PM S HRI KExviRIyA VI DYALAYA GACHIBO WLI, GPRA CAMPUS, HYD- 32
$\mathcal{S A M P L E} \operatorname{PAPER} \mathcal{T E S T} 08 \mathcal{F O R} \mathcal{B O} \mathcal{A R D} \operatorname{EXAM} 2024$
( $\mathcal{A N W E R S ) ~}$
$\mathcal{S U B I} \mathcal{E C T}: ~ \mathscr{M A T H E M A T}$ ICS
$\mathcal{M A X}$. $\operatorname{MAR} \mathcal{R} S: 80$
CLASS : $X$
$\mathcal{D I R A T I O \mathcal { N }}: 3 \mathcal{H R S}$

## General Instruction:

1. This Question Paper has 5 Sections A-E.
2. Section $\mathbf{A}$ has 20 MCQs carrying 1 mark each.
3. Section $\mathbf{B}$ has 5 questions carrying 02 marks each.
4. Section $\mathbf{C}$ has 6 questions carrying 03 marks each.
5. Section $\mathbf{D}$ has 4 questions carrying 05 marks each.
6. Section $\mathbf{E}$ has 3 case based integrated units of assessment ( 04 marks each) with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi=22 / 7$ wherever required if not stated.

## SECTION - A

## Questions 1 to 20 carry 1 mark each.

1. The pair of linear equations $2 x+3 y=5$ and $4 x+6 y=10$ is
(a) inconsistent
(b) consistent
(c) dependent consistent
(d) none of these

Ans: (c)
$\frac{a_{1}}{a_{2}}=\frac{2}{4}=\frac{1}{2}, \frac{b_{1}}{b_{2}}=\frac{3}{6}=\frac{1}{2}, \frac{c_{1}}{c_{2}}=\frac{-5}{-10}=\frac{1}{2}$
$\Rightarrow \frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}$
Therefore, the pair of linear equations has infinity many solutions and hence dependent consistent
2. Points $A(3,1), B(5,1), C(a, b)$ and $D(4,3)$ are vertices of a parallelogram $A B C D$. The values of $a$ and $b$ are respectively
(a) $a=6, b=3$ (b) $a=2, b=1$
(c) $a=4, b=2$
(d) None of these

Ans: (a) $a=6, b=3$

ABCD is a parallelogram.


Since, the diagonals of a parallelogram bisect each other.

$$
\begin{aligned}
& \therefore\left(\frac{3+a}{2}, \frac{1+b}{2}\right)=\left(\frac{4+5}{2}, \frac{3+1}{2}\right) \\
& \Rightarrow \frac{3+a}{2}=\frac{9}{2} \Rightarrow 3+a=9 \\
& \Rightarrow a=6 \\
& \text { and } \frac{1+b}{2}=\frac{4}{2} \Rightarrow 1+b=4 \Rightarrow b=3
\end{aligned}
$$

Hence, $a=6$ and $b=3$.
3. If $\triangle \mathrm{ABC} \sim \triangle \mathrm{EDF}$ and $\triangle \mathrm{ABC}$ is not similar to $\triangle \mathrm{DEF}$, then which of the following is not true?
(a) $\mathrm{BC} \cdot \mathrm{EF}=\mathrm{AC} \cdot \mathrm{FD}$
(b) $\mathrm{AB} \cdot \mathrm{EF}=\mathrm{AC} \cdot \mathrm{DE}$
(c) $\mathrm{BC} \cdot \mathrm{DE}=\mathrm{AB} \cdot \mathrm{EF}$
(d) $\mathrm{BC} \cdot \mathrm{DE}=\mathrm{AB} \cdot \mathrm{FD}$

Ans: (c) $\mathrm{BC} \cdot \mathrm{DE}=\mathrm{AB} \cdot \mathrm{EF}$
4. If $\sec \mathrm{A}=15 / 7$ and $\mathrm{A}+\mathrm{B}=90^{\circ}$, find the value of $\operatorname{cosec} \mathrm{B}$.
(a) $8 / 7$
(b) $12 / 7$
(c) $7 / 15$
(d) $15 / 7$

Ans: (d)
5. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600 . If one number is 280 , then the other number is
(a) 20
(b) 28
(c) 60
(d) 80

