PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

SAMPLE PAPER TEST 04 FOR BOARD EXAM 2024

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**

CLASS : X **DURATION : 3 HRS**

General Instruction:

1. This Question Paper has 5 Sections A-E.
2. **Section A** has 20 MCQs carrying 1 mark each.
3. **Section B** has 5 questions carrying 02 marks each.
4. **Section C** has 6 questions carrying 03 marks each.
5. **Section D** has 4 questions carrying 05 marks each.
6. **Section E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. The point on the x-axis which is equidistant from (– 4, 0) and (10, 0) is:
(a) (7, 0) (b) (5, 0) (c) (0, 0) (d) (3, 0)
Ans. (d) (3, 0)

2. If a cylinder is covered by two hemispheres shaped lid of equal shape, then the total curved surface area of the new object will be
(a) $4\pi rh + 2\pi r^2$ (b) $4\pi rh – 2\pi r^2$ (c) $2\pi rh + 4\pi r^2$ (d) $2\pi rh + 4\pi r$
Ans: (c) 2πrh + 4πr2
Curved surface area of cylinder = $2\pi rh$
The curved surface area of hemisphere = $2\pi r^2$
Here, we have two hemispheres.
So, total curved surface area = $2\pi rh + 2(2\pi r^2) = 2\pi rh + 4\pi r^2$

3. If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a =
(a) 1 (b) 2 (c) 3 (d) 4
Ans. (d) 4

4. The sum of exponents of prime factors in the prime-factorisation of 196 is:
(a) 3 (b) 4 (c) 5 (d) 6
Ans. (b) 4

5. The values of k for which the quadratic equation $2x^2 – kx + k = 0$ has equal roots is
(a) 0 only (b) 8 only (c) 0,8 (d) 4
Ans: (c) 0,8
For equal roots, $D = b^2 – 4ac = 0$
$\Rightarrow (-k)^2 – 4(2)(k) = 0$
$\Rightarrow k^2 – 8k = 0 \Rightarrow k(k – 8) = 0 \Rightarrow k= 0,8$

6. A number x is chosen at random from the numbers -3, -2, -1, 0, 1, 2, 3 the probability that $|x| < 2$ is
(a) 1/7 (b) 2/7 (c) 3/7 (d) 5/7
Ans: (c) 3/7

Total possible number of events (n) = 7
Now for $|x| < 2$, possible values of x = -1, 0, 1
∴ Required probability = 3/7

7. If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$ then x + y is:
(a) 3 (b) 2 (c) 1 (d) 1/2
Ans: (a) 3

8. If ½ is a root of the equation $x^2 + kx - 5/4 = 0$, then the value of k is
(a) 2 (b) – 2 (c) ¼ (d) ½
Ans: (a) 2
If ½ is a root of the equation
$x^2 + kx - 5/4 = 0$ then, substituting the value of ½ in place of x should give us the value of k.
Given, $x^2 + kx - 5/4 = 0$ where, x = ½
$(½)^2 + k(½) - (5/4) = 0$
⇒ (k/2) = (5/4) – ¼ ⇒ k = 2

9. The pair of equations x + 2y + 5 = 0 and –3x – 6y + 1 = 0 have
(a) a unique solution (b) exactly two solutions
(c) infinitely many solutions (d) no solution
Ans: (d) no solution
$a_1 = 1$; $b_1 = 2$; $c_1 = 5$
$a_2 = -3$; $b_2 = -6$; $c_2 = 1$
$a_1/a_2 = -1/3$
$b_1/b_2 = -2/6 = -1/3$
$c_1/c_2 = 5/1 = 5$
Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$
Therefore, the pair of equation has no solution.

10. The point which lies on the perpendicular bisector of the line segment joining point A (–2, –5) and B (2, 5) is:
(a) (0, 0) (b) (0, –1) (c) (–1, 0) (d) (1, 0)
Ans. (a) (0, 0)

11. A card is selected at random from a well shuffled deck of 52 playing cards. The probability of its being a face card is
(a) 3/13 (b) 4/13 (c) 6/13 (d) 9/13
Ans: (a) 3/13
Total number of outcomes = 52
Number of face cards = 12
The probability of its being a face card = 12/52 = 3/13

12. The ratio in which the line segment joining the points P(-3, 10) and Q(6, –8) is divided by O(-1, 6) is:
(a) 1:3 (b) 3:4 (c) 2:7 (d) 2:5
Ans: (c) 2:7
Let k :1 be the ratio in which the line segment joining P(-3, 10) and Q(6, -8) is divided by point O(-1, 6).
By the section formula, we have -1 = (6k – 3)/(k + 1)
⇒ –k – 1 = 6k – 3
⇒ 7k = 2 ⇒ k = 2/7
Hence, the required ratio is 2:7.

