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SAMPLE PAPER TEST 10 FOR BOARD EXAM 2024

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**

CLASS : X **DURATION : 3 HRS**

General Instruction:

1. This Question Paper has 5 Sections A-E.
2. **Section A** has 20 MCQs carrying 1 mark each.
3. **Section B** has 5 questions carrying 02 marks each.
4. **Section C** has 6 questions carrying 03 marks each.
5. **Section D** has 4 questions carrying 05 marks each.
6. **Section E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, the other number is

(*a*) 31 (*b*) 71 (*c*) 61 (*d*) 91

Ans: (*d*) 91

$a \times b = HCF(a, b) \times LCM(a, b) \Rightarrow 26 \times b = 13 \times 182 \Rightarrow b = 13 \times 182 / 26 = 91$

2. If p and q are positive integers such that $p = a^3b^2$ and $q = a^2b$, where 'a' and 'b' are prime numbers, then the HCF (p, q) is …..

(a) a^2b (b) a^2b^2 (c) a^3b^2 (d) a^3b^3

Ans: (a) a^2b

3. The quadratic equations $x^2 - 4x + k = 0$ has distinct real roots if

(a) k = 4 (b) k > 4 (c) k = 16 (d) k < 4

Ans. (d) k < 4

4. The number of polynomials having zeroes as -2 and 5 is

(a) 1 (b) 2 (c) 3 (d) more than 3

Ans: (d) more than 3

5. The pair of equations y = 0 and y = -7 has

(a) one solution (b) two solutions (c) infinitely many solutions (d) no solution

Ans. (d) no solution

6. The line segment joining the points A (5, 3) and B (-3, 11) is divided by the point C (3,5) in the ratio

(a) 1:3 (b) 3:1 (c) 2:3 (d) 3:2

Ans: (a) 1:3

7. ∆ABC is such that AB = 3 cm, BC = 2 cm, CA = 2.5 cm. If ∆ABC~∆DEF and EF=4cm, then perimeter of ∆DEF is

(a) 7.5 cm (b) 15cm (c) 22.5 cm (d) 30 cm

Ans: (b) 15cm

8. The value of sin 30° cos 60° + sin 60° cos 30° is:
(a) 0 (b) 1 (c) 2 (d) 4
Ans: (b) 1
sin 30° cos 60° + sin 60° cos 30°
$$=\left(\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}\right)=\frac{1}{4}+\frac{3}{4}=\frac{1+3}{4}=\frac{4}{4}=1$$

9. If 2 sin 2θ = √3, then find the value of θ.
(a) 30° (b) 60° (c) 90° (d) 45°
Ans: (a) 30°
$2\sin 2\theta = \sqrt{3} \Rightarrow \sin 2\theta = \sqrt{3}/2 = \sin 60^0$
$\Rightarrow 2\theta = 60^0 \Rightarrow \theta = 30^0$

10. A girl walks 200m towards East and then 150m towards North. The distance of the girl from the starting point is
(a) 350m (b) 250m (c) 300 m (d) 325 m
Ans: (b) 250 m

11. Consider the data:

Class	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Frequency	4	5	13	20	14	7	4

The difference of the upper limit of the median class and the lower limit of the modal class is
(a) 0 (b) 19 (c) 20 (d) 38
Ans: (c) 20

12. From an external point Q, the length of the tangents to a circle is 5 cm and the distance of Q from the centre is 8 cm. The radius of the circle is
(a) 39 cm (b) 3 cm (c) √39 cm (d) 7 cm
Ans: (c) √39 cm

13. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then:
(a) $R_1 + R_2 = R$ (b) $R_1^2 + R_2^2 = R^2$ (c) $R_1 + R_2 < R$ (d) $R_1 + R_2 < R_2$
Ans. (b) $R_1^2 + R_2^2 = R^2$
According to the given condition,
Area of circle with radius R = Area of circle with radius R1 + Area of circle with radius R2
$\Rightarrow \pi R^2 = \pi R_1^2 + \pi R_2^2$
$\Rightarrow R^2 = R_1^2 + R_2^2$

14. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal, then the ratio of the slant height of the cone to the height of the cylinder is:
(a) 2 : 1 (b) 1 : 2 (c) 1 : 3 (d) 3 : 1
Ans. (a) 2 : 1