Ans: (d) According to the question, $\mathrm{LCM}+\mathrm{HCF}=600$
Since LCM $=14 \times$ HCF
$\Rightarrow 14 \times \mathrm{HCF}+\mathrm{HCF}=600 \Rightarrow 15 \times \mathrm{HCF}=600 \Rightarrow \mathrm{HCF}=600 \div 15=40$
$\Rightarrow$ LCM $=600-\mathrm{HCF}=600-40=560$
We know that $\operatorname{HCF}(a, b) \times \operatorname{LCM}(a, b)=a \times b$
$\Rightarrow$ Other number $=40 \times 560 / 280=80$
6. When 2120 is expressed as the product of its prime factors we get
(a) $2 \times 5^{3} \times 53$
(b) $2^{3} \times 5 \times 53$
(c) $5 \times 7^{2} \times 31$
(d) $5^{2} \times 7 \times 33$

Ans: (b) $2120=2 \times 2 \times 2 \times 5 \times 53=2^{3} \times 5 \times 53$
7. If $p$ and $q$ are the zeroes of the quadratic polynomial $f(x)=2 x^{2}-7 x+3$, find the value of $p+q$ -pq is
(a) 1
(b) 2
(c) 3
(d) None of these

Ans: (b) 2
8. ABCD is a trapezium with $\mathrm{AD} \| \mathrm{BC}$ and $\mathrm{AD}=4 \mathrm{~cm}$. If the diagonals AC and BD intersect each other at O such that $\mathrm{AO} / \mathrm{OC}=\mathrm{DO} / \mathrm{OB}=1 / 2$, then $\mathrm{BC}=$
(a) 6 cm
(b) 7 cm
(c) 8 cm
(d) 9 cm

Ans: (c) 8 cm
$A D \| B C$
and $\frac{A O}{O C}=\frac{D O}{O B}=\frac{1}{2}$
$\therefore \frac{A D}{B C}=\frac{A O}{O C} \Rightarrow \frac{4}{B C}=\frac{1}{2} \Rightarrow B C=8 \mathrm{~cm}$

9. If the angle between two radii of a circle is $140^{\circ}$, then the angle between the tangents at the ends of the radii is
(a) $90^{\circ}$
(b) $50^{\circ}$
(c) $70^{\circ}$
(d) $40^{\circ}$

Ans: (d) $40^{\circ}$
10. The number of revolutions made by a circular wheel of radius 0.7 m in rolling a distance of 176 m is
(a) 22
(b) 24
(c) 75
(d) 40

Ans: (d) 40
Number of revolutions $=\frac{\text { total distance }}{\text { circumference }}=\frac{176}{2 \times \frac{22}{7} \times 0.7}=40$
11. In $\triangle A B C$, right angled at $B, A B=5 \mathrm{~cm}$ and $\sin C=1 / 2$. Determine the length of side $A C$.
(a) 10 cm
(b) 15 cm
(c) 20 cm
(d) none of these

Ans: (a)

$\sin C=\frac{A B}{A C} \Rightarrow \frac{1}{2}=\frac{5}{A C} \Rightarrow A C=10 \mathrm{~cm}$
12. In the $\triangle A B C, D$ and $E$ are points on side $A B$ and $A C$ respectively such that $D E \| B C$. If $A E=2$ $\mathrm{cm}, \mathrm{AD}=3 \mathrm{~cm}$ and $\mathrm{BD}=4.5 \mathrm{~cm}$, then CE equals
(a) 1 cm
(b) 2 cm
(c) 3 cm
(d) 4 cm

Ans: (c) 3 cm
$\frac{A D}{B D}=\frac{A E}{C E} \Rightarrow \frac{3}{4.5}=\frac{2}{C E} \Rightarrow C E=3 \mathrm{~cm}$
13. The median class of the following data is:

| Marks | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students | 8 | 10 | 12 | 22 | 30 | 18 |

(a) 20-30
(b) $30-40$
(c) $40-50$
(d) $50-60$

Ans: (b)
14. Two dice are thrown simultaneously. What is the probability of getting doublet?
(a) $1 / 36$
(b) $1 / 6$
(c) $5 / 6$
(d) $11 / 36$

Ans: (b) $1 / 6$
Number of Possible outcomes are 36
Number of favourable outcomes $=6$
Probability $=6 / 36=1 / 6$
15. If $4 \tan \theta=3$, then the value of $\frac{4 \sin \theta-\cos \theta}{4 \sin \theta+\cos \theta}$ is
(a) $1 / 2$
(b) $1 / 3$
(c) $1 / 4$
(d) $1 / 5$

