$
Let the radius of two spheres be r 1 and r 2 .
Given, the ratio of the volume of two spheres $=64: 27$
$\frac{V_{1}}{V_{2}}=\frac{\frac{4}{3} \pi r_{1}^{3}}{\frac{4}{3} \pi r_{2}^{3}}=\frac{r_{1}^{3}}{r_{2}^{3}}=\frac{64}{27} \Rightarrow \frac{r_{1}}{r_{2}}=\frac{4}{3}$
Let the surface areas of the two spheres be S1 and S2.
$\therefore \frac{S_{1}}{S_{2}}=\frac{4 \pi r_{1}^{2}}{4 \pi r_{2}{ }^{2}}=\frac{r_{1}^{2}}{r_{2}{ }^{2}}=\left(\frac{4}{3}\right)^{2}=\frac{16}{9}$
16. The area of the square that can be inscribed in a circle of radius 8 cm is
(a) $256 \mathrm{~cm}^{2}$
(b) $128 \mathrm{~cm}^{2}$
(c) $64 \sqrt{ } 2 \mathrm{~cm}^{2}$
(d) $64 \mathrm{~cm}^{2}$

Ans: (b) $128 \mathrm{~cm}^{2}$
Radius of circle $=8 \mathrm{~cm}$
$\Rightarrow$ Diameter $=8 \times 2=16 \mathrm{~cm}$
The diameter of circle $=$ diagonal of square $=16 \mathrm{~cm}=\mathrm{a} \sqrt{2}$
$\Rightarrow a=16 / \sqrt{2}=8 \sqrt{ } 2$
Area of square $=(\text { side })^{2}=(8 \sqrt{ } 2)^{2}=128 \mathrm{~cm}^{2}$
17. Two dice are thrown at the same time and the product of numbers appearing on them is noted. The probability that the product is a prime number is
(a) $1 / 3$
(b) $1 / 6$
(c) $1 / 5$
(d) $5 / 6$

Ans: (b) 1/6
Total number of possible outcomes $=36$
Now for the product of the numbers on the dice is prime number can be have in these possible ways $=(1,2),(2,1),(1,3),(3,1),(5,1),(1,5)$
So, number of possible ways $=6$
$\therefore$ Required probability $=6 / 36=1 / 6$
18. In $\triangle \mathrm{ABC}, \mathrm{DE} \| \mathrm{AB}$, If $\mathrm{CD}=3 \mathrm{~cm}, \mathrm{EC}=4 \mathrm{~cm}, \mathrm{BE}=6 \mathrm{~cm}$, then DA is equal to
(a) 7.5 cm
(b) 3 cm
(c) 4.5 cm
(d) 6 cm

Ans: (c) 4.5 cm
$D E \| A B \Rightarrow \frac{C D}{D A}=\frac{C E}{E B} \Rightarrow \frac{3}{D A}=\frac{4}{6} \Rightarrow D A=\frac{9}{2}=4.5 \mathrm{~cm}$
DIRECTION: In the question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R).
Choose the correct option
19. Assertion (A): The point $(0,4)$ lies on $y$-axis.

Reason (R): The x co-ordinate on the point on $y$-axis is zero.
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of Assertion (A)
(c) Assertion (A) is true but reason $(R)$ is false.
(d) Assertion (A) is false but reason(R) is true.

Ans: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
20. Assertion (A): If $\operatorname{HCF}(90,144)=18$, then $\operatorname{LCM}(90,144)=720$

Reason (R): $\operatorname{HCF}(\mathrm{a}, \mathrm{b}) \times \operatorname{LCM}(\mathrm{a}, \mathrm{b})=\mathrm{a} \times \mathrm{b}$
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

Ans: (a) Both Assertion (A) and Reason (R) are true and Reason ( $R$ ) is the correct explanation of Assertion (A).
HCF $\times$ LCM $=$ Product of two numbers
$\Rightarrow L C M=\frac{90 \times 144}{18}=5 \times 144=720$