15. For the following distribution:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of Students	3	12	27	57	75	80

the modal class is
(a) 10 – 20 (b) 20 – 30 (c) 30 – 40 (d) 50 – 60
Ans: (c) 30 – 40

16. The area of a circle that can be inscribed in a square of side 6 cm is:
(a) 36 π cm^2 (b) 18 π cm^2 (c) 12 π cm^2 (d) 9 π cm^2
Ans. (d) 9 π cm^2

17. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought?
(a) 40 (b) 240 (c) 480 (d) 750
Ans. (c) 480

18. If sin A = 1/2, cos B = 1, 0 < A, B $\le \pi/2$, then the value of cot (A + B) is:
(a) $\sqrt{3}$ /2 (b) 1/2 (c) 0 (d) $\sqrt{3}$
Ans: (d) $\sqrt{3}$
$\sin A = 1/2 = \sin 30^0 \Rightarrow A = 30^0$
$\cos B = 1 = \cos 0^0 \Rightarrow B = 0^0$
Now, $\cot(A+B) = \cot(30^0 + 0^0) = \cot 30^0 = \sqrt{3}$

Direction: In the question number 19 & 20, A statement of Assertion (A) is followed by a statement of Reason(R). Choose the correct option

19. **Assertion (A):** The value of y is 3, if the distance between the points P(2, -3) and Q(10, y) is 10.
Reason (R): Distance between two points is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of Assertion (A)
(c) Assertion (A) is true but reason(R) is false.
(d) Assertion (A) is false but reason(R) is true.
Ans: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

20. **Assertion (A):** If HCF (90, 144) = 18, then LCM (90, 144) = 720
Reason (R): HCF (a, b) x LCM (a, b) = a x b
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.
Ans: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

SECTION-B
Questions 21 to 25 carry 2 marks each

21. For what value of p will the following pair of linear equations have infinitely many solutions?
(p – 3)x + 3y = p; px + py = 12
Ans: Consider equations (p – 3)x + 3y = p
and px + py = 12
For infinitely many solutions,
$$\frac{p-3}{p} = \frac{3}{p} = \frac{p}{12} \quad ...(i)$$

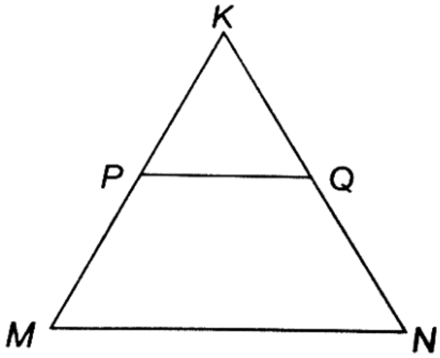
Consider, $\frac{3}{p} = \frac{p}{12} \Rightarrow p^2 = 36 \Rightarrow p = \pm 6$

For p = 6, from (i), $\frac{p-3}{p} = \frac{3}{p} = \frac{p}{12}$ is true

For p = – 6, from (i), $\frac{p-3}{p} = \frac{3}{p} = \frac{p}{12}$ is false.

Hence, for p = 6, pair of linear equations has infinitely many solutions.

22. In Figure, PQ is parallel to MN. If $\frac{KP}{PM} = \frac{4}{13}$ and KN = 20.4 cm. Find KQ.



Ans: In ΔKMN, we have PQ ∥ MN

$\therefore \frac{KP}{PM} = \frac{KQ}{QN}$ [Basic proportionality Theorem]

$\Rightarrow \frac{KP}{PM} = \frac{KQ}{KN - KQ} \Rightarrow \frac{4}{13} = \frac{KQ}{20.4 - KQ}$

⇒ 4(20.4 − KQ) = 13KQ

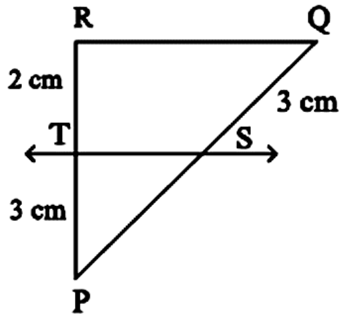
⇒ 81.6 − 4KQ = 13KQ

⇒ 17KQ = 81.6

$\Rightarrow \ KQ = \frac{81.6}{17} = 4.8cm$

OR

In the below figure, if ST ∥ QR. Find PS.