Ans: (a) $1 / 2$
Dividing Numerator and Denominator by $\cos \theta$, we get
$\frac{4 \sin \theta-\cos \theta}{4 \sin \theta+\cos \theta}=\frac{4 \tan \theta-1}{4 \tan \theta+1}=\frac{4 \times \frac{3}{4}-1}{4 \times \frac{3}{4}+1}=\frac{3-1}{3+1}=\frac{2}{4}=\frac{1}{2}$
16. The area of the square that can be inscribed in a circle of radius 8 cm is
(a) $256 \mathrm{~cm}^{2}$
(b) $128 \mathrm{~cm}^{2}$
(c) $64 \sqrt{ } 2 \mathrm{~cm}^{2}$
(d) $64 \mathrm{~cm}^{2}$

Ans: (b) $128 \mathrm{~cm}^{2}$
17. The ratio of the total surface area to the lateral surface area of a cylinder with base radius 80 cm and height 20 cm is
(a) $1: 2$
(b) $2: 1$
(c) $3: 1$
(d) $5: 1$

Ans: (d) $5: 1$
$\frac{\text { Total surface area }}{\text { Lateral surface area }}=\frac{2 \pi r(h+r)}{2 \pi r h}=\frac{h+r}{h}=\frac{(20+80)}{20}=\frac{100}{20}=\frac{5}{1}$
18. The mean and mode of a frequency distribution are 28 and 16 respectively. The median is
(a) 22
(b) 23.5
(c) 24
(d) 24.5

Ans: (c) 24
We know that, Mode $=3$ Median -2 Mean
$\Rightarrow 3$ Median $=$ Mode +2 Mean
$\Rightarrow 3$ Median $=16+2 \times 28 \Rightarrow$ Median $=72 / 3=24$
DIRECTION: In the question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R).
19. Statement A (Assertion): The value of y is -6 , for which the distance between the points $\mathrm{P}(2$, $3)$ and $\mathrm{Q}(10, y)$ is 10 .
Statement R(Reason) : Distance between two given points A ( $\mathrm{x}_{1}, \mathrm{y}_{1}$ ) and $\mathrm{B}\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ is given by $\mathrm{AB}=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Ans: (d) Assertion (A) is false but reason (R) is true.
20. Statement A (Assertion): The number $6^{\mathrm{n}}$ never end with digit 0 for any natural number n .. Statement R(Reason) : The number $9^{\mathrm{n}}$ never end with digit 0 for any natural number n .
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Ans: (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

## SECTION - B

## Questions 21 to 25 carry 2 marks each.

21. In the given figure, find the value of x in terms of $\mathrm{a}, \mathrm{b}$ and c .


Ans: In $\Delta \mathrm{s}$ LMK and PNK, we have
$\angle \mathrm{M}=\angle \mathrm{N}=50^{\circ}$ and $\angle \mathrm{K}=\angle \mathrm{K}$
So, by AA similarity criterion, $\triangle \mathrm{LMK} \sim \Delta \mathrm{PNK}$
Thus, $\frac{L M}{P N}=\frac{K M}{K N} \Rightarrow \frac{a}{x}=\frac{b+c}{c} \Rightarrow x=\frac{a c}{b+c}$
22. XY and MN are the tangents drawn at the end points of the diameter DE of the circle with centre O. Prove that XY \| MN.
Ans: Since, XY is the tangent to the circle at the point D .
$\Rightarrow \mathrm{OD} \perp \mathrm{XY} \Rightarrow \angle \mathrm{EDX}=90^{\circ}$
Also, MN is the tangent to the circle at E .

$\Rightarrow \mathrm{OE} \perp \mathrm{MN} \Rightarrow \angle \mathrm{DEN}=90^{\circ}$
As, $\angle \mathrm{EDX}=\angle \mathrm{DEN}\left(\right.$ each $\left.90^{\circ}\right)$
which are alternate interior angles.
$\Rightarrow \mathrm{XY} \| \mathrm{MN}$
23. A rope by which a cow is tethered is increased from 16mto 23 m . How much additional ground does it have now to graze?
Ans: Given : length of rope $(\mathrm{r})=16 \mathrm{~m}$
Increased length of rope $(\mathrm{R})=23 \mathrm{~m}$

Hence the additional area cow can graze $=\pi R^{2}-\pi r^{2}=\pi\left(R^{2}-r^{2}\right)$
$=\frac{22}{7}\left(23^{2}-16^{2}\right)=\frac{22}{7}(529-256)$
$=\frac{22}{7} \times 273=858 \mathrm{~m}^{2}$

## OR

In the below figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm . If $\mathrm{OD}=2$ cm , find the area of the (i) quadrant OACB , (ii) shaded region.