## SECTION - B

## Questions 21 to 25 carry 2 marks each.

21. The length of the minute hand of a clock is 14 cm . Find the area swept by the minute hand in 5 minutes.
Ans: We know that the minute hand completes one rotation in 1 hour or 60 minutes.
Length of the minute hand $(\mathrm{r})=14 \mathrm{~cm}$
Area swept by minute hand in 1 minute $=\pi r^{2} / 60$
Thus, area swept by minute hand in 5 minutes $=\left(\pi r^{2} / 60\right) \times 5=\pi r^{2} / 12$
$=1 / 12 \times 22 / 7 \times 14 \times 14 \mathrm{~cm}^{2}$
$=154 / 3 \mathrm{~cm}^{2}$

## OR

In a circle of radius 21 cm , an arc subtends an angle of $60^{\circ}$ at the centre. Find (i) the length of the arc (ii) area of the sector formed by the arc
Ans: Here, $\mathrm{r}=21 \mathrm{~cm}, \theta=60^{\circ}$


Area of the segment $\mathrm{APB}=$ Area of sector $\mathrm{AOPB}-$ Area of $\triangle \mathrm{AOB}$
(i) Length of the Arc, $\mathrm{APB}=\theta / 360^{\circ} \times 2 \pi \mathrm{r}$
$=60^{\circ} / 360^{\circ} \times 2 \times 22 / 7 \times 21 \mathrm{~cm}$
$=22 \mathrm{~cm}$
(ii) Area of the sector, $\mathrm{AOBP}=\theta / 360^{\circ} \times \pi \mathrm{r}^{2}$
$=60^{\circ} / 360^{\circ} \times 22 / 7 \times 21 \times 21 \mathrm{~cm}^{2}=231 \mathrm{~cm}^{2}$
22. For what value of k will the following system of linear equations have no solution?
$3 \mathrm{x}+\mathrm{y}=1 ;(2 \mathrm{k}-1) \mathrm{x}+(\mathrm{k}-1) \mathrm{y}=2 \mathrm{k}+1$
Ans: $3 \mathrm{x}+\mathrm{y}-1=0$
$(2 \mathrm{k}-1) \mathrm{x}+(\mathrm{k}-1) \mathrm{y}-2 \mathrm{k}-1=0$
$\frac{a_{1}}{a_{2}}=\frac{3}{2 k-1} ; \frac{b_{1}}{b_{2}}=\frac{1}{k-1} ; \frac{c_{1}}{c_{2}}=\frac{-1}{-2 k-1}=\frac{1}{2 k+1}$
For no solutions, $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}}$
$\Rightarrow \frac{3}{2 k-1}=\frac{1}{k-1} \neq \frac{1}{2 k+1} \Rightarrow \frac{3}{2 k-1}=\frac{1}{k-1} \Rightarrow 3 \mathrm{k}-3=2 \mathrm{k}-1 \Rightarrow \mathrm{k}=2$
23. From a point $P$, two tangents $P A$ and $P B$ are drawn to a circle $C(0, r)$. If $O P=2 r$, then find $\angle A P B$. Prove that triangle APB is an equilateral triangle.


Ans: Let $\angle A P O=\theta$
$\Rightarrow \sin \theta=\frac{O A}{O P}=\frac{1}{2}=\sin 30^{\circ} \Rightarrow \theta=30^{\circ}$
$\Rightarrow \angle A P O=2 \theta=2\left(30^{\circ}\right)=60^{\circ}$
Also, $\angle P A B=\angle P B A=60^{\circ} \quad(\because P A=P B)$
$\Rightarrow \triangle \mathrm{APB}$ is an equilateral triangle.
24. If $\tan (A+B)=\sqrt{3}$ and $\tan (A-B)=\frac{1}{\sqrt{3}} ; 0^{\circ}<A+B \leq 90^{\circ} ; A>B$, find $A$ and $B$.