13. A box contains cards numbered 6 to 50. A card is drawn at random from the box. The probability that the drawn card has a number which is a perfect square is :

(a) 1/45 (b) 2/15 (c) 4/45 (d) 1/9

Ans. (d) 1/9

P(perfect Square)= 5/45 = 1/9

14. In a circle of diameter 42cm, if an arc subtends an angle of 60º at the centre, then the length of the arc is:

(a) 22/7 cm (b) 11cm (c) 22 cm (d) 44 cm

Ans: (c) 22 cm

15. If the lines 3x + 2ky – 2 = 0 and 2x + 5y + 1 = 0 are parallel, then what is the value of k?

(a) 4/15 (b) 15/4 (c) ⅘ (d) 5/4

Ans: (b) 15/4

The condition for parallel lines is a1/a2 = b1/b2 ≠ c1/c2

Hence, 3/2 = 2k/5

⇒ k = 15/4

16. For the following distribution:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of students	3	12	27	57	75	80

the modal class is

(a) 10-20 (b) 20-30 (c) 30-40 (d) 50-60

Ans: (c) 30-40

17. The distance of the point P (2, 3) from the x-axis is

(a) 2 (b) 3 (c) 1 (d) 5

Ans: (b) 3

We know that, (x, y) is a point on the Cartesian plane in first quadrant.

Then, x = Perpendicular distance from Y – axis and

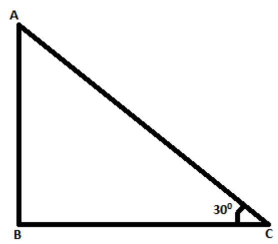
y = Perpendicular distance from X – axis

Therefore, the perpendicular distance from X-axis = y coordinate = 3

18. A circus artist is climbing a 30 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the distance of the pole to the peg in the ground, if the angle made by the rope with the ground level is 30º.

(a) 20√3 m (b)15√3 m (c)10√3 m (d) 20 m

Ans: (b)15√3 m



$$\cos 30^\circ = \frac{BC}{AC} \Rightarrow BC = \cos 30^\circ \times AC = \frac{\sqrt{3}AC}{2} = \frac{30\sqrt{3}}{2} = 15\sqrt{3}\text{ m}$$

Direction : In the question number 19 & 20 , A statement of Assertion (A) is followed by a statement of Reason(R) . Choose the correct option

19. Assertion (A): The largest number that divide 70 and125 which leaves remainder 5 and 8 is 13

Reason (R): HCF (65,117) =13

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A
(c) A is true and R is false
(d) A is false and R is true
Ans: (a) Both A and R are true and R is the correct explanation of A

20. Assertion (A): In ΔABC, DE || BC such that AD = (7x - 4) cm, AE = (5x - 2) cm, DB = (3x + 4) cm and EC = 3x cm than x equal to 5.
Reason (R): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distant point, than the other two sides are divided in the same ratio.
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.
Ans: (d) Assertion (A) is false but Reason (R) is true.

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$$

$\Rightarrow 21x^2 - 12x = 15x^2 + 20x - 6x - 8$
$\Rightarrow 6x^2 - 26 + 8x = 0 \Rightarrow 3x^2 - 13x + 4 = 0$
$\Rightarrow 3x^2 - 12x - x + 4 = 0 \Rightarrow 3x(x - 4) - 1(x - 4) = 0$
$\Rightarrow (x - 4)(3x - 1) = 0 \Rightarrow x = 4, 1/3$
Neglecting x = 1/3 as AD will become negative, we have x = 4
So, A is false but R is true.

SECTION-B
Questions 21 to 25 carry 2M each

21. Find the value of m for which the pair of linear equations:
2x + 3y – 7 = 0 and (m – 1) x + (m + 1) y = (3m – 1) has infinitely many solutions
Ans: For infinitely many solutions the condition is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{m-1} = \frac{3}{m+1} = \frac{7}{3m-1}$$

Now, 2(m + 1) = 3(m – 1) ⇒ m = 5
and 3(3m – 1) = 7(m + 1) ⇒ m = 5
Hence, for m = 5, the system has infinitely many solutions.

22. Find the zeroes of the quadratic polynomials $p(t) = 5t^2 + 12t + 7$ and verify the relationship between the zeroes and the coefficients.
Ans: $5t^2 + 12t + 7 = 0 \Rightarrow 5t^2 + 5t + 7t + 7 = 0$
⇒ 5t (t + 1) + 7(t + 1) = 0 ⇒ (t + 1)(5t + 7) = 0
⇒ t + 1 = 0 ⇒ t = -1
5t + 7 = 0 ⇒ 5t = -7 ⇒ t = -7/5
Therefore, zeroes are (-7/5) and -1
Now, Sum of the zeroes = – (coefficient of x) ÷ coefficient of x2
α + β = – b/a
⇒ (- 1) + (- 7/5) = – (12)/5
⇒ – 12/5 = – 12/5
Product of the zeroes = constant term ÷ coefficient of x2
α β = c/a
⇒ (- 1)(- 7/5) = 7/5
⇒ 7/5 = 7/5

23. Two dice are thrown at the same time. Find the probability of getting (i) same number on both dice (ii) different numbers on both dice.

