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SAMPLE PAPER TEST 09 FOR BOARD EXAM 2024

(ANSWERS)

SUBJECT: MATHEMATICS — **MAX. MARKS : 80**

CLASS : X — **DURATION : 3 HRS**

General Instruction:

1. This Question Paper has 5 Sections A-E.
2. **Section A** has 20 MCQs carrying 1 mark each.
3. **Section B** has 5 questions carrying 02 marks each.
4. **Section C** has 6 questions carrying 03 marks each.
5. **Section D** has 4 questions carrying 05 marks each.
6. **Section E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. If LCM(x, 18) = 36 and HCF(x, 18) = 2, then x is:
(a) 2 (b) 3 (c) 4 (d) 5 1
Ans. (c) 4
LCM × HCF = Product of two numbers
⇒ 36 × 2 = 18 × x
⇒ x = 4

2. In ΔABC right angled at B, if tanA = $\sqrt{3}$, then then cosA cosC – sinAsinC =
(a) –1 (b) 0 (c) 1 (d) $\sqrt{3}$ / 2
Ans: (b) 0
tan A = $\sqrt{3}$ = tan 60°, so, ∠A = 60°,
Hence, ∠C = 30°.
So, cos A cos C – sin A sin C $= \frac{1}{2}\times\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\times\frac{1}{2}=0$

3. If $2\sin^2 \beta - \cos^2 \beta = 2$, then β is:
(a) 0° (b) 90° (c) 45° (d) 30° 1
Ans. (b) 90°
$2 \sin^2 \beta - \cos^2 \beta = 2$
Then, $2 \sin^2 \beta - (1 - \sin^2 \beta) = 2$
⇒ $3\sin^2 \beta = 3$ or $\sin^2 \beta = 1$ ⇒ β is 90°.

4. The ratio of LCM and HCF of the least composite and the least prime numbers is:
(a) 1: 2 (b) 2: 1 (c) 1: 1 (d) 1: 3 1
Ans. (b) 2: 1
Least composite number is 4 and the least prime number is 2
LCM(4, 2): HCF (4, 2) = 4: 2 = 2: 1

5. The value of k for which the lines 5x + 7y = 3 and 15x + 21y = k coincide is:
(a) 9 (b) 5 (c) 7 (d) 18
Ans: (a) 9

For coincident lines we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{5}{15} = \frac{7}{21} = \frac{-3}{-k} \Rightarrow \frac{1}{3} = \frac{3}{k} \Rightarrow k = 9$

6. Write a quadratic polynomial whose sum of zeroes is $\frac{-1}{4}$ and product of zeroes is $\frac{1}{4}$.

(a) $4x^2 + x + 1$ (b) $x^2 + 4x - 1$
(c) $2x^2 + 3x - 1$ (d) $x^2 - 2x + 1$

Ans: (a) $4x^2 + x + 1$

Sum of zeroes = $\frac{-1}{4}$ and Product of zeroes = $\frac{1}{4}$

∴ Quadratic Polynomial is p(x) = x^2 – (sum of zeroes) x + product of zeroes

∴ $p(x) = x^2 - \left(\frac{-1}{4}\right) x + \frac{1}{4} = 0 \Rightarrow p(x) = \frac{1}{4}(4x^2 + x + 1)$

Hence, required quadratic polynomial is $4x^2 + x + 1$

7. The vertices of a parallelogram in order are A(1, 2), B(4, y), C(x, 6) and D(3, 5). Then (x, y) is:

(a) (6, 3) (b) (3, 6) (c) (5, 6) (d) (1, 4)

Ans: (a) (6, 3)

∴ Mid-point of diagonal AC = Mid-point of BD

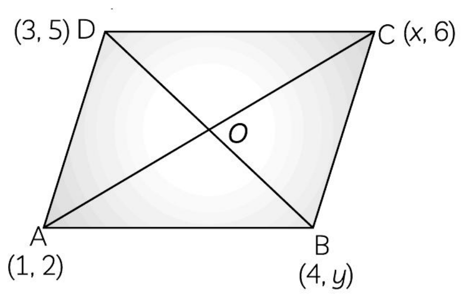
$\Rightarrow \left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{4+3}{2}, \frac{y+5}{2}\right)$

$\Rightarrow \frac{1+x}{2} = \frac{4+3}{2}$; $\frac{2+6}{2} = \frac{y+5}{2}$

$\Rightarrow x + 1 = 7$; $8 = y + 5$

$\Rightarrow x = 6$; $y = 3$

∴ $(x, y) = (6, 3)$



8. A horse is tied to a pole with 28 m long rope. The perimeter of the field where the horse can graze is (Take π = 22/7)

(a) 60 cm (b) 85 cm (c) 124 cm (d) 176 cm

Ans: (d) 176 cm

Horse can graze in the field which is a circle of radius 28 cm.