Ans: (i) Area of the quadrant $\mathrm{OACB}=\frac{1}{4} \pi r^{2}=\frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}=\frac{77}{8}=9.625 \mathrm{~cm}^{2}$
(ii) Area of the $\triangle \mathrm{BOD}=(1 / 2) \times \mathrm{OB} \times \mathrm{OD}=(1 / 2) \times 3.5 \times 2=3.5 \mathrm{~cm}^{2}$

Area of the shaded region $=$ Area of the quadrant $\mathrm{OACB}-$ Area of the $\triangle \mathrm{BOD}$

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=9.625-3.5=6.125 \mathrm{~cm}^{2}
$$

24. In figure, $A B C D$ is a rectangle. Find the values of $x$ and $y$.


Ans: $\mathrm{AB}=\mathrm{DC}$ and $\mathrm{BC}=\mathrm{AD}$
$\Rightarrow \mathrm{x}+\mathrm{y}=30$
and $\mathrm{x}-\mathrm{y}=14$...(ii)
Adding (i) and (ii), we get $2 \mathrm{x}=44 \Rightarrow \mathrm{x}=22$
$\Rightarrow \mathrm{y}=30-22=8$
Thus, $x=22$ and $y=8$
25. Find $A$ and $B$, if $\sin (A+2 B)=\sqrt{3} / 2$ and $\cos (A+B)=1 / 2$.

Ans: Given : $\sin (\mathrm{A}+2 \mathrm{~B})=\sin 60^{\circ}$
$\Rightarrow \mathrm{A}+2 \mathrm{~B}=60^{\circ} \ldots$ (i)
$\cos (\mathrm{A}+\mathrm{B})=\cos 60^{\circ}$
$\Rightarrow A+B=60^{\circ}$
Subtracting equation (i) and (ii), we get $\mathrm{B}=0^{\circ}$
Putting the value of $B$ in equation (ii), we get,
$\mathrm{A}=60^{\circ}-0^{\circ}=60^{\circ}$
So, $\mathrm{A}=60^{\circ}$ and $\mathrm{B}=0^{\circ}$.

## OR

If $(1+\cos A)(1-\cos A)=3 / 4$, find the value of $\tan A$.
Ans: $(1+\cos \mathrm{A})(1-\cos \mathrm{A})=3 / 4$
$\Rightarrow 1-\cos ^{2} \mathrm{~A}=3 / 4 \Rightarrow \cos ^{2} \mathrm{~A}=1-3 / 4=1 / 4 \Rightarrow \cos \mathrm{~A}= \pm 1 / 2$
Also, $1-\cos ^{2} \mathrm{~A}=3 / 4 \Rightarrow \sin ^{2} \mathrm{~A}=3 / 4 \Rightarrow \sin \mathrm{~A}= \pm \sqrt{ } 3 / 2$
$\Rightarrow \tan A=\sin A / \cos A= \pm \sqrt{ } 3$

## SECTION - C

## Questions 13 to 22 carry 3 marks each.

26. A part of monthly hostel charges in a college is fixed and the remaining depends on the number of days one has taken food in the mess. When a student ' A ' takes food for 22 days, he has to pay Rs. 1380 as hostel charges; whereas a student 'B', who takes food for 28 days, pays Rs. 1680 as hostel charges. Find the fixed charges and the cost of food per day.
Ans: Let the fixed hostel charges be Rs. x and the cost of food per day be Rs. y .
According to the question, we get
$x+22 y=1380 \ldots$..i)
and $x+28 y=1680 \ldots$...(ii)
Subtracting (i) from (ii), we get
$6 y=300 \Rightarrow y=300 \div 6=50$
Putting $y=50$ in (i), we get
$\mathrm{x}+22(50)=1380 \Rightarrow \mathrm{x}+1100=1380 \Rightarrow \mathrm{x}=280$
$\therefore$ Fixed hostel charges $=$ Rs. 280 and cost of the food per day $=$ Rs. 50.