Ans: Total number of possible outcomes = 36

(i) Same number on both dice.

Number of possible outcomes = 6

Therefore, the probability of getting same number on both dice = 6/36 = 1/6

(ii) Different number on both dice.

Number of possible outcomes = 36 – 6 = 30

Therefore, the probability of getting different number on both dice = 30/36 = 5/6

OR

Cards marked with number 3, 4, 5, …, 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the selected card bears (i) a perfect square number (ii) a single digit number

Ans: Total number of cards = 48

(i) Total number of perfect squares = 6

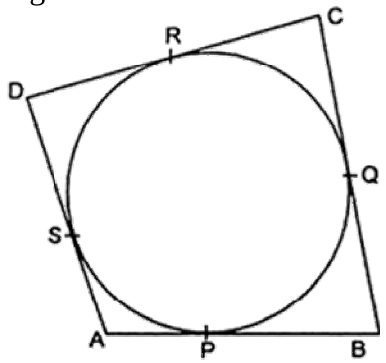
∴ Required Probability = 6/48 = 1/8

(ii) Total single digit numbers = 7

∴ Required Probability = 7/48

24. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.

Ans: We know that the lengths of tangents drawn from an exterior point to a circle are equal.



AP = AS ... (i) [tangents from A]

BP = BQ ... (ii) [tangents from B]

CR = CQ ... (iii) [tangents from C]

DR = DS ... (iv) [tangents from D]

AB + CD = (AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS) [using (1), (ii), (iii), (’v)]

= (AS + DS) + (BQ + CQ)

=AD+ BC.

Hence, AB + CD = AD + BC.

25. Find the points on the ***x***–axis which are at a distance of 2**√**5 from the point (7, –4). How many such points are there?

Ans: Let coordinates of the point=(x, 0)(given that the point lies on x axis)

$x_1 = 7$, y_1=-4and $x_2 = x$, y_2=0

Distance =$\sqrt{(x_2 – x_1)^2 + (y_2 – y_1)^2}$

According to the question, $2\sqrt{5}=\sqrt{(x – 7)^2+ (0 – 4(-4))^2}$

Squaring L.H.S and R.H.S, we get $20 = x^2 + 49 – 14x + 16$

$\Rightarrow 20=x^2 + 65 – 14x \Rightarrow x^2 – 14x + 45 = 0 \Rightarrow x^2 – 9x – 5x + 45 = 0$

$\Rightarrow x(x – 9) – 5(x – 9) = 0 \Rightarrow (x – 9) (x – 5) = 0 \Rightarrow x – 9 = 0, x – 5 = 0$

$\Rightarrow x = 9$ or $x = 5$

Therefore, coordinates of points (9, 0) or (5, 0)

OR

If A and B are (-2, -2) and (2, -4) respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB and P lies on the line segment AB.

Ans: Given that AP = $\frac{3}{7}$ AB ⇒ PB = $\frac{4}{7}$ AB

Therefore, Point P divides AB internally in the ratio 3 : 4

Using section formula, we get

Coordinates of $P = \left(\frac{3\times(2)+4\times(-2)}{3+4}, \frac{3\times(-4)+4\times(-2)}{3+4}\right) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$

SECTION-C

Questions 26 to 31 carry 3 marks each

26. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. Find the minimum distance each should walk so that each can cover the same distance in complete steps.

Ans: Step measures of three persons are 40 cm, 42 cm and 45 cm.

The minimum distance each should walk so that each can cover the same distance in complete steps is the LCM of 40 cm, 42 cm and 45 cm.

Prime factorisation of 40, 42 and 45 gives

40 = 23 × 5, 42 = 2 × 3 × 7, 45 = 32 × 5

LCM (40, 42, 45) = Product of the greatest power of each prime factor involved in the numbers

= 23 × 32 × 5 × 7 = 8 × 9 × 35 = 72 × 35 = 2520 cm.

OR

Show that 5 + 2√7 is an irrational number, where √7 is given to be an irrational number.

Ans: Let 5 + 2√7 is a rational number such that

5 + 2√7 = *a*, where *a* is a rational number

⇒ 2√7 = *a* – 5 ⇒ $\sqrt{7} = \frac{a-5}{2}$

Since a is a rational number and 2, 5 are integers, therefore $\frac{a-5}{2}$ is a rational number

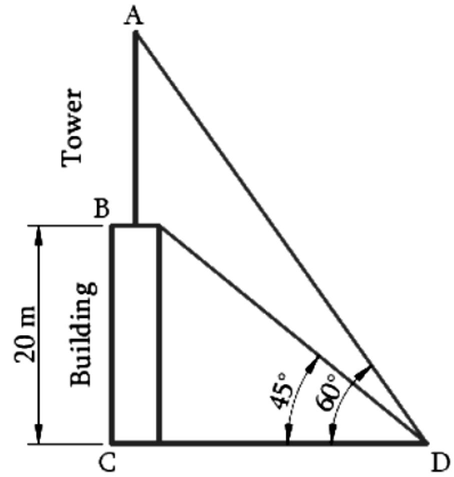
⇒ √7 is a rational number which contradicts the fact that √7 is an irrational number