Ans: $\tan (A+B)=\sqrt{ } 3=\tan 60^{\circ}$
$\Rightarrow \mathrm{A}+\mathrm{B}=60^{\circ}$
$\tan (\mathrm{A}-\mathrm{B})=1 / \sqrt{ } 3=\tan 30^{\circ} \Rightarrow \mathrm{A}-\mathrm{B}=30^{\circ}$
Adding equation (i) and (ii),
$2 \mathrm{~A}=90^{\circ} \Rightarrow \mathrm{A}=45^{\circ}$
Putting the value of A in equation (i), $45^{\circ}+\mathrm{B}=60^{\circ}$
$\Rightarrow B=60^{\circ}-45^{\circ} \Rightarrow B=15^{\circ}$

## OR

If $x \sin ^{3} \theta+y \cos ^{3} \theta=\sin \theta \cos \theta$ and $x \sin \theta=y \sin \theta$ then find $x^{2}+y^{2}$.
Ans:
We have, $x \sin ^{3} \theta+y \cos ^{3} \theta=\sin \theta \cos \theta$
$(x \sin \theta) \sin ^{2} \theta+(y \cos \theta) \cos ^{2} \theta=\sin \theta \cos \theta$
$\Rightarrow x \sin \theta\left(\sin ^{2} \theta\right)+(x \sin \theta) \cos ^{2} \theta=\sin \theta \cos \theta$
$\Rightarrow x \sin \theta\left(\sin ^{2} \theta+\cos ^{2} \theta\right)=\sin \theta \cos \theta$
$\Rightarrow x \sin \theta=\sin \theta \cos \theta \Rightarrow x=\cos \theta$
Now, $x \sin \theta=y \cos \theta \Rightarrow \cos \theta \sin \theta=y \cos \theta \Rightarrow y=\sin \theta$
Hence, $x^{2}+y^{2}=\cos ^{2} \theta+\sin ^{2} \theta=1$
25. $A B C D$ is a trapezium in which $A B \| C D$ and its diagonals intersect each other at the point $O$.

Using a similarity criterion of two triangles, show that $\frac{O A}{O C}=\frac{O B}{O D}$
Ans: Given : ABCD is a trapezium, AB II CD


In $\triangle \mathrm{AOB}$ and $\triangle \mathrm{COD}$
$\angle \mathrm{OBA}=\angle \mathrm{ODC}---$ eq. 1 (alt.angles are equal)
$\angle \mathrm{OAB}=\angle \mathrm{OCD}---$ eq. 2 (alt.angles are equal)
Therefore, $\triangle \mathrm{AOB} \sim \Delta \mathrm{COD}$ ( A.A Similarity)
Hence, $\frac{O A}{O C}=\frac{O B}{O D}$

## SECTION - C

## Questions 26 to 31 carry 3 marks each.

26. The sum of the digits of a two-digit number is 9 . Also 9 times this number is twice the number obtained by reversing the order of the digits. Find the number.
Ans: Let the tens digits and unit digit of the number be $x$ and $y$ respectively. Then, the number will be $10 x+y$
Number after reversing the digits is $10 y+x$
According to the question,
$x+y=9 \ldots$ (i)
$9(10 x+y)=2(10 y+x)$
$\Rightarrow 88 x-11 y=0 \Rightarrow 8 x+y=0 \ldots$
Adding equation (i) and (ii), we get
$9 x=9 \Rightarrow x=1$
Putting the value in equation (i), we get $y=8$
Hence, the number is 18.

## OR

Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?
Ans: Let $x$ be the number of right answers and $y$ be the number of wrong answers.
According to the question,
$3 \mathrm{x}-\mathrm{y}=40$
and, $2 x-y=25$
On subtraction, we get: $x=15$
putting the value of $x$ in (i), we get $3(15)-y=40 y=5$
Number of right answers $=15$ answers
Number of wrong answers $=5$ answers.
Total Number of questions $=5+15=20$
27. Prove that $(\sin A+\operatorname{cosec} A)^{2}+(\cos A+\sec A)^{2}=7+\tan ^{2} A+\cot ^{2} A$

Ans: L.H.S $=(\sin A+\operatorname{cosec} A)^{2}+(\cos A+\sec A)^{2}$
$=\sin ^{2} A+\operatorname{cosec}^{2} A+2 \sin A \operatorname{cosec} A+\cos ^{2} A+\sec ^{2} A+2 \cos A \sec A$
$=\sin ^{2} A+\cos ^{2} A+\operatorname{cosec}^{2} A+\sec ^{2} A+2 \sin A \times 1 / \sin A+2 \cos A \times 1 / \cos A$
Since, $\left(\sin ^{2} A+\cos ^{2} A=1\right)$
$\left(\sec ^{2} \mathrm{~A}=1+\tan ^{2} \mathrm{~A}, \operatorname{cosec}^{2} \mathrm{~A}=1+\cot ^{2} \mathrm{~A}\right)$
$=1+1+\cot ^{2} \mathrm{~A}+1+\tan ^{2} \mathrm{~A}+2+2$
$=7+\tan ^{2} \mathrm{~A}+\cot ^{2} \mathrm{~A}=$ RHS
28. Prove that $\sqrt{5}$ is an irrational number.