So, required perimeter = 2πr = 2π (28) cm = 2 × 22/7 × 28 cm = 176 cm

9. Two dice are thrown at the same time. The probability of getting not doublet is

(a) 1/3 (b) 1/6 (c) 1/5 (d) 5/6

Ans: (d) 5/6

Total number of possible outcomes = 36

Number of outcomes with doublets = 6 i.e. (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

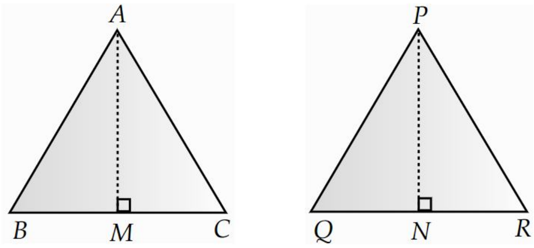
So, number of favourable outcomes = 36 – 6 = 30

∴ Required probability = 30/36 = 5/6

10. ΔABC~ΔPQR. If AM and PN are altitudes of ΔABC and ΔPQR respectively and $AB^2 : PQ^2 = 4 : 9$, then AM: PN =

(a) 16: 81 (b) 4: 9 (c) 3: 2 (d) 2: 3

Ans: (d) 2 : 3



We have, $\Delta ABC \sim \Delta PQR$

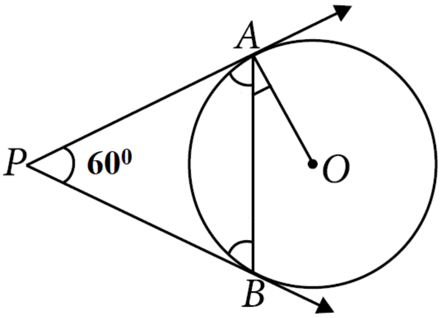
$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{AM}{PN} \quad \text{(corresponding sides of similar triangle)}$$

But $\frac{AB^2}{PQ^2} = \frac{4}{9} \Rightarrow \frac{AB}{PQ} = \frac{2}{3}$

i.e., $\frac{AB}{PQ} = \frac{AM}{PN} = \frac{2}{3}$

Hence, $AM : PN = 2 : 3$.

11. In the given figure, PA and PB are tangents to the circle with centre O. If ∠APB = 60°, then ∠OAB is



(a) 30° (b) 60° (c) 90° (d) 15°

Ans: (a) 30°

PA = PB (Tangents drawn from external point are equal)

⇒ ∠ABP = ∠BAP = x (Angles opposite to equal sides are equal)

In ΔAPB, 60° + x + x = 180°

⇒ 2x = 120° ⇒ x = 60°

Now, ∠OAP = 90° (∵ Tangent is perpendicular to the radius through the point of contact)

∴ ∠OAB = 90° – 60° = 30°

12. If the difference of Mode and Median of a data is 24, then the difference of median and mean is

(a) 8 (b) 12 (c) 24 (d) 36

Ans: (b) 12

mode – median = 24 (given)

∴ mode = 24 + median

Since, mode = 3 median – 2 mean [By empirical relation]

∴ 24 + median = 3 median – 2 mean

⇒ 2 median – 2 mean = 24

⇒ median – mean = 12

13. For the following distribution:

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

the sum of lower limits of the median class and modal class is

(a) 15 (b) 25 (c) 30 (d) 35

Ans: (b) 25

Since, $N = 66$, then $\frac{N}{2} = 33$

and cumulative frequency greater than or equal to 33 lies in class 10 – 15

So, median class is 10 – 15

$\therefore$ Lower limit of median class is 10

and highest frequency is 20 lie in class 15 – 20

So, modal class is 15 – 20.

$\therefore$ Lower limit of modal class is 15.

Class	Frequency (f)	c.f.
0 – 5	10	10
5 – 10	15	25
10 – 15	12	37
15 – 20	20	57
20 – 25	9	66
	$N = 66$	

Hence, sum of lower limits of the median and modal class is 10 + 15 = 25.

14. If 5 tan θ = 4, then the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$ is

(a) 1/6 (b) 1/7 (c) 1/4 (d) 1/5

Ans: (a) 1/6

$$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{5\tan\theta - 3}{5\tan\theta + 2}$$ [Dividing numerator and denominator by cos θ]

$$= \frac{5\times\frac{4}{5} - 3}{5\times\frac{4}{5} + 2} = \frac{4-3}{4+2} = \frac{1}{6}$$

15. The ratio of the volumes of two spheres is 8 : 27. The ratio between their surface areas is

(a) 2 : 3 (b) 4 : 27 (c) 8 : 9 (d) 4 : 9

Ans: (d) 4 : 9

$$\frac{\text{Volume of sphere with radius } r}{\text{Volume of sphere with radius } R} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{8}{27} \Rightarrow \frac{r^3}{R^3} = \frac{8}{27} \Rightarrow \left(\frac{r}{R}\right)^3 = \left(\frac{2}{3}\right)^3 \Rightarrow \frac{r}{R} = \frac{2}{3}$$

$$\text{Ratio between their surface areas } = \frac{4\pi r^2}{4\pi R^2} = \frac{r^2}{R^2} = \left(\frac{r}{R}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

16. The area of the circle that can be inscribed in a square of 6cm is

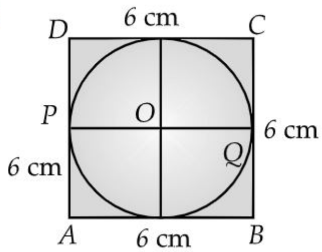
(a) 36π cm^2 (b) 18π cm^2 (c) 12π cm^2 (d) 9π cm^2

Ans: (d) 9π cm^2

ABCD is a square of side 6 cm. PQ is a diameter of given circle such that $PQ = AB = 6$ cm

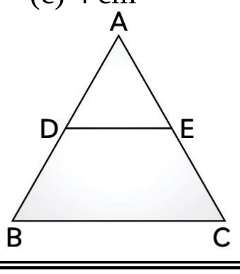
$\therefore$ Radius $(r) = \frac{6}{2} = 3$ cm

Area of the circle $= \pi r^2 = \pi(3)^2 = 9\pi$ cm^2.