Therefore, our assumption is wrong

Hence, 5 + 2√7 is an irrational number

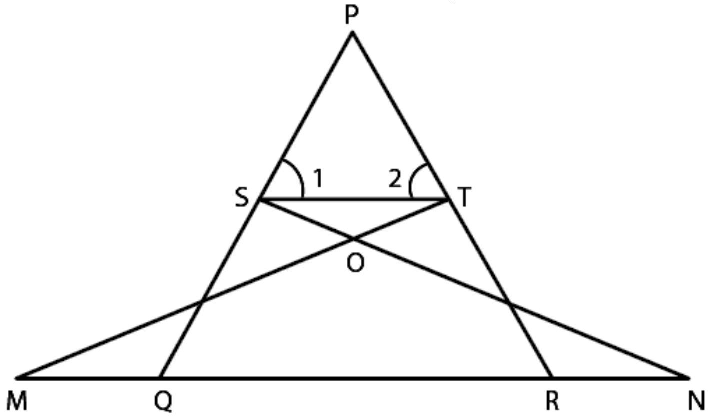
27. From a point on a ground, the angle of elevation of bottom and top of a transmission tower fixed on the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Ans: Let the height of the building is BC, the height of the transmission tower which is fixed at the top of the building be AB.



D is the point on the ground from where the angles of elevation of the bottom B and the top A of the transmission tower AB are 45° and 60° respectively.
The distance of the point of observation D from the base of the building C is CD.
Combined height of the building and tower = AC = AB + BC
In ΔBCD, tan 45° = BC/CD
⇒ 1 = 20/CD
⇒ CD = 20
In ΔACD, tan 60° = AC/CD
⇒ √3 = AC/20
⇒ AC = 20√3
Height of the tower, AB = AC - BC
⇒ AB = 20√3 - 20 m = 20 (√3 - 1) m

28. In the below figure, if ∠1 =∠2 and ΔNSQ = ΔMTR, then prove that ΔPTS ~ ΔPRQ.



Ans: According to the question, Δ NSQ ≅ ΔMTR and ∠1 = ∠2
Since, ΔNSQ = ΔMTR
So, SQ = TR ….(i)
Also, ∠1 = ∠2 ⇒ PT = PS….(ii) [sides opposite to equal angles]
From Equation (i) and (ii),
PS/SQ = PT/TR
⇒ ST || QR (By converse of basic proportionality theorem)
∴ ∠1 = ∠PQR and ∠2 = ∠PRQ (corresponding angles)
In ΔPTS and ΔPRQ.
∠P = ∠P [Common angles]
∠1 = ∠PQR (proved)
∠2 = ∠PRQ (proved)
∴ ΔPTS – ΔPRQ [By AAA similarity criteria]
Hence proved

29. If cosecθ + cotθ = p, then prove that $\cos\theta = \frac{p^2-1}{p^2+1}$

Ans: Given cosecθ + cotθ = p ….. (1)

$\Rightarrow (cosec\theta - \cot\theta)(cosec\theta + \cot\theta) = 1 \Rightarrow (cosec\theta - \cot\theta)\,p = 1$

$\Rightarrow \cos ec\theta - \cot\theta = \frac{1}{p}$ ……. (2)

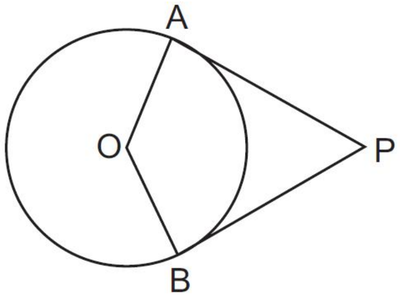
Adding (1) and (2), we get

$$cosec\theta = \frac{p + \frac{1}{p}}{2} = \frac{p^2+1}{2p}; \cot\theta = \frac{p - \frac{1}{p}}{2} = \frac{p^2-1}{2p}$$

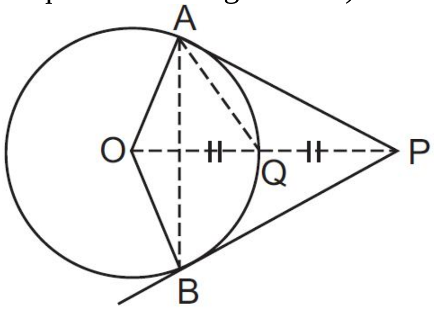
Now, $\cos\theta = \frac{\cot\theta}{\cos ec\theta} = \frac{\frac{p^2-1}{2p}}{\frac{p^2+1}{2p}} = \frac{p^2-1}{p^2+1}$