Ans: By Basic proportionality theorem, $\frac{PS}{QS} = \frac{PT}{RT}$

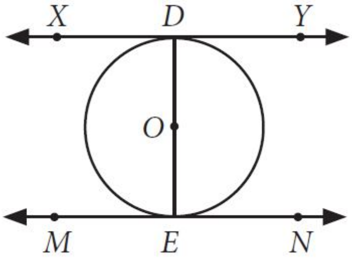
$\Rightarrow \frac{PS}{3} = \frac{3}{2} \ \Rightarrow PS = \frac{9}{2}cm$

23. If tan (A + B) = $\sqrt{3}$ and tan (A – B) = $\frac{1}{\sqrt{3}}$; 0° < A+B ≤ 90°; A > B, find A and B.

Ans: tan(A + B) = √3 = tan 60°

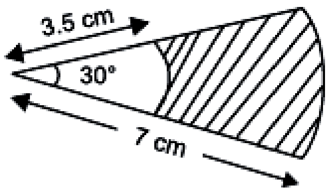
$\Rightarrow A + B = 60° \dashrightarrow$ (i)
$\tan(A - B) = 1/\sqrt{3} = \tan 30°$
$\Rightarrow A - B = 30° \dashrightarrow$ (ii)
Adding equation (i) and (ii),
$2A = 90°$
$\Rightarrow A = 45°$
Putting the value of A in equation (i),
$45° + B = 60°$
$\Rightarrow B = 60° - 45°$
$\Rightarrow B = 15°$

24. XY and MN are the tangents drawn at the end points of the diameter DE of the circle with centre O. Prove that XY || MN.



Ans: Since, XY is the tangent to the circle at the point D.
$\Rightarrow OD \perp XY \Rightarrow \angle EDX = 90°$
Also, MN is the tangent to the circle at E.
$\Rightarrow OE \perp MN \Rightarrow \angle DEN = 90°$
As, $\angle EDX = \angle DEN$ (each 90°)
which are alternate interior angles.
$\Rightarrow XY \| MN$

25. In the given figure, sectors of two concentric circles of radii 7 cm and 3.5 cm are given. Find the area of the shaded region. (Use $\pi = \frac{22}{7}$)



Ans: Area of the shaded region = area of the sector of 30° with radius 7 cm – area of the sector 30° with radius 3.5 cm

$$= \left[\frac{30°}{360°} \times \pi \times 7^2 - \frac{30°}{360°} \times \pi \times (3.5)^2\right] cm^2$$

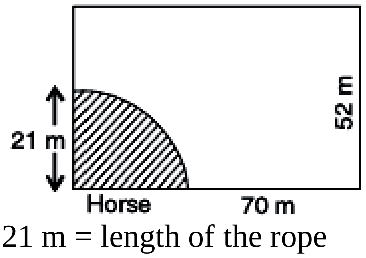
$$= \frac{30°}{360} \times \pi\left[7^2 - \left(\frac{7}{2}\right)^2\right]$$

$$= \frac{1}{12} \times \frac{22}{7} \times 7^2\left[1 - \frac{1}{(2)^2}\right] cm^2 = \frac{1}{12} \times 22 \times 7\left[1 - \frac{1}{4}\right] cm^2$$

$$= \frac{1}{12} \times 22 \times 7 \times \frac{3}{4} cm^2 = \frac{77}{8} cm^2 = 9.625 \text{ sq. cm}$$

OR

A horse is placed for grazing inside a rectangular field 70 m by 52 m and is tethered to one corner by a rope 21 m long. On how much area can it graze?
Ans : Area of the portion that horse can graze = area of the shaded portion.



Shaded portion is a sector of radius 21 m = length of the rope
Angle of this sector = angle of the corners of the rectangle = 90°
Area of the shaded portion that horse can graze

$$= \frac{\theta}{360°} \times \pi r^2 = \frac{90°}{360°} \times \frac{22}{7} \times (21)^2 m^2$$
$$= \frac{1}{4} \times \frac{22}{7} \times 21 \times 21\ m^2 = \frac{11}{2} \times 3 \times 21\ m^2 = 346.5\ m^2$$