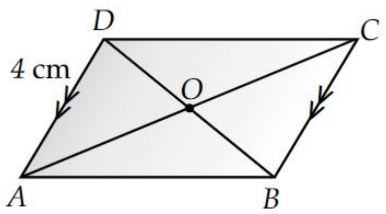
17. In the figure, if DE || BC, AD = 3 cm, BD = 4 cm and BC = 14 cm, then DE equals :

(a) 7 cm (b) 6 cm (c) 4 cm (d) 3 cm



Ans: (b) 6 cm
∵ DE || BC
∴ ∠ADE = ∠ABC [corresonding angles] ....(i)
Now, in ΔADE and ΔABC,
∠ADE = ∠ABC [Proved in (i)]
∠A = ∠A [Common angle]
∴ ΔADE ~ ΔABC [By AA similarity axiom]
$\therefore \frac{AD}{AB} = \frac{DE}{BC}$ [∵ Corresponding sides of similar triangles are proportional]
$\Rightarrow \frac{AD}{AD+BD} = \frac{DE}{BC} \Rightarrow \frac{3}{3+4} = \frac{DE}{14} \Rightarrow \frac{3}{7} = \frac{DE}{14} \Rightarrow DE = 6$

18. ABCD is a trapezium with AD ∥ BC and AD = 4cm. If the diagonals AC and BD intersect each other at O such that AO/OC = DO/OB =1/2, then BC =
(a) 6cm (b) 7cm (c) 8cm (d) 9cm
Ans: (c) 8cm
$AD \parallel BC$
and $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$
$\therefore \frac{AD}{BC} = \frac{AO}{OC} \Rightarrow \frac{4}{BC} = \frac{1}{2} \Rightarrow BC = 8$ cm



DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**.
Choose the correct option

19. Assertion (A): The number 6^n, n being a natural number, ends with the digit 5.
Reason (R): The number 9^n cannot end with digit 0 for any natural number n.
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.
Ans: (d) Assertion (A) is false but reason (R) is true.

20. Assertion (A): The point (–1, 6) divides the line segment joining the points (–3, 10) and (6, –8) in the ratio 2 : 7 internally.
Reason (R): Given three points, i.e. A, B, C form an equilateral triangle, then AB = BC = AC.
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.
Ans: (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

SECTION – B

Questions 21 to 25 carry 2 marks each.

21. If sin(A + B) = 1 and cos(A – B) = √3/2, 0°< A + B ≤ 90° and A > B, then find the measures of angles A and B.

Ans: $\sin(A + B) = 1 = \sin 90^0$, so $A + B = 90^0$ ……….(i)
$\cos(A – B) = \sqrt{3}/2 = \cos 30^0$, so $A – B = 30^0$ ………(ii)
From (i) & (ii) $\angle A = 60°$ and $\angle B = 30°$

OR

Find an acute angle θ when $\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$

Ans: $\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$

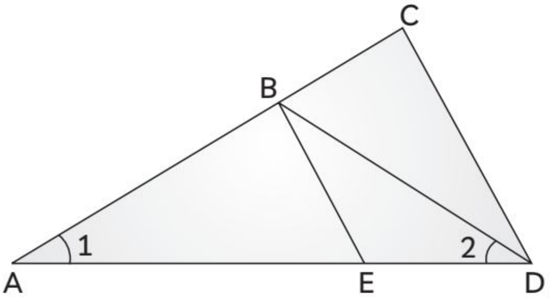
Dividing the numerator and denominator of LHS by cos θ, we get

$$\frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Which on simplification (or comparison) gives

$$\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$$

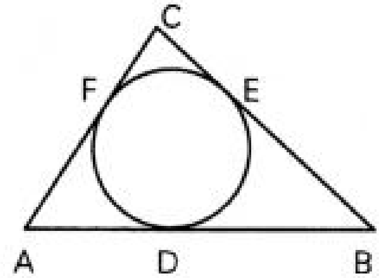
22. In the given figure below, AD/AE=AC/BD and ∠1=∠2. Show that Δ BAE~ ΔCAD .



Ans: In ΔABC, ∠1 = ∠2
∴ AB = BD ………(i)
Given, $\frac{AD}{AE}=\frac{AC}{BD}$
Using equation (i), we get $\frac{AD}{AE}=\frac{AC}{AB}$ ……….(ii)
In ΔBAE and ΔCAD, by equation (ii), $\frac{AC}{AB}=\frac{AD}{AE}$
and ∠A= ∠A (common)
∴ ΔBAE ~ ΔCAD [By SAS similarity criterion]

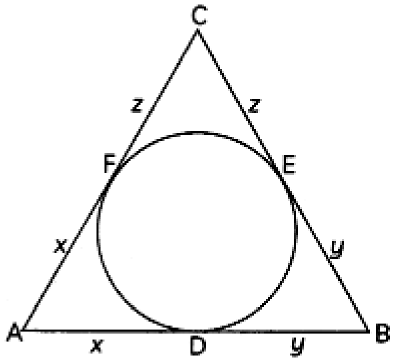
23. If 217x + 131y = 913, 131x + 217y = 827, then find the value of x and y
Ans: Adding the two equations and dividing by 348, we get : x + y = 5
Subtracting the two equations and dividing by 86, we get : x – y = 1
Solving these two new equations, we get, x = 3 and y = 2