30. If 2x + y = 23 and 4x – y = 19, find the values of 5y – 2x and y/x – 2.
Ans: Given equations are 2x + y = 23 …(i)
4x – y = 19 …(ii)
On adding both equations, we get 6x = 42
⇒ x = 7
Put the value of x in Eq. (i), we get
2(7) + y = 23
⇒ y = 23 – 14
⇒ y = 9
Hence 5y – 2x = 5(9) – 2(7) = 45 – 14 = 31
y/x – 2 = 9/7 -2 = -5/7

31. In the given figure, OP is equal to diameter of the circle. Prove that ABP is an equilateral triangle.



Ans: Join OP and let it meets the circle at point Q.
Since OP = 2r (Diameter of the circle)
⇒ OQ = QP = r
Consider ΔAOP in which OA ⊥ AP and OP is the hypotenuse.
∴ OQ = AQ = OA
(Mid-point of the hypotenuse is equidistant from the vertices)
⇒ OAQ is an equilateral triangle.
⇒ ∠AOQ = 60° (Each angle of an equilateral triangle is 60°)

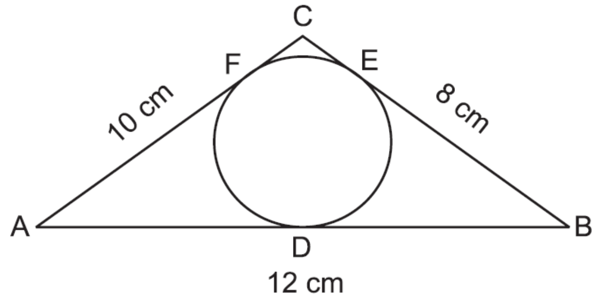


Consider right-angled triangle OAP.
∠AOQ = 60° (Proved above)
∠OAP = 90° ⇒ ∠APO = 30°
∠APB = 2∠APO = 2 × 30° = 60°
Also PA = PB (Tangents to a circle from an external point are equal.)
⇒ ∠PAB = ∠PBA (Angles opposite to equal sides in ΔPAB)
In ΔABP, ∠APB = 60°
⇒ ∠PAB = ∠PBA = $\frac{180^0 - 60^0}{2} = 60^0$
⇒ Each angle of DPAB is 60°

⇒ PAB is an equilateral triangle.

OR

A circle is inscribed in a ΔABC having sides 8 cm, 10 cm and 12 cm as shown in the following figure. Find AD, BE and CF.



Ans: Let AD = x_1, BE = x_2 and CF = x_3;
then AF = AD = x_1, BD = BE = x_2
and CE = CF = x_3.
∴ $x_1 + x_2 = 12$; $x_2 + x_3 = 8$; $x_1 + x_3 = 10$ (1)
Adding,
$2(x_1 + x_2 + x_3) = 30$
⇒ $x_1 + x_2 + x_3 = 15$
Solve for x_1, x_2 and x_3 to get
AD = 7 cm, BE = 5 cm, CF = 3 cm

SECTION-D
Questions 32 to 35 carry 5M each

32. Two pipes running together can fill a cistern in $3\frac{1}{13}$ hours. If one pipe takes 3 hours more than the other to fill it, find the time in which each pipe would fill the cistern.

Ans: Let time taken by faster pipe to fill the cistern be x hrs.
Therefore, time taken by slower pipe to fill the cistern = (x + 3) hrs
Since the faster pipe takes x minutes to fill the cistern.
∴ Portion of the cistern filled by the faster pipe in one hour = $\frac{1}{x}$
Portion of the cistern filled by the slower pipe in one hour = $\frac{1}{x+3}$
Portion of the cistern filled by the two pipes together in one hour = $\frac{1}{\frac{40}{13}} = \frac{13}{40}$
According to question, $\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40} \Rightarrow \frac{x+3+x}{x(x+3)} = \frac{13}{40}$
⇒ 40 (2x + 3) = 13x (x + 3) ⇒ 80x + 120 = 13x2 + 39x
⇒ 13x2 – 41x – 120 = 0 ⇒ 13x2 – 65x + 24x – 120 = 0
⇒ 13x (x – 5) + 24 (x – 5) = 0 ⇒ (x – 5) (13x + 24) = 0
Either x – 5 = 0 or 13x + 24 = 0
⇒ x = 5 as x = −24/13 not possible.
Hence, the time taken by the two pipes is 5 hours and 8 hours respectively.