## OR

The ratio of income of two persons is $9: 7$ and the ratio of their expenditure is $4: 3$, if each of them manage to save Rs. 2000/month. Find their monthly incomes.
Ans: Let the income of first person be $9 x$ and the income of second person be $7 x$. Further, let the expenditures of first and second persons be $4 y$ and $3 y$ respectively. Then, Saving of the first person $=9 x-4 y$
Saving of the second person $=7 x-3 y$
According to question,
$9 x-4 y=2000$ or $9 x-4 y-2000=0 \ldots$..(i)
and $7 \mathrm{x}-3 \mathrm{y}=2000$ or $7 \mathrm{x}-3 \mathrm{y}-2000=0 \ldots$..(ii)
Solving (i) and (ii), we get $x=2000$ and $y=4000$
Thus, monthly income of first person $=9 \times 2000=$ Rs. 18000
Monthly income of second person $=7 \times 4000=$ Rs. 28000
27. Prove that: $\frac{\sin \theta-\cos \theta+1}{\sin \theta+\cos \theta-1}=\sec \theta+\tan \theta$

And: LHS $=\frac{\tan \theta-1+\sec \theta}{\tan \theta+1-\sec \theta}$ (Dividing numerator and denominator by $\cos \theta$ )
$=\frac{\tan \theta+\sec \theta-1}{\tan \theta+1-\sec \theta}$
$=\frac{\tan \theta+\sec \theta-\left(\sec ^{2} \theta-\tan ^{2} \theta\right)}{\tan \theta+1-\sec \theta}$
$=\frac{(\sec \theta+\tan \theta)(1-\sec \theta+\tan \theta)}{\tan \theta+1-\sec \theta}$
$=\sec \theta+\tan \theta=$ RHS
28. Prove that $\sqrt{ } 5$ is and irrational number.

Ans: Let $\sqrt{5}$ is a rational number then we have $\sqrt{5}=\frac{p}{q}$, where p and q are co-primes.
$\Rightarrow p=\sqrt{5} q$
Squaring both sides, we get $p^{2}=5 q^{2}$
$\Rightarrow \mathrm{p}^{2}$ is divisible by $5 \Rightarrow \mathrm{p}$ is also divisible by 5
So, assume $p=5 \mathrm{~m}$ where m is any integer.
Squaring both sides, we get $\mathrm{p}^{2}=25 \mathrm{~m}^{2}$
But $p^{2}=5 q^{2}$
Therefore, $5 \mathrm{q}^{2}=25 \mathrm{~m}^{2} \Rightarrow \mathrm{q}^{2}=5 \mathrm{~m}^{2}$
$\Rightarrow q^{2}$ is divisible by $5 \Rightarrow q$ is also divisible by 5
From above we conclude that p and q have one common factor i.e. 5 which contradicts that p and $q$ are co-primes.
Therefore, our assumption is wrong.
Hence, $\sqrt{5}$ is an irrational number.
29. Find the zeroes of the quadratic polynomial $6 x^{2}-7 x-3$ and verify the relationship between the zeroes and the coefficients of the polynomial.
Ans: $6 x^{2}-7 x-3=0$
$\Rightarrow 6 x^{2}-9 x+2 x-3=0$
$\Rightarrow 3 \mathrm{x}(2 \mathrm{x}-3)+1(2 \mathrm{x}-3)=0$
$\Rightarrow(3 \mathrm{x}+1)(2 \mathrm{x}-3)=0$
$\Rightarrow \mathrm{x}=\frac{-1}{3}, \frac{3}{2}$
Now, $\alpha+\beta=\frac{-1}{3}+\frac{3}{2}=\frac{-2+9}{6}=\frac{7}{6}$ and $\frac{-b}{a}=\frac{7}{6} \Rightarrow \alpha+\beta=\frac{-b}{a}$

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\alpha \beta=\frac{-1}{3} \times \frac{3}{2}=\frac{-1}{2} \text { and } \frac{c}{a}=\frac{-1}{2} \Rightarrow \alpha \beta=\frac{c}{a}
$$

30. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the center
Ans: Given: $X Y$ and $X^{\prime} Y^{\prime}$ are two parallel tangents to the circle wth centre $O$ and $A B$ is the tangent at the point C , which intersects XY at A and $\mathrm{X}^{\prime} \mathrm{Y}^{\prime}$ at B .