24. A circle is inscribed in a ΔABC having AB= 10cm, BC = 12cm and CA = 8cm and touching these sides at D, E, F respectively. Find the lengths of AD, BE and CF



Ans: Let AD = AF = x cm,
BD = BE = y cm

and CE = CF = z cm



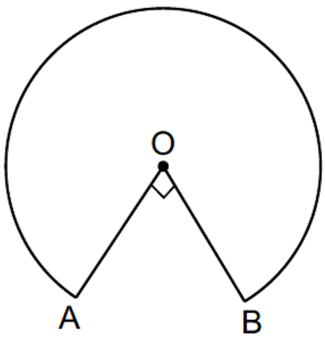
Now, x + y = AB = 10 cm
y + z = BC = 12 cm
z + x = CA = 8 cm
Adding all we get 2(x+ y +z) = 30 ⇒ x + y + z = 15
Subtracting, we get z = 5 cm, x = 3 cm and y = 7 cm
Hence, AD = 3 cm, BE = 7 cm and CF = 5 cm.

25. The length of the minute hand of a clock is 6cm. Find the area swept by it when it moves from 5:25 pm to 6:00 pm.
Ans: We know that, in 60 minutes, the tip of minute hand moves 360°
In 1 minute, it will move = 360°/60 = 6°
∴ From 5:25 pm to 6:00 pm i.e. 35 min, it will move through = 35 × 6° = 210°
∴ Area of swept by the minute hand in 35 min = Area of sector with sectorial angle θ of 210° and radius of 6 cm

$$= \frac{210}{360} \times \pi \times 6^2 = \frac{7}{12} \times \frac{22}{7} \times 6 \times 6 = 66 \text{ cm}^2$$

OR

In the given figure, the shape of the top of a table is that of a sector of a circle with centre O and ∠AOB = 90°. If AO = OB = 42 cm, then find the perimeter of the top of the table is [Take π = 22/7]



Ans: Perimeter = length of major arc + 2r

$$= \frac{270°}{360°} \times 2 \times \pi r + 2r$$

$$= \frac{3}{2} \times \frac{22}{7} \times 42 + 2 \times 42$$

$$= 198 + 84 = 282 \text{ cm}$$

SECTION – C

Questions 26 to 31 carry 3 marks each.

26. A train covered a certain distance at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr; it would have taken 6 hours more than the scheduled time. Find the length of the journey.

Ans: Let the actual speed of the train be x km/hr and let the actual time taken be y hours.

Distance covered is xy km If the speed is increased by 6 km/hr, then time of journey is reduced by 4 hours i.e., when speed is (x+6)km/hr, time of journey is (y−4) hours.

∴ Distance covered = (x + 6)(y − 4)

$\Rightarrow xy = (x + 6)(y - 4) \Rightarrow -4x + 6y - 24 = 0 \Rightarrow -2x + 3y - 12 = 0$ …………(i)

Similarly $xy = (x - 6)(y + 6) \Rightarrow 6x - 6y - 36 = 0 \Rightarrow x - y - 6 = 0$ …………(ii)

Solving (i) and (ii) we get x=30 and y=24

Putting the values of x and y in equation (i), we obtain

Distance = (30 × 24)km =720km.

Hence, the length of the journey is 720km.

OR

Anuj had some chocolates, and he divided them into two lots A and B. He sold the first lot at the rate of ₹2 for 3 chocolates and the second lot at the rate of ₹1 per chocolate, and got a total of ₹400. If he had sold the first lot at the rate of ₹1 per chocolate, and the second lot at the rate of ₹4 for 5 chocolates, his total collection would have been ₹460. Find the total number of chocolates he had.

Ans: Let the number of chocolates in lot A be x

And let the number of chocolates in lot B be y

∴ total number of chocolates =x+y

Price of 1 chocolate = ₹ 2/3 , so for x chocolates = $\frac{2}{3}x$

and price of y chocolates at the rate of ₹ 1 per chocolate =y.

∴ by the given condition $\frac{2}{3}x + y = 400 \Rightarrow 2x + 3y = 1200$ ....(i)

Similarly, $x + \frac{4}{5}y = 460 \Rightarrow 5x + 4y = 2300$ .... (ii)

Solving (i) and (ii) we get x = 300 and y = 200

∴ x + y = 300 + 200 = 500

So, Anuj had 500 chocolates.

27. Prove that: $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \sec\theta + \tan\theta$

Ans: LHS = $\frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$ (Dividing numerator and denominator by cosθ)

$$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{\tan\theta + 1 - \sec\theta}$$

$= \sec\theta + \tan\theta$ = RHS

28. Given that √5 is irrational, prove that 2 + 3√5 is irrational.

Ans: Let us assume 2 + 3√5 is rational, then it must be in the form of p/q where p and q are co-prime integers and q ≠0

i.e. 2 + 3√5 = $\frac{p}{q}$

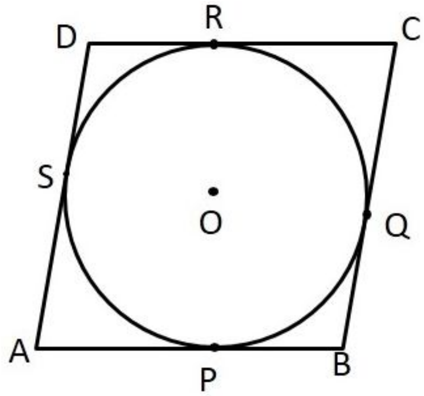
So $\sqrt{5} = \dfrac{p-2q}{3q}$ …(i)
Since p, q, 5 and 2 are integers and q ≠ 0,
RHS of equation (i) is rational.
But LHS of (i) is √5 which is irrational. This is not possible.
This contradiction has arisen due to our wrong assumption that 2 + 3√5 is rational.
So, 2 + 3√5 is irrational.