Ans: Let $\sqrt{5}$ is a rational number then we have $\sqrt{5}=\frac{p}{q}$, where p and q are co-primes.
$\Rightarrow p=\sqrt{5} q$
Squaring both sides, we get $p^{2}=5 q^{2}$
$\Rightarrow \mathrm{p}^{2}$ is divisible by $5 \Rightarrow \mathrm{p}$ is also divisible by 5

So, assume $\mathrm{p}=5 \mathrm{~m}$ where m is any integer.
Squaring both sides, we get $\mathrm{p}^{2}=25 \mathrm{~m}^{2}$
But $p^{2}=5 q^{2}$
Therefore, $5 \mathrm{q}^{2}=25 \mathrm{~m}^{2} \Rightarrow \mathrm{q}^{2}=5 \mathrm{~m}^{2}$
$\Rightarrow q^{2}$ is divisible by $5 \Rightarrow q$ is also divisible by 5
From above we conclude that p and q have one common factor i.e. 5 which contradicts that p and q are co-primes.
Therefore, our assumption is wrong.
Hence, $\sqrt{5}$ is an irrational number.
29. Find the zeroes of the quadratic polynomial $x^{2}-2 x-8$ and verify the relationship between the zeroes and the coefficients of the polynomial.
Ans: Given, $f(x)=x^{2}-2 x-8$
The zeroes of $f(x)$ are given by, $f(x)=0$
$\Rightarrow \mathrm{x}^{2}+2 \mathrm{x}-4 \mathrm{x}-8=0 \Rightarrow \mathrm{x}(\mathrm{x}+2)-4(\mathrm{x}+2)=0 \Rightarrow(\mathrm{x}+2)(\mathrm{x}-4)=0$
$\Rightarrow \mathrm{x}=-2$ (or) $\mathrm{x}=4$
Hence, the zeros of $f(x)=x^{2}-2 x-8$ are $\alpha=-2$ and $\beta=4$
$\alpha+\beta=-2+4=2=\frac{-b}{a}=2$
$\alpha \beta=-2 \times 4=-8=\frac{c}{a}=-8$
30. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

Ans: Given, To Prove, Constructions and Figure - $1 \frac{1}{2}$ marks
Correct Proof - $11 / 2$ marks

## OR

In the figure XY and $\mathrm{X}^{\prime} \mathrm{Y}^{\prime}$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C interesting XY at A and $\mathrm{X}^{\prime} \mathrm{Y}^{\prime}$ at B , what is the measure of $\angle \mathrm{AOB}$.


Ans: Join OC. Since, the tangents drawn to a circle from an external point are equal.
$\therefore \mathrm{AP}=\mathrm{AC}$


In $\Delta \mathrm{PAO}$ and $\Delta \mathrm{AOC}$, we have:
$\mathrm{AO}=\mathrm{AO}$ [Common]
$\mathrm{OP}=\mathrm{OC}$ [Radii of the same circle $]$
$\mathrm{AP}=\mathrm{AC}$
$\Rightarrow \triangle \mathrm{PAO} \cong \triangle \mathrm{AOC}$ [SSS Congruency]
$\therefore \angle \mathrm{PAO}=\angle \mathrm{CAO}=\angle 1$
$\angle \mathrm{PAC}=2 \angle 1$
Similarly $\angle \mathrm{CBQ}=2 \angle 2$
Again, we know that sum of internal angles on the same side of a transversal is $180^{\circ}$.
$\therefore \angle \mathrm{PAC}+\angle \mathrm{CBQ}=180^{\circ}$
$\Rightarrow 2 \angle 1+2 \angle 2=180^{\circ}$ [From (1) and (2)]
$\Rightarrow \angle 1+\angle 2=180^{\circ} / 2=90^{\circ}$
Also $\angle 1+\angle 2+\angle \mathrm{AOB}=180^{\circ}$ [Sum of angles of a triangle]
$\Rightarrow 90^{\circ}+\angle \mathrm{AOB}=180^{\circ}$
$\Rightarrow \angle \mathrm{AOB}=180^{\circ}-90^{\circ}$
$\Rightarrow \angle \mathrm{AOB}=90^{\circ}$.
31. Two dice are thrown at the same time. What is the probability that the sum of the two numbers appearing on the top of the dice is
(i) 8 ?
(ii) 7 ?
(iii) less than or equal to 12 ?