In $\triangle \mathrm{OAP}$ and $\triangle \mathrm{OAC}$
$\mathrm{AP}=\mathrm{AC}($ Tangents from to same point A$)$
$\mathrm{PO}=\mathrm{OC}($ Radii of the same circle $)$
$\mathrm{OA}=\mathrm{OA}($ Common side)
so, $\triangle \mathrm{OAP}=\triangle \mathrm{OAC}$ ( SSS congruence criterion)
$\therefore \angle \mathrm{AOP}=\angle \mathrm{AOC}=\angle 1(\mathrm{CPCT})$
Similarly, $\angle \mathrm{BOQ}=\angle \mathrm{BOC}=\angle 2$
Now, POQ is a diameter of the circle.
Hence, it is a straight line.
$\therefore \angle 1+\angle 1+\angle 2+\angle 2=180^{\circ}$
$2(\angle 1+\angle 2)=180^{\circ}$
$\therefore \angle 1+\angle 2=90^{\circ}$
$\therefore \angle \mathrm{AOB}=90^{\circ}$.

## OR

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.
Ans: Let ABCD be the quadrilateral circumscribing a circle at the center $O$ such that it touches the circle at the point $P, Q, R, S$. Let join the vertices of the quadrilateral $A B C D$ to the center of the circle


In $\triangle \mathrm{OAP}$ and $\Delta \mathrm{OAS}$
$\mathrm{AP}=\mathrm{AS}$ ( Tangents from to same point A )
$\mathrm{PO}=\mathrm{OS}$ ( Radii of the same circle)
$\mathrm{OA}=\mathrm{OA}$ ( Common side)
so, $\triangle \mathrm{OAP}=\triangle \mathrm{OAS}$ (SSS congruence criterion)
$\therefore \angle \mathrm{POA}=\angle \mathrm{AOS}$ (CPCT)
$\Rightarrow \angle 1=\angle 8$
Similarly, $\angle 2=\angle 3, \angle 4=\angle 5$ and $\angle 6=\angle 7$
$\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7+\angle 8=360^{\circ}$
$\Rightarrow(\angle 1+\angle 8)+(\angle 2+\angle 3)+(\angle 4+\angle 5)+(\angle 6+\angle 7)=360^{\circ}$
$\Rightarrow 2(\angle 1)+2(\angle 2)+2(\angle 5)+2(\angle 6)=360^{\circ}$
$\Rightarrow(\angle 1)+(\angle 2)+(\angle 5)+(\angle 6)=180^{\circ}$
$\therefore \angle \mathrm{AOD}+\angle \mathrm{COD}=180^{\circ}$
Similarly, $\angle \mathrm{BOC}+\angle \mathrm{DOA}=180^{\circ}$
31. One card is drawn at random from a well-shuffled deck of 52 playing cards. Find the probability that the card drawn is (i) either a red card or a king, (ii) neither a red card nor a queen.
Ans: Total number of cards $=52$
(i) Number of either red card or a king card $=28$

Required Probability $=\frac{28}{52}=\frac{7}{13}$
(ii) Number of cards neither a red card or a queen card $=52-28=24$

Required Probability $=\frac{24}{52}=\frac{6}{13}$

## SECTION - D

Questions 32 to 35 carry 5 marks each.
32. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top which is open, is 5 cm . It is filled with water upto the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of water flows out. Find the number of lead shots dropped into the vessel.
Ans: We have, height of the conical vessel, $h=8 \mathrm{~cm}$
and radius of the conical vessel, $\mathrm{r}=5 \mathrm{~cm}$
$\therefore$ Volume of water filled in the vessel cone $=\frac{1}{3} \pi r^{2} h=\frac{1}{3} \pi \times 5^{2} \times 8=\frac{200}{3} \pi \mathrm{~cm}^{3}$
Also, we have radius of a spherical lead shot $=0.5 \mathrm{~cm}$
$\therefore$ Volume of each lead shot $=\frac{4}{3} \pi r^{3}=\frac{4}{3} \pi \times(0.5)^{3} \mathrm{~cm}^{3}$
$\therefore$ Volume of lead shots dropped $=$ Volume of water that overflows $=\frac{1}{4} \times \frac{200}{3} \pi \mathrm{~cm}^{3}=\frac{50}{3} \pi \mathrm{~cm}^{3}$
$\therefore$ Number of lead shots dropped $=\frac{\frac{50}{3} \pi}{\frac{4}{3} \pi \times 0.5 \times 0.5 \times 0.5}=100$
Hence required number of lead shots is 100 .