29. Find the zeroes of the polynomial $x^2 + \frac{1}{6}x - 2$, and verify the relation between the coefficients and the zeroes of the polynomial.

Ans: Now we have given the polynomial: $x^2 + \frac{1}{6}x - 2 = 0$
Simplifying it, we get $6x^2 + x - 12 = 0$
$\Rightarrow 6x^2 - 8x + 9x - 12 = 0$
$\Rightarrow (6x^2 - 8x) + (9x - 12) = 0$
$\Rightarrow 2x(3x - 4) + 3(3x - 4) = 0$
$\Rightarrow (3x - 4)(2x + 3) = 0$
$\Rightarrow x = 4/3$ or $x = -3/2$
Here, a = 6, b = 1, c = –12
Sum of zeroes = $\frac{4}{3} + \left(-\frac{3}{2}\right) = \frac{8-9}{6} = \frac{-1}{6} = \frac{-b}{a}$
Product of zeroes = $\frac{4}{3} \times \left(-\frac{3}{2}\right) = \frac{-12}{6} = \frac{c}{a}$

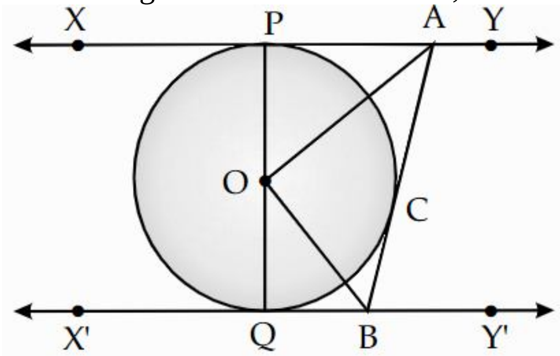
30. Two coins are tossed simultaneously. What is the probability of getting
(i) At least one head? (ii) At most one tail? (iii) A head and a tail?
Ans: Total number of outcomes = 4
(i) Number of outcomes with at least one head = 3
∴ Required probability = 3/4
(ii) Number of outcomes with at most one tail = 3
∴ Required probability = 3/4
(iii) Number of outcomes with a head and a tail = 2
∴ Required probability = 2/4 = 1/2

31. Prove that a parallelogram circumscribing a circle is a rhombus
Ans: We have ABCD, a parallelogram which circumscribes a circle (i.e., its sides touch the circle) with centre O.
Since tangents to a circle from an external point are equal in length,
∴ AP = AS, BP = BQ, CR = CQ and DR = DS
Adding, we get
(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
⇒ AB + CD = AD + BC
But AB = CD [opposite sides of ABCD]
and BC = AD
∴ AB + CD = AD + BC ⇒ 2 AB = 2 BC
⇒ AB = BC
Similarly AB = DA and DA = CD
Thus, AB = BC = CD = AD
Hence ABCD is a rhombus.

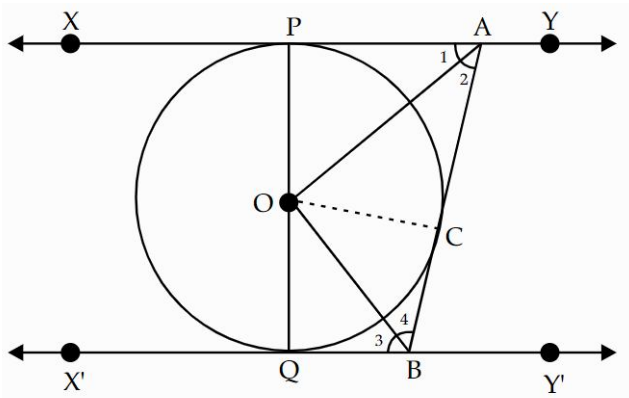


OR

In the figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C interesting XY at A and X'Y' at B, what is the measure of ∠AOB.



Ans: Join OC. Since, the tangents drawn to a circle from an external point are equal.
∴ AP = AC

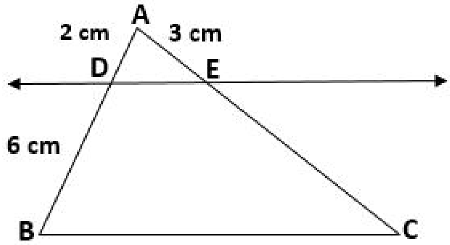


In Δ PAO and Δ AOC, we have:
AO = AO [Common]
OP = OC [Radii of the same circle]
AP = AC
⇒ Δ PAO ≅ Δ AOC [SSS Congruency]
∴ ∠PAO = ∠CAO = ∠1
∠PAC = 2 ∠1 ...(1)
Similarly ∠CBQ = 2 ∠2 ...(2)
Again, we know that sum of internal angles on the same side of a transversal is 180°.
∴ ∠PAC + ∠CBQ = 180°
⇒ 2 ∠1 + 2 ∠2 = 180° [From (1) and (2)]
⇒ ∠1 + ∠2 = 180°/2 = 90° ...(3)
Also ∠1 + ∠2 + ∠AOB = 180° [Sum of angles of a triangle]
⇒ 90° + ∠AOB = 180°
⇒ ∠AOB = 180° − 90° ⇒ ∠AOB = 90°.