Ans: (i) Number of outcomes with sum of the numbers $8=5$
$\therefore$ Required Probability $=5 / 36$
(ii) Number of outcomes with sum of the numbers $7=6$
$\therefore$ Required Probability $=6 / 36=1 / 6$
(iii) Number of outcomes with sum of the numbers less than or equal to $12=36$
$\therefore$ Required Probability $=36 / 36=1$

## SECTION - D

## Questions 32 to 35 carry 5 marks each.

32. State and Prove Basic Proportionality Theorem.

Ans: Statement - 1 mark
Given, To prove, Construction and figure of 2 marks
Proof of 2 marks
33. A person on tour has Rs. 360 for his expenses. If he extends his tour for 4 days, he has to cut down his daily expenses by Rs.3. Find the original duration of the tour.
Ans: Let days be the original duration of the tour.
Total expenditure on tour ₹ 360
Expenditure per day ₹ 360 /x
Duration of the extended tour $(x+4)$ days
Expenditure per day according to the new schedule ₹ $360 /(x+4)$
Given that daily expenses are cut down by ₹ 3
As per the given condition, $\frac{360}{x}-\frac{360}{x+4}=3$
$\Rightarrow 360\left(\frac{1}{x}-\frac{1}{x+4}\right)=3$
$\Rightarrow\left(\frac{1}{x}-\frac{1}{x+4}\right)=\frac{3}{360}=\frac{1}{120}$
$\Rightarrow \frac{x+4-x}{x(x+4)}=\frac{1}{120} \Rightarrow \frac{4}{x(x+4)}=\frac{1}{120}$
$\Rightarrow \mathrm{x}(\mathrm{x}+4)=480$
$\Rightarrow \mathrm{x}^{2}+4 \mathrm{x}=480$
$\Rightarrow \mathrm{x}^{2}+4 \mathrm{x}-480=0$
$\Rightarrow \mathrm{x}^{2}+24 \mathrm{x}-20 \mathrm{x}-480=0$
$\Rightarrow \mathrm{x}(\mathrm{x}+24)-20(\mathrm{x}+24)=0$
$\Rightarrow \mathrm{x}-20=0$ or $\mathrm{x}+24=0$
$\Rightarrow \mathrm{x}=20$ or $\mathrm{x}=-24$
Since the number of days cannot be negative. So, $x=20$
Therefore, the original duration of the tour was 20 days

## OR

Rs. 6500 were divided equally among a certain number of persons. Had there been 15 more persons, each would have got Rs. 30 less. Find the original number of persons.
Ans: Let the original number of persons be $x$
Total money which was divided = Rs. 6500
Each person share $=$ Rs. 6500/x
According to the question, $\frac{6500}{x}-\frac{6500}{x+15}=30$
$\Rightarrow \frac{6500 x+97500-6500 x}{x(x+15)}=30$
$\Rightarrow \frac{97500}{x(x+15)}=30 \Rightarrow \frac{3250}{x(x+15)}=1$
$\Rightarrow \mathrm{x}^{2}+15 \mathrm{x}-3250=0$
$\Rightarrow \mathrm{x}^{2}+65 \mathrm{x}-50 \mathrm{x}-3250=0$
$\Rightarrow \mathrm{x}(\mathrm{x}+65)-50(\mathrm{x}+65)=0$
$\Rightarrow(\mathrm{x}+65)(\mathrm{x}-50)=0$
$\Rightarrow \mathrm{x}=-65,50$
Since the number of persons cannot be negative, hence the original numbers of person is 50
34. Ramesh made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and its radius is 30 cm . Find the total surface area of the bird-bath.