SECTION-C
Questions 26 to 31 carry 3 marks each

26. Given that √3 is irrational, prove that (2 + 5√3) is an irrational number.
Ans: Let 2 + 5√3 be a rational number such that
2 + 5√3 = *a*, where *a* is a non-zero rational number.
$\Rightarrow 5\sqrt{3} = a - 2 \Rightarrow \sqrt{3} = \frac{a-2}{5}$
Since 5 and 2 are integers and a is a rational number, therefore $\frac{a-2}{5}$ is a rational number
⇒ √3 is a rational number which contradicts the fact that √3 is an irrational number.
Therefore, our assumption is wrong.
Hence 2 + 5√3 is an irrational number

27. Find the zeroes of the polynomial $x^2 + \frac{1}{6}x - 2$, and verify the relation between the coefficients and the zeroes of the polynomial.
Ans: The polynomial can be rewritten as $\frac{1}{6}(6x^2 + x - 12)$
On factoring, $6x^2 + x - 12 = 6x^2 + 9x - 8x - 12$
= 3x(2x + 3) - 4(2x + 3)
= (3x - 4)(2x + 3)
So, 1/6 $(6x^2 + 6x - 12) = 0$
(3x - 4)(2x + 3) = 0
3x - 4 = 0 ⇒ 3x = 4 ⇒ x = 4/3
2x + 3 = 0 ⇒ 2x = -3 ⇒ x = -3/2
Therefore, the zeros of the polynomial are 4/3 and -3/2.
Sum of the roots:
LHS: $\alpha + \beta$ =4/3 + (-3/2) = 8 - 9/6 = -1/6
RHS: -coefficient of x/coefficient of x^2 = -1/6
Product of the roots
LHS: $\alpha\beta$ = (4/3)(-3/2) = -12/6 = -2
RHS: constant term/coefficient of x^2 = -12/6 = -2.

28. A number consists of two digits. Where the number is divided by the sum of its digits, the quotient is 7. If 27 is subtracted from the number, the digits interchange their places, find the number.
Ans : Let digit at unit place be x, and at tenth place be y.
∴ Number = 10y + x
According to the question, $\frac{10y+x}{y+x} = 7 \Rightarrow 6x - 3y = 0$
⇒ 2x – y = 0 ...(i)
Again according to the question, (10y + x) – 27 = 10x + y
⇒ 9x – 9y = – 27 ⇒ x – y = – 3 ...(ii)
Solving for x and y, we get
x = 3 and y = 6
∴ Number is 63.

OR

Students of a class are made to stand in rows. If 4 students are extra in a row, there would be two rows less. If 4 students are less in a row, there would be four more rows. Find the number of students in the class.
Ans : Let number of students in a row be x and number of rows be y.
⇒ Total number of students = x . y
From condition 1 : (x + 4) (y – 2) = xy
⇒ xy – 2x + 4y – 8 = xy – 2x + 4y = 8 ...(i)
From condition 2 : (x – 4) (y + 4) = xy xy + 4x – 4y – 16 = xy
⇒ 4x – 4y = 16 ...(ii)
Adding (i) and (ii), we get 2x = 24 ⇒ x = 12
Substituting in (i), we get y = 8.
∴ Total number of students = xy = 12 × 8 = 96.

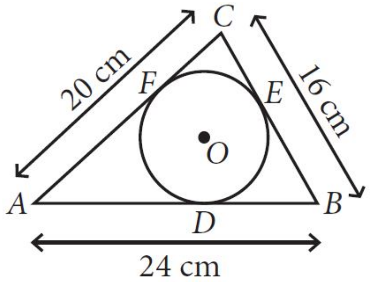
29. Prove that : sin θ (1 + tan θ) + cos θ (1 + cot θ) = sec θ + cosec θ.
Ans: LHS = sin θ (1 + tan θ) + cos θ (1 + cot θ)
= sin θ + sin θ . tan θ + cos θ + cos θ . cot θ

$$= \sin\theta + \sin\theta . \frac{\sin\theta}{\cos\theta} + \cos\theta + \cos\theta . \frac{\cos\theta}{\sin\theta} = \sin\theta + \frac{\cos^2\theta}{\sin\theta} + \frac{\sin^2\theta}{\cos\theta} + \cos\theta$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta} + \frac{\sin^2\theta + \cos^2\theta}{\cos\theta} = \frac{1}{\sin\theta} + \frac{1}{\cos\theta}$$