SECTION – D

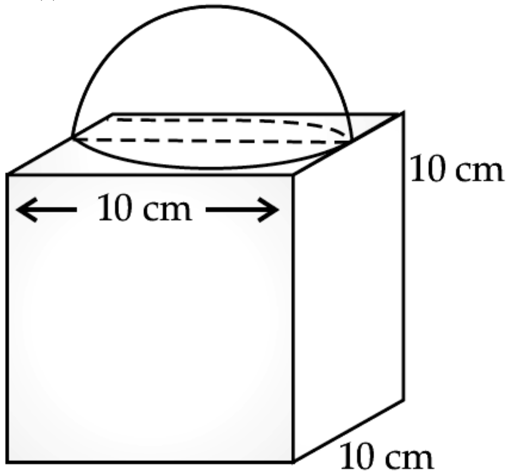
Questions 32 to 35 carry 5 marks each.

32. Prove that if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio.
In the figure, find EC if AD/DB = AE/EC using the above theorem.



Ans: For the Theorem :
Given, To prove, Construction and figure of 2 marks
Proof of 2 marks
Using Thales theorem, we get $\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{2}{6} = \frac{3}{EC} \Rightarrow \frac{1}{3} = \frac{3}{EC} \Rightarrow EC = 9cm$ 1 mark

33. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have? Find the cost of painting the total surface area of the solid so formed, at the rate of Rs. 5 per 100 sq. cm. [Use π = 3.14]
Ans: Side of the cubical block (l) = 10 cm.



The hemisphere is surmounted on it.
The largest diameter the hemisphere can have =
side of the cubical block
Diameter of the hemisphere = 10 cm
Radius of the hemisphere (r) = 5 cm
Total surface area of the solid formed
= TSA of the cubical + CSA of the hemisphere - Area of the base of the hemisphere
$= 6l^2 + 2\pi r^2 - \pi r^2 = 6l^2 + \pi r^2$
$= 6 \times (10)^2 + 3.14 \times (5)^2$
$= 6 \times 100 + 3.14 \times 25$
= 600 + 78.50
= 678.5 cm^2
Rate of painting = Rs. 5 per 100 cm^2
Cost pf painting the solid formed = Rs. 5/100 × 678.5
= Rs. 33.925 = Rs. 33.93 (approx)

OR

Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively decided to provide place and the canvas for 1500 tents and share the whole expenditure equally. The lower part of each tent is cylindrical with base radius 2.8 m and height 3.5 m and the upper part is conical with the same base radius, but of height 2.1 m. If the canvas used to make the tents costs ₹120 per m^2, find the amount shared by each school to set up the tents.
Ans: Radius of the base of cylinder (r) = 2.8 m = Radius of the base of the cone (r)
Height of the cylinder (h)=3.5 m

Height of the cone (H)=2.1 m.
Slant height of conical part (l)=$\sqrt{(r^2 + H^2)} = \sqrt{[(2.8)^2 + (2.1)^2]} = \sqrt{(7.84 + 4.41)} = \sqrt{12.25} = 3.5$ m
Area of canvas used to make tent = CSA of cylinder + CSA of cone = $2\pi rh + \pi rl$

$= \pi r(2h + l) = \frac{22}{7}\times 2.8\times(7+3.5) = 22\times 0.4\times 10.5 = 92.4m^2$

Cost of 1500 tents at ₹120 per sq.m = 1500 × 120 × 92.4 = 1,66,32,000
Share of each school to set up the tents = 16632000/50 = ₹3,32,640

34. The median of the following data is 868. Find the values of x and y, if the total frequency is 100

Class	Frequency
800 – 820	7
820 – 840	14
840 – 860	x
860 – 880	25
880 – 900	y
900 – 920	10
920 – 940	5

Ans:

Class	Frequency	Frequency
800 – 820	7	7
820 – 840	14	21
840 – 860	x	x + 21
860 – 880	25	x + 46
880 – 900	y	x + y + 46
900 – 920	10	x + y + 56
920 – 940	5	x + y + 61

From table, we have x + y + 61 = 100 ⇒ x + y = 100 – 61 ⇒ x + y = 39
Here, median = 868, therefore median class is 860 – 880
So, l = 860, cf = x + 21, f = 25, h = 20, n/2 = 50

Now, $Median = l + \left(\frac{\frac{n}{2} - cf}{f}\times h\right) \Rightarrow 868 = 860 + \left(\frac{50-(x+21)}{25}\times 20\right)$

$\Rightarrow 868 - 860 = \left(\frac{50 - x - 21)}{5}\times 4\right) \Rightarrow 8 = \frac{29-x}{5}\times 4$

$\Rightarrow 40 = (29-x)4 \Rightarrow 29 - x = 10 \Rightarrow x = 29 - 10 = 19$

$\Rightarrow y = 39 - 19 = 20$

35. Two pipes running together can fill a cistern in $3\frac{1}{13}$ hours. If one pipe takes 3 hours more than the other to fill it, find the time in which each pipe would fill the cistern.
Ans: Let time taken by faster pipe to fill the cistern be x hrs.
Therefore, time taken by slower pipe to fill the cistern = (x + 3) hrs
Since the faster pipe takes x minutes to fill the cistern.