Ans: Let h be height of the cylinder, and r the common radius of the cylinder and hemisphere.
Then, the total surface area = CSA of cylinder + CSA of hemisphere

$$
\begin{aligned}
& =2 \pi \mathrm{rh}+2 \pi \mathrm{r}^{2}=2 \pi \mathrm{r}(\mathrm{~h}+\mathrm{r}) \\
& =2 \times \frac{22}{7} \times 30(145+30) \mathrm{cm}^{2} \\
& =2 \times \frac{22}{7} \times 30 \times 175 \mathrm{~cm}^{2} \\
& =33000 \mathrm{~cm}^{2}=3.3 \mathrm{~m}^{2}
\end{aligned}
$$

## OR

A tent is in shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively and the slant height of the top is 2.8 m . Find the area
of canvas used for making the tent. Also find the cost of canvas of the tent at the rate of 500 per $\mathrm{m}^{2}$.
Ans: Radius $=2 \mathrm{~m}$, Slant height $\mathrm{l}=2.8 \mathrm{~m}$, height $\mathrm{h}=2.1 \mathrm{~m}$
Cost of canvas per $\mathrm{m}^{2}=$ Rs. 500
Area of canvas used $=$ CSA of cone + CSA of cylinder

$$
\begin{aligned}
& =\pi \mathrm{rl}+2 \pi \mathrm{rh} \\
& =22 / 7 \times 2 \times 2.8+2 \times 22 / 7 \times 2 \times 2.1 \\
& =17.6+26.4 \\
& =44 \mathrm{~m}^{2}
\end{aligned}
$$

Cost of the canvas of tent $=44 \times 500$

$$
=\text { Rs.22,000 }
$$

35. The following frequency distribution gives the monthly consumption of 68 consumers of a locality. Find median, mean and mode of the data and compare them.

| Monthly consumption of <br> electricity (in units) | Number of consumers |
| :---: | :---: |
| $65-85$ | 4 |
| $85-105$ | 5 |
| $105-125$ | 13 |
| $125-145$ | 20 |
| $145-165$ | 14 |
| $165-185$ | 8 |
| $185-205$ | 4 |

Ans: For mean , median, mode
To calculate $\mathrm{x}_{\mathrm{i}}$, cumulative frequency, identifying highest frequency
Formulae for mean, median, mode

$$
\begin{aligned}
\text { Mean }= & a+\frac{\sum f_{i} d_{i}}{\sum_{i} f_{i}}=135+\frac{140}{68} \\
= & 137.05
\end{aligned}
$$

$$
\text { Median }=l+\left(\frac{\frac{n}{2}-c f}{f} \times h\right)=125+\left(\frac{34-22}{20} \times 20\right)=125+12=137
$$

$$
\text { Mode }=l+\left(\frac{f_{1}-f_{0}}{2 f_{1}-f_{0}-f_{2}} \times h\right)=125+\left(\frac{20-13}{40-13-14} \times 20\right)
$$

$$
=125+\left(\frac{7}{13} \times 20\right)=125+10.77=135.77
$$

## SECTION - E(Case Study Based Questions)

## Questions 36 to 38 carry 4 marks each.

## 36. Case Study-2

In order to conduct sports day activities in your school, lines have been drawn with chalk powder at a distance of 1 m each in a rectangular shaped ground ABCD. 100 flower pots have been placed at the distance of 1 m from each other along AD , as shown in the following figure. Niharika runs $\left(\frac{1}{4}\right)$ th distance AD on the 2 nd line and posts a green Flag. Preet runs $\left(\frac{1}{5}\right)$ th distance AD on the eighth line and posts are red flags. Taking A as the origin AB along x -axis and AD along y -axis, answer the following questions:
(i) Find the coordinates of the green flag.
(ii) Find the distance between the two flags.
(iii) If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?
(2)

OR
(iii) If Joy has to post a flag at one fourth distance from the green flag, in the line segment joining the green and red flags, then where should he post his flag?
(2)