## OR

A copper wire of diameter 8 mm is evenly wrapped on a cylinder of length 24 cm and diameter 49 cm to cover the whole surface. Find (i) the length of the wire (ii) the volume of the wire.
Ans: The thickness of wire $=$ its diameter $=8 \mathrm{~mm}=0.8 \mathrm{~cm}$.
And, the length of the cylinder $=24 \mathrm{~cm}$
$\therefore$ Number of turns of the wire required to cover the whole surface of the cylinder
$=$ Length of the cylinder/Diameter of the wire $=24 / 0.8=30$
Since, diameter of the cylinder $=49 \mathrm{~cm}$
$\therefore$ Radius of the cylinder, $\mathrm{r}=49 / 2 \mathrm{~cm}$
(i) Length of wire wrapped in 1 round $=$ Circumference of the cylinder
$=2 \pi r=2 \times \frac{22}{7} \times \frac{49}{2}=154 \mathrm{~cm}$
Length of wire wrapped in 30 rounds $=30 \times 154 \mathrm{~cm}=4620 \mathrm{~cm}$
(ii) Since radius (r) of wire $=\frac{0.8}{2}=0.4 \mathrm{~cm}$ and its length or height $(\mathrm{h})=4620 \mathrm{~cm}$
$\therefore$ Volume of the wire $=\pi r^{2} h=\frac{22}{7} \times(0.4)^{2} \times 4620=2323.2 \mathrm{~cm}^{3}$
33. Prove that if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio.
Using the above theorem prove that a line through the point of intersection of the diagonals and parallel to the base of the trapezium divides the non parallel sides in the same ratio.

Ans: For the Theorem :
Given, To prove, Construction and figure of $11 / 2$ marks
Proof of $11 / 2$ marks
Let ABCD be a trapezium $\mathrm{DC} \| \mathrm{AB}$ and EF is a line parallel to AB and hence to DC .
Join AC, meeting EF in G.
In $\triangle A B C$, we have GF \|AB

$$
\frac{C G}{G A}=\frac{C F}{F B} \quad[\text { By BPT }] \ldots(1)
$$

In $\triangle A D C$, we have $E G \| D C$

$$
\begin{equation*}
\frac{D E}{E A}=\frac{C G}{G A} \tag{ByBPT}
\end{equation*}
$$

From (1) \& (2), we get, $\frac{D E}{E A}=\frac{C F}{F B}$

34. Two water taps together can fill a tank in $9 \frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. (NCERT Exercise 4.3 Q9)

## OR

A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m . Find its length and breadth. NCERT Quadratic Equations Example-12, page no. 84
35. If the median of the distribution given below is 28.5 , find the values of $x$ and $y$.

| Class | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 5 | $x$ | 20 | 15 | $y$ | 5 | $\mathbf{6 0}$ |

Ans: Here, median $=28.5, \mathrm{n}=60$

| Class interval | Frequency $\left(f_{i}\right)$ | Cumulative frequency $(c f)$ |
| :---: | :---: | :---: |
| $0-10$ | 5 | 5 |
| $10-20$ | $x$ | $5+x$ |
| $20-30$ | 20 | $25+x$ |
| $30-40$ | 15 | $40+x$ |
| $40-50$ | $y$ | $40+x+y$ |
| $50-60$ | 5 | $45+x+y$ |
| Total | $\Sigma \boldsymbol{f}_{\boldsymbol{i}}=\mathbf{6 0}$ |  |

Since the median $=28.5$, therefore, median class is $20-30$
$\therefore \frac{n}{2}=30, l=20, h=10, c f=5+x, f=20$
$\therefore \quad$ Median $=l+\left(\frac{\frac{n}{2}-c f}{f}\right) \times h \quad \Rightarrow \quad 28.5=20+\left(\frac{30-(5+x)}{20}\right) \times 10$
$\Rightarrow 28.5=20+\frac{25-x}{20} \times 10 \Rightarrow 28.5=20+\frac{25-x}{2} \quad \Rightarrow \quad 57=40+25-x$
$\Rightarrow 57=65-x \Rightarrow x=65-57=8$
Also, $45+x+y=60 \Rightarrow y=7$

## SECTION - E(Case Study Based Questions)

## Questions 35 to 37 carry 4 marks each.

36. Anita's mother start a new shoe shop. To display the shoes, she put 3 pairs of shoes in 1st row, 5 pairs in 2 nd row, 7 pairs in 3rd row and so on.


On the basis of above information, answer the following questions.
(i) If she puts a total of 120 pairs of shoes, then find the number of rows required.
(ii) What is the difference of pairs of shoes in 17th row and 10th row.