∴ Portion of the cistern filled by the faster pipe in one hour = $\frac{1}{x}$

Portion of the cistern filled by the slower pipe in one hour = $\frac{1}{x+3}$

Portion of the cistern filled by the two pipes together in one hour = $\frac{13}{40}$

According to the question, $\frac{1}{x}+\frac{1}{x+3}=\frac{13}{40} \Rightarrow \frac{x+3+x}{x(x+3)}=\frac{13}{40}$

$\Rightarrow 40\ (2x + 3) = 13x\ (x + 3) \Rightarrow 80x + 120 = 13x^2 + 39x$

$\Rightarrow 13x^2 - 41x - 120 = 0 \Rightarrow 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow 13x\ (x - 5) + 24\ (x - 5) = 0 \Rightarrow (x - 5)\ (13x + 24) = 0$

$\Rightarrow$ Either $x - 5 = 0$ or $13x + 24 = 0 \Rightarrow x = 5$ as $x = -24/13$ not possible.

Hence, the time taken by the two pipes is 5 hours and 8 hours respectively.

OR

In a flight of 600km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr from its usual speed and the time of the flight increased by 30 min. Find the scheduled duration of the flight.

Ans: Let the usual speed of plane be x km/hr

and the reduced speed of the plane be (x – 200) km/hr

Distance =600 km [Given]

According to the question,

(time taken at reduced speed) – (Schedule time) = 30 minutes = 0.5 hours.

$\Rightarrow \frac{600}{x-200}-\frac{600}{x}=\frac{1}{2}$

Which on simplification gives: $x^2 - 200x - 240000 = 0$

$\Rightarrow x^2 - 600x + 400x - 240000 = 0$

$\Rightarrow x(x - 600) + 400(x - 600) = 0 \Rightarrow (x - 600)(x + 400) = 0 \qquad \Rightarrow x = 600$ or $x = -400$

But speed cannot be negative.

∴ The usual speed is 600 km/hr and

the scheduled duration of the flight is 600/600 =1hour

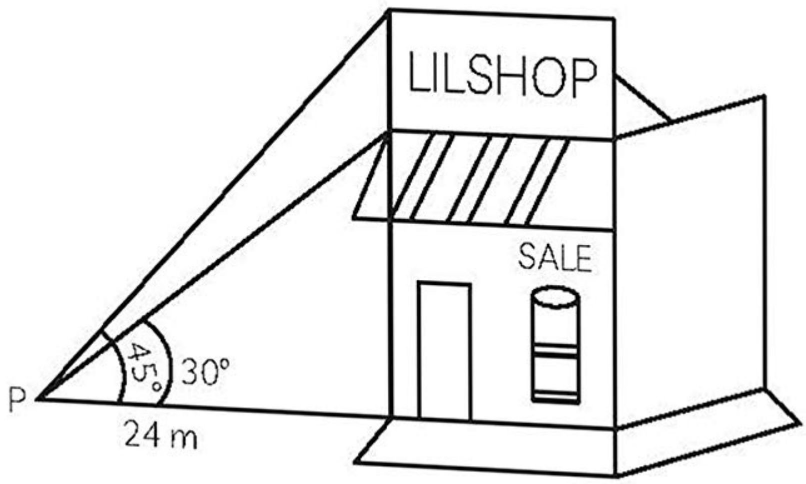
SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. Case Study – 1

Anita purchased a new building for her business. Being in the prime location, she decided to make some more money by putting up an advertisement sign for a rental ad income on the roof of the building.

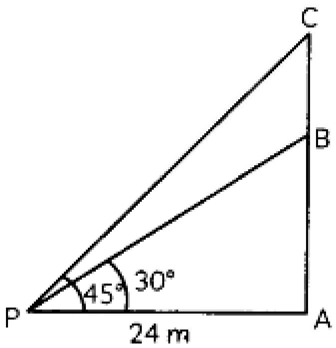
From a point P on the ground level, the angle of elevation of the roof of the building is 30° and the angle of elevation of the top of the sign board is 45°. The point P is at a distance of 24 m from the base of the building.



On the basis of the above information, answer the following questions:
(i) Find the height of the building (without the sign board). (2)

OR

Find the height of the building (with the sign board) (2)
(ii) Find the height of the sign board. (1)
(iii) Find the distance of the point P from the top of the sign board. (1)
Ans: (i) In ΔAPC,



tan 30° = AB/AP
⇒ 1/√3 = AB/24
⇒ AB = 24/√3 m = 13.85 m = 14 m (approx)
OR
Considering, the diagram in the above question, AC as the new height of the shop including the sign-baard.
In ΔAPC,
tan 45° = AC/AP
⇒ 1 = AC/24
⇒ AC = 24 m

(ii) From Q (i) and Q (ii).
Length of sign board, BC = AC – AB
= 24 – 14
= 10 m

(iii) In ΔAPC,
cos 45° = AP/AC
⇒ 1/√2 = 24/AC
⇒ PC = 24√2 m

37. Case Study-2

The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.