Ans: (i) Position of the red flag is $\left(2, \frac{1}{4} \times 100\right)=(2,25)$
(ii) Distance between the two flags $=\sqrt{(36+25)}=\sqrt{61} \mathrm{~cm}$
(iii) Position of the blue flag $=\left(\frac{2+8}{2}, \frac{25+20}{2}\right)$
$=(5,22.5)$

## OR



Required point is $(3.5,23.75)$

## 37. Case Study - 3

Lakshaman Jhula is located 5 kilometers north-east of the city of Rishikesh in the Indian state of Uttarakhand. The bridge connects the villages of Tapovan to Jonk. Tapovan is in Tehri Garhwal district, on the west bank of the river, while Jonk is in Pauri Garhwal district, on the east bank. Lakshman Jhula is a pedestrian bridge also used by motorbikes. It is a landmark of Rishikesh. A group of Class X students visited Rishikesh in Uttarakhand on a trip. They observed from a point $(\mathrm{P})$ on a river bridge that the angles of depression of opposite banks of the river are $60^{\circ}$ and $30^{\circ}$ respectively. The height of the bridge is about 18 meters from the river.

Based on the above information answer the following questions.
(i) Find the distance PA.
(ii) Find the distance PB
(iii) Find the width AB of the river.

## OR

(iii) Find the height BQ if the angle of the elevation from P to Q be $30^{\circ}$. (2)

Ans: (i) $\sin 60^{\circ}=\frac{P C}{P A}$
$\Rightarrow \frac{\sqrt{3}}{2}=\frac{18}{P A} \Rightarrow P A=\frac{36}{\sqrt{3}}=12 \sqrt{3} m$
(ii) $\sin 30^{\circ}=\frac{P C}{P B}$
$\Rightarrow \frac{1}{2}=\frac{18}{P B} \Rightarrow P B=36 m$
(iii) $\tan 60^{\circ}=\frac{P C}{A C} \Rightarrow \sqrt{3}=\frac{18}{A C} \Rightarrow A C=6 \sqrt{3} m$
$\tan 30^{\circ}=\frac{P C}{C B} \Rightarrow \frac{1}{\sqrt{3}}=\frac{18}{C B} \Rightarrow C B=18 \sqrt{3} m$
Width $\mathrm{AB}=\mathrm{AC}+\mathrm{CB}=6 \sqrt{3} m+18 \sqrt{3} m=24 \sqrt{3} m$

## OR

$\mathrm{RB}=\mathrm{PC}=18 \mathrm{~m} \& \mathrm{PR}=\mathrm{CB}=18 \sqrt{ } 3 \mathrm{~m}$
$\tan 30^{\circ}=\frac{Q R}{P R} \Rightarrow \frac{1}{\sqrt{3}}=\frac{Q R}{18 \sqrt{3}} \Rightarrow Q R=18 m$
$\mathrm{QB}=\mathrm{QR}+\mathrm{RB}=18+18=36 \mathrm{~m}$.
Hence height BQ is 36 m .

## 38. Case Study-1

Mohan takes a loan from a bank for his car. Mohan replays his total loan of Rs. 118000 by paying every month starting with the first instalment of Rs.1000. If he increases the instalment by Rs. 100 every month.

(i) What is the first term and common difference of given question.
(ii) The amount paid buy him in $30^{\text {th }}$ instalment.
(iii) The amount paid by him in the 30 instalments is
(OR)
(iii) What amount does he still have to pay after $30^{\text {th }}$ instalment?

Ans: (i) Given , $a_{1}=1000$
Common difference, $\mathrm{d}=100$
Total loan=Rs.1,18,000
(ii) $\mathrm{a}_{30}=\mathrm{a}+29 \mathrm{~d}$

$$
\begin{aligned}
& =1000+29 \times 100 \\
& =3900
\end{aligned}
$$

Amount paid in $30^{\text {th }}$ installment is Rs. 3900
(iii) $\mathrm{S}_{30}=\frac{30}{2}[2 \times 1000+(30-1) \times 100]$

$$
\begin{aligned}
& =15 \times 4900 \\
& =73,500
\end{aligned}
$$

Amount paid in 30 installments is Rs.73,500 (OR)
The amount he still have to pay after 30 installments=Rs. 118000 - Rs. 73,500 $=$ Rs. 44,500