Ans: Number of pairs of shoes in 1st, 2nd, 3rd row, ... are 3, 5, 7, ...
So, it forms an A.P. with first term $\mathrm{a}=3, \mathrm{~d}=5-3=2$
(i) Let n be the number of rows required.
$\therefore \mathrm{Sn}=120 \Rightarrow(\mathrm{n} / 2)[2(3)+(\mathrm{n}-1) 2]=120$
$\Rightarrow \mathrm{n}^{2}+2 \mathrm{n}-120=0 \Rightarrow \mathrm{n}^{2}+12 \mathrm{n}-10 \mathrm{n}-120=0$
$\Rightarrow(\mathrm{n}+12)(\mathrm{n}-10)=0 \Rightarrow \mathrm{n}=10$

So, 10 rows required to put 120 pairs.
(ii) No. of pairs in 17 th row $=\mathrm{a}_{17}=3+16(2)=35$

No. of pairs in 10th row $=a_{10}=3+9(2)=21$
$\therefore$ Required difference $=35-21=14$
37. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is $60^{\circ}$. After 30 seconds, the angle of elevation reduces to $30^{\circ}$ (see the below figure).


Based on the above information, answer the following questions. (Take $\sqrt{ } 3=1.732$ )
(i) Find the distance travelled by the balloon during the interval.
(ii) Find the speed of the balloon.

Ans: (i) In the figure, let C be the position of the observer (the girl).
A and P are two positions of the balloon.
CD is the horizontal line from the eyes of the (observer) girl.
Here $\mathrm{PD}=\mathrm{AB}=88.2 \mathrm{~m}-1.2 \mathrm{~m}=87 \mathrm{~m}$
In right $\triangle A B C$, we have $\frac{A B}{B C}=\tan 60^{\circ}$
$\Rightarrow \frac{87}{B C}=\sqrt{3} \Rightarrow B C=\frac{87}{\sqrt{3}} \mathrm{~m}$
In right $\triangle P D C$, we have $\frac{P D}{C D}=\tan 30^{\circ}$
$\Rightarrow \frac{87}{C D}=\frac{1}{\sqrt{3}} \Rightarrow C D=87 \sqrt{3}$
Now, $B D=C D-B C=87 \sqrt{3}-\frac{87}{\sqrt{3}}$

$$
=58 \sqrt{3} \mathrm{~m}
$$



Thus, the required distance between the two positions of the balloon $=58 \sqrt{3} \mathrm{~m}$
$=58 \times 1.73=100.46 \mathrm{~m}$ (approx.)
(ii) Speed of the balloon $=$ Distance/time $=100.46 / 30=3.35 \mathrm{~m} / \mathrm{s}($ approx. $)$
38. In the sport of cricket the Captain sets the field according to a plan. He instructs the players to take a position at a particular place. There are two reasons to set a cricket field-to take wickets and to stop runs being scored.
The following graph shows the position of players during a cricket match.
(i) Find the coordinate of the point on $y$-axis which are equidistant from the points representing the players at Cover $\mathrm{P}(2,-5)$ and Mid-wicket $\mathrm{Q}(-2,9)$
(ii) Find the ratio in which x -axis divides the line segment joining the points Extra Cover $\mathrm{S}(3,-$ $3)$ and Fine Leg ( $-2,7$ ).


Ans: (i) Let $A(0, y)$ be any point on the $y$-axis.
Since $A(0, y)$ is equidistant from $P(2,-5)$ and $Q(-2,9)$
So $A P=A Q \Rightarrow A P^{2}=A Q^{2}$
$\Rightarrow(2)^{2}+(y+5)^{2}=(2)^{2}+(y-9)^{2} \Rightarrow y^{2}+10 y+25=y^{2}-18 y+81$
$\Rightarrow 28 y=81-25 \Rightarrow 28 y=56$
$\Rightarrow y=28 / 56=2$
So, the point is $(0,2)$
(ii) Let point $\mathrm{P}(\mathrm{x}, 0)$ divides the line segment joining the points A and B in the ratio $\mathrm{k}: 1$

$$
\begin{array}{lll} 
& k & 1 \\
\hdashline A(3,-3) & P(x, 0) & B(-2,7)
\end{array}
$$

Using section formula,
Coordinates of $P$ are $\left(\frac{-2 k+3}{k+1}, \frac{7 k-3}{k+1}\right)$
$y$-coordinate of $P=\frac{7 k-3}{k+1}=0$
$\Rightarrow 7 k=3 \Rightarrow k=\frac{3}{7}$
Hence, the point P divides the line segment in the ratio $3: 7$.