(i) If the first circular row has 30 seats, how many seats will be there in the 10th row? (1)

(ii) For 1500 seats in the auditorium, how many rows need to be there? (2)

OR

If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row? (2)

(iii) If there were 17 rows in the auditorium, how many seats will be there in the middle row?(1)

Ans: (i) Since each row is increasing by 10 seats, so it is an AP with first term a = 30, and common difference d =10. So number of seats in 10th row = a_{10} = a + 9d

= 30 + 9 ×10 = 120

(ii) $S_n = \frac{n}{2}$ (2a + (n – 1)d) ⇒ 1500 = $\frac{n}{2}$ (2 × 30 + (n-1)10)

⇒ 3000 = 50n + 10n^2 ⇒ n^2 + 5n – 300 = 0

⇒ n^2 + 20n – 15n – 300 = 0 ⇒ (n + 20) (n – 15) =0

Rejecting the negative value, n = 15

OR

No. of seats already put up to the 10th row = S_{10}

$S_{10} = \frac{10}{2}$ {2 × 30 + (10-1)10)} = 5(60 + 90) = 750

So, the number of seats still required to be put are 1500 – 750 = 750

(iii) If no. of rows =17

then the middle row is the 9th row

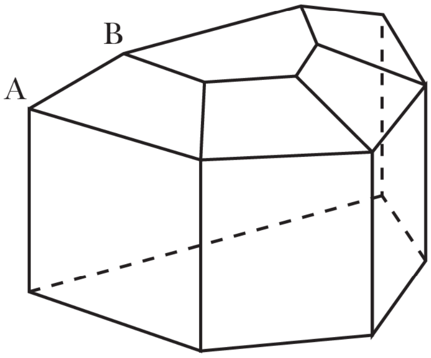
a_9= a + 8d = 30 + 80 = 110 seats

38. Case Study-3

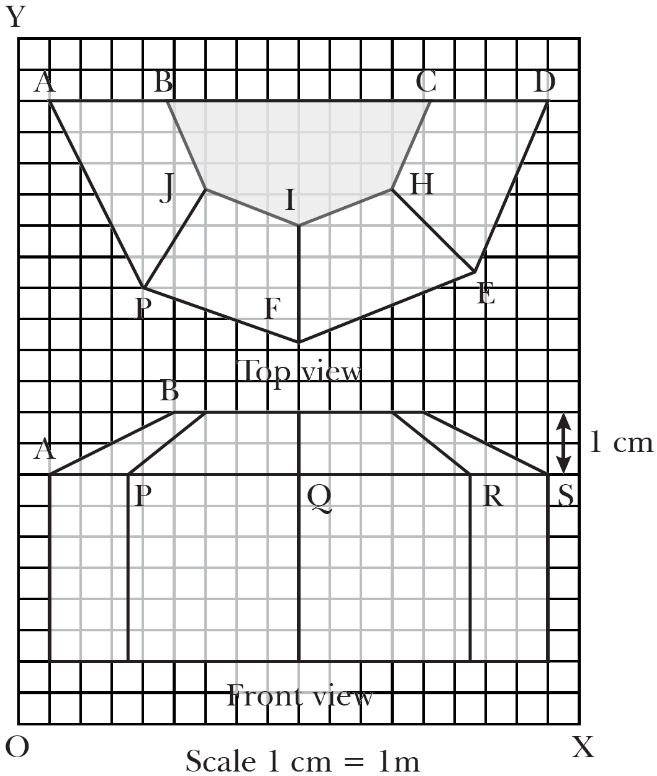
The diagrams show the plans for a sun room. It will be built onto the wall of a house. The four walls of the sunroom are square clear glass panels. The roof is made using

• Four clear glass panels, trapezium in shape, all the same size

• One tinted glass panel, half a regular octagon in shape



Not to scale



Scale 1 cm = 1m

(i) Find the mid-point of the segment joining the points J (6, 17) and I (9, 16). (1)
(ii) Find the distance between the points A and S. (1)
(iii) Find the co-ordinates of the point which divides the line segment joining the points A and B in the ratio 1:3 internally. (2)

OR

(iii) If a point (x,y) is equidistant from the Q(9,8) and S(17,8),then find the relation between x and y. (2)

Ans: (i) Mid-point of JI = $\left(\frac{6+9}{2}, \frac{17+16}{2}\right) = \left(\frac{15}{2}, \frac{33}{2}\right)$

(ii) Distance between A and S = 16 boxes.
(iii) Coordinates of A and B are (1, 8) and (5, 10) respectively.
Coordinates of point dividing AB in the ratio 1 : 3 internally are:

$$x = \frac{1\times5+3\times1}{1+3}, y = \frac{1\times10+3\times8}{1+3} \Rightarrow x = \frac{8}{4} = 2, y = \frac{34}{4} = 8.5$$

Co-ordinates of required points be (2, 8.5)

OR

(iii) Let P (x,y) is equidistant from the Q(9,8) and S(17,8) then we have
$PQ = PS \Rightarrow PQ^2 = PS^2$
$\Rightarrow (x-9)^2 + (y-8)^2 = (x-17)^2 + (y-8)^2$
$\Rightarrow (x-9)^2 = (x-17)^2$
$\Rightarrow x^2 - 18x + 81 = x^2 - 34x + 289$
$\Rightarrow 34x - 18x + 81 - 289 = 0$
$\Rightarrow 16x - 208 = 0$
$\Rightarrow x - 13 = 0$