OR

If Zeba was younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now? [NCERT Exemplar]
Ans: Let the present age of Zeba be x years.
Age before 5 years = (x – 5) years
According to given condition, $(x - 5)^2 = 5x + 11$

$\Rightarrow x^2 + 25 - 10x = 5x + 11 \Rightarrow x^2 - 10x - 5x + 25 - 11 = 0$
$\Rightarrow x^2 - 15x + 14 = 0 \Rightarrow x^2 - 14x - x + 14 = 0$
$\Rightarrow x(x - 14) - 1(x - 14) = 0 \Rightarrow (x - 1)(x - 14) = 0$
$\Rightarrow x - 1 = 0$ or $x - 14 = 0$
$\Rightarrow x = 1$ or $x = 14$
But present age cannot be 1 year.
Hence, Present age of Zeba is 14 years.

33. State and prove Basic Proportional Theorem.
Ans: Statement – 1 mark
Given, To Prove, Construction and Figure – 2 marks
Correct Proof – 2 marks

34. A survey regarding the heights (in cm) of 50 girls of class Xth of a school was conducted and the following data was obtained. Find the mean, median and mode of the given data.

Heights (in cm)	120 – 130	130 – 140	140 – 150	150 – 160	160 – 170
No. of Girls	2	8	12	20	8

Ans:

Height (in cm)	Number of girls	Cumulative frequency
120 – 130	2	2
130 – 140	8	10
140 – 150	$12 = f_0$	$22 = c.f.$
150 – 160	$20 = f_1$	42
160 – 170	$8 = f_2$	50
Total	50	

$n = 50 \Rightarrow \frac{n}{2} = 25$

$\therefore$ Median class = 150 – 160
$l = 150$, $c.f. = 22$, $f = 20$, $h = 10$

$\therefore$ Median $= l + \frac{\frac{n}{2} - c.f.}{f} \times h$

$= 150 + \frac{25 - 22}{20} \times 10 = 150 + 1.5 = 151.5$

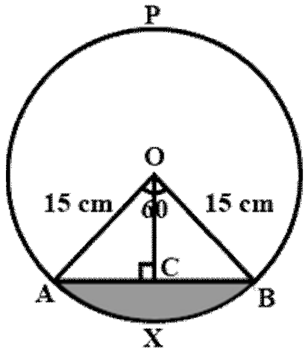
Modal class = 150 – 160
$l = 150$, $h = 10$, $f_1 = 20$, $f_0 = 12$, $f_2 = 8$

Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 150 + \frac{20 - 12}{2 \times 20 - 12 - 8} \times 10 = 150 + 4 = 154$

Now, Mode = 3 Median – 2 Mean
$\Rightarrow 154 = 3 \times 151.5 - 2 \text{ Mean} \Rightarrow 154 - 454.5 = -2 \text{ Mean}$
$\Rightarrow 300.5 = 2 \text{ Mean} \Rightarrow \text{Mean} = \frac{300.5}{2} = 150.25$

35. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and √3 = 1.73)
Ans: Here, O is the centre of circle, AB is a chord
AXB is a major arc, OA = OB = radius = 15 cm
Arc AXB subtends an angle 60° at O.

Area of sector $AOB = \frac{60}{360} \times \pi \times r^2 = \frac{60}{360} \times 3.14 \times (15)^2 = 117.75 cm^2$



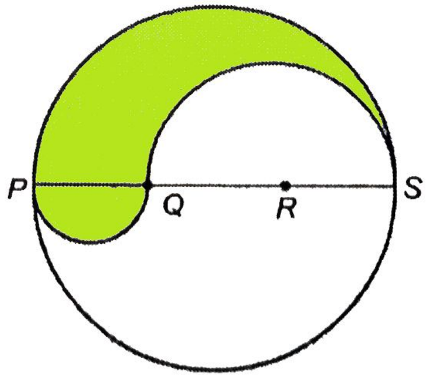
Area of minor segment (Area of Shaded region) = Area of sector AOB − Area of △ AOB
By trigonometry, AC = 15sin30º and OC = 15cos30º
Also, AB = 2AC
∴ AB = 2 × 15sin30º = 15 cm

$$\therefore OC = 15\cos 30^0 = 15\frac{\sqrt{3}}{2} = 15\times\frac{1.73}{2} = 12.975$$

∴ Area of △AOB = 0.5×15×12.975 = 97.3125cm^2
∴ Area of minor segment (Area of Shaded region) =117.75 − 97.3125 = 20.4375 cm^2
Area of major segment = Area of circle − Area of minor segment
= (3.14×15×15) − 20.4375 = 686.0625 cm^2

OR

PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in below figure. Find the perimeter and area of the shaded region



Ans: Here, PS = 12 cm

as PQ = QR = RS = $\frac{1}{3}$ × PS = $\frac{1}{3}$ ×12 = 4 cm

and QS = 2PQ ⇒ QS = 2 × 4 = 8 cm
Area of shaded region: A = area of a semicircle with PS as diameter + area of a semicircle with PQ as diameter – the area of a semicircle with QS as diameter;

$= \frac{1}{2}$ [3.14 × 6^2 + 3.14 × 2^2 - 3.14 × 4^2]

$= \frac{1}{2}$ [3.14 ×36 + 3.14 ×4 – 3.14 ×16]

$= \frac{1}{2}$ [3.14 (36 + 4 – 16)]

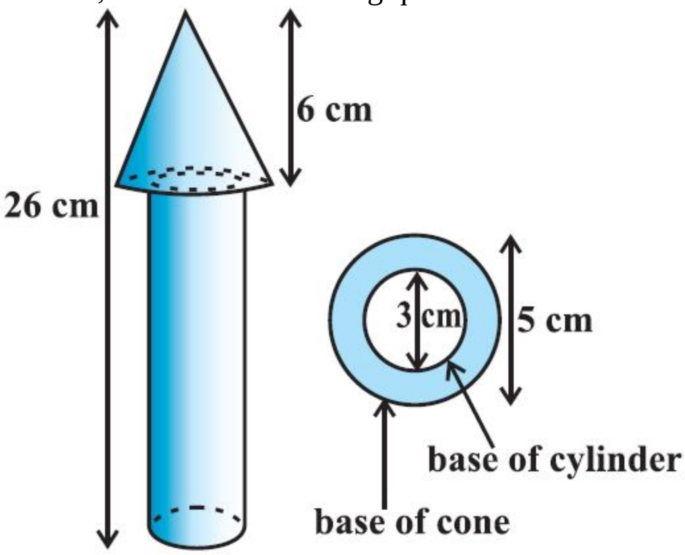
$= \frac{1}{2}$ (3.14 × 24) = $\frac{1}{2}$ × 75.36 = 37.68 cm^2

The area of shaded region = 37.68 cm^2.
The perimeter of the shaded region = Arc of the semicircle of radius 6 +Arc of the semicircle of radius 4 + Arc of the semicircle of radius 2
= (6π + 4π + 2π) = 12π

$= 12 \times \frac{22}{7} = \frac{264}{7}$ = 37.71 cm

SECTION-E (Case Study Based Questions)
Questions 36 to 38 carry 4M each

36. In a toys manufacturing company, wooden parts are assembled and painted to prepare a toy. One specific toy is in the shape of a cone mounted on a cylinder. For the wood processing activity center, the wood is taken out of storage to be sawed, after which it undergoes rough polishing, then is cut, drilled and has holes punched in it. It is then fine polished using sandpaper. For the retail packaging and delivery activity center, the polished wood sub-parts are assembled together, then decorated using paint. The total height of the toy is 26 cm and the height of its conical part is 6 cm. The diameters of the base of the conical part is 5 cm and that of the cylindrical part is 3 cm. On the basis of the above information, answer the following questions:



(a) If its cylindrical part is to be painted yellow, find the surface area need to be painted. [1]
(b) If its conical part is to be painted green, find the surface area need to be painted. [2]

OR

(b) Find the volume of the wood used in making this toy. [2]
(c) If the cost of painting the toy is 3 paise per sq cm, then find the cost of painting the toy. (Use π = 3.14) [1]

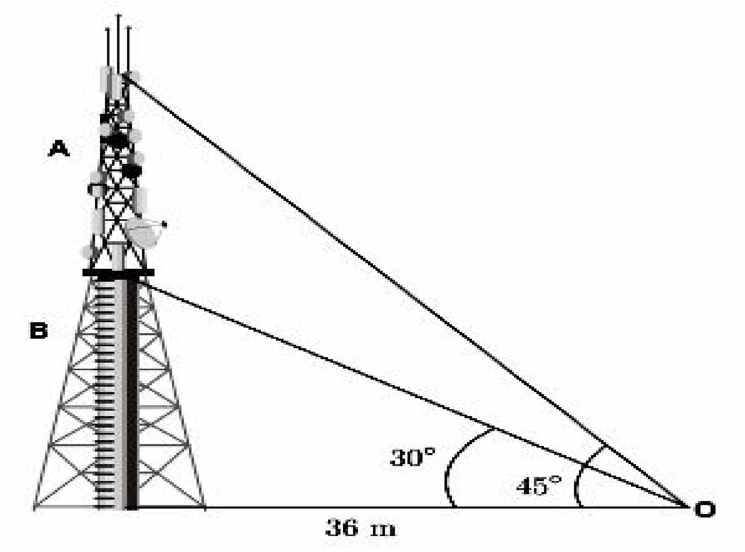
Ans: Let the radius of cone be r, slant height of cone be l, height of cone be h, radius of cylinder be r′ and height of cylinder be h′.
Then r = 2.5 cm, h = 6 cm, r′ = 1.5 cm, h′ = 26 – 6 = 20 cm and
Slant height, $l = \sqrt{r^2 + h^2} = \sqrt{2.5^2 + 6^2} = \sqrt{6.25 + 36} = \sqrt{42.25} = 6.5cm$
(a) Area to be painted yellow = CSA of the cylinder + area of one base of the cylinder
$= 2\pi r'h' + \pi(r')^2 = \pi r'(2h' + r') = (3.14 \times 1.5)(2 \times 20 + 1.5)$ cm^2
$= 4.71 \times 41.5$ cm^2
$= 195.465$ cm^2
(b) Area to be painted green = CSA of the cone + base area of the cone – base area of the cylinder
$= \pi rl + \pi r^2 - \pi(r')^2 = \pi[(2.5 \times 6.5) + (2.5)^2 - (1.5)^2]$ cm^2
$= \pi[20.25]$ cm^2 $= 3.14 \times 20.25$ cm^2
$= 63.585$ cm^2

OR

Volume of wood used in making the toy = Volume of cone + Volume of cylinder
$= \frac{1}{3}\pi r^2 h + \pi r'^2 h' = \pi\left[\frac{1}{3}r^2 h + r'^2 h'\right] = 3.14\left[\frac{1}{3} \times 2.5 \times 2.5 \times 6 + 1.5 \times 1.5 \times 20\right]$
$= 3.14(12.5 + 45) = 180.55 cm^3$
(c) Total area of painting = 195.465 + 63.585 = 259.05 cm^2
Cost of painting 1 cm^2 = Re. 0.03
Total cost of painting = Rs. 0.03 x 256.05
= Rs. 7.77

37. Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure, including microwave dishes. They are among the tallest human-made structures. There are 2 main types: guyed and self-supporting structures. On a similar concept, a radio station tower was built in two sections A and B.

Tower is supported by wires from a point O. Distance between the base of the tower and point O is 36 m. From point O, the angle of elevation of the top of section B is 30° and the angle of elevation of the top of section A is 45°.



(i) What is the height of the section B? (1)

(ii) What is the height of the section A? (1)

(iii) What is the length of the wire structure from the point O to the top of section A? (2)

OR

(iii) What is the length of the wire structure from the point O to the top of section B? (2)

Ans: Given, that the distance between the base of the tower and point O = 36 m

(i)Consider ΔOCB, $\tan 30^0 = \frac{BC}{OC} \Rightarrow \frac{BC}{36} = \frac{1}{\sqrt{3}}$

Hence, BC= $12\sqrt{3}$ = 20.78 m

(ii)In ΔOAC, $\tan 45^0 = \frac{AB+BC}{OC} \Rightarrow \frac{AC}{36} = 1 \Rightarrow AC = 36$m

∴ Height of section A = $36 - 12\sqrt{3} = 12(3 - \sqrt{3})$ m

(iii) length of the wire structure from the point O to the top of the section A

$\cos 45^0 = \frac{36}{OA} \Rightarrow OA = 36\sqrt{2}$M

OR

length of the wire structure from the point O to the top of the section B

$\cos 30^0 = \frac{OC}{OB} \quad \Rightarrow \frac{\sqrt{3}}{2} = \frac{36}{OB} \quad \Rightarrow OB = 72 / \sqrt{3} = 24\sqrt{3}$m

38. Mohan is an auto driver. His autorickshaw was too old and he had to spend a lot of money on repair and maintenance every now and then. One day he got to know about the EV scheme of the Government of India where he can not only get a good exchange bonus but also avail heavy discounts on the purchase of an electric vehicle. So, he took a loan of 71,18,000 from a reputed bank and purchased a new autorickshaw.

Mohan repays his total loan of 118000 rupees by paying every month starting with the first instalment of 1000 rupees.

(i) If he increases the instalment by 100 rupees every month, then what amount will be paid by him in the 30th instalment? [1]
(ii) If he increases the instalment by 100 rupees every month, then what amount of loan does he still have to pay after 30th instalment? [2]

OR

(ii)If he increases the instalment by 200 rupees every month, then what amount would he pay in 40th instalment? [2]
(iii) If he increases the instalment by 100 rupees every month, then what amount will be paid by him in the 100th instalment [1]
Ans: (i) Clearly, the amount of installment in the first month = Rs. 1000, which increases by Rs. 100 every month
therefore, installment amount in second month = Rs. 1100, third month = Rs. 1200, fourth month = Rs. 1300 .....
which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100
Now, amount paid in the 30th installment,
$a_{30} = 1000 + (30 - 1)100 = 3900$ [$\because a_n = a + (n - 1)d$]
(ii) Amount paid in 30 instalments,
$S_{30} = \frac{30}{2}[2 \times 1000 + (30 - 1)100] = 73500$
Hence, remaining amount of loan that he has to pay = 118000 - 73500 = Rs. 44500

OR

If he increases the instalment by 200 rupees every month, amount would he pay in 40th instalment
Then a = 1000, d = 200 and n= 40
$a_{40} = a + 39d$
$\Rightarrow a_{40} = 1000 + 39(200)$
$\Rightarrow a_{40} = 880$
(iii) Here, a = 1000 and common difference, d = Rs. 100
Amount paid in the 100th instalments
$a_{100} = 1000 + (100 - 1)100 = 10900$ [$\because a_n = a + (n - 1)d$]