= cosec θ + sec θ = sec θ + cosec θ = RHS

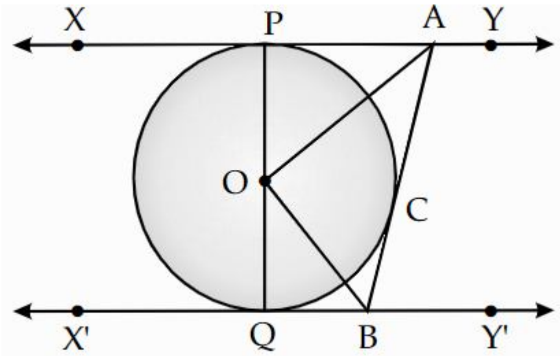
30. A circle is inscribed in a ΔABC having sides 16 cm, 20 cm and 24 cm as shown in figure. Find AD, BE and CF.



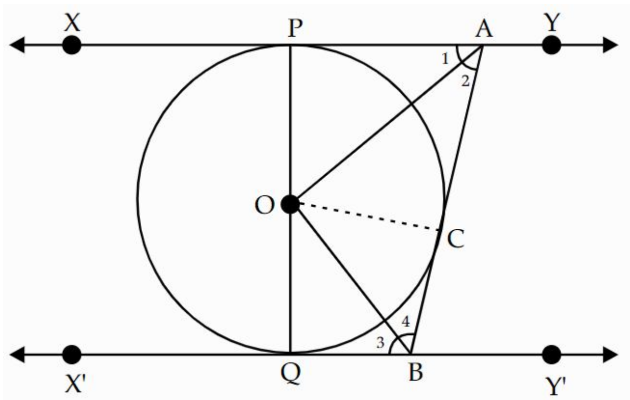
Ans: Since, tangents drawn from an external point to a circle are equal.
∴ AD = AF = x (say)
BD = BE = y (say)
CE = CF = z (say)
According to the question,
AB = x + y = 24 cm ...(i)
BC = y + z = 16 cm ...(ii)
AC = x + z = 20 cm ...(iii)
Subtracting (iii) from (i), we get
y – z = 4 ...(iv)
Adding (ii) and (iv), we get
2y = 20 ⇒ y = 10 cm
Substituting the value of y in (i) and (ii), we get
x = 14 cm, z = 6 cm
Hence, AD = 14 cm, BE = 10 cm and CF = 6 cm.

OR

In the figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C interesting XY at A and X'Y' at B, prove that ∠AOB is a right angle.



Ans: Join OC. Since, the tangents drawn to a circle from an external point are equal.
∴ AP = AC
In Δ PAO and Δ AOC, we have:
AO = AO [Common]
OP = OC [Radii of the same circle]
AP = AC
⇒ Δ PAO ≅ Δ AOC [SSS Congruency]
∴ ∠PAO = ∠CAO = ∠1
∠PAC = 2 ∠1 ...(1)
Similarly ∠CBQ = 2 ∠2 ...(2)



Again, we know that sum of internal angles on the same side of a transversal is 180°.
∴ ∠PAC + ∠CBQ = 180°
⇒ 2 ∠1 + 2 ∠2 = 180° [From (1) and (2)]
⇒ ∠1 + ∠2 = 180°/2 = 90° ...(3)
Also ∠1 + ∠2 + ∠AOB = 180° [Sum of angles of a triangle]
⇒ 90° + ∠AOB = 180°
⇒ ∠AOB = 180° − 90° ⇒ ∠AOB = 90°.

31. Two dice are thrown at the same time. What is the probability that the sum of the two numbers appearing on the top of the dice is
(i) at least 9? (ii) 7? (iii) less than or equal to 6?
Ans: (i) Number of outcomes with sum of the numbers is at least 9 = 10
∴ Required Probability = 10/36 = 5/18
(ii) Number of outcomes with sum of the numbers 7 = 6
∴ Required Probability = 6/36 = 1/6
(iii) Number of outcomes with sum of the numbers less than or equal to 6 = 36
∴ Required Probability = 15/36 = 5/12

SECTION-D
Questions 32 to 35 carry 5 marks each

32. State and Prove Basic Proportionality Theorem.

Ans: Statement - 1 mark
Given, To Prove, Construction, Figure – 2 marks
Proof – 2 marks

33. The median of the following data is 137. Find the values of x and y, If the total frequency is 68.

Class intervals	65 – 85	85 – 105	105 – 125	125 – 145	145 – 165	165 – 185	185 – 205
Frequency	4	x	13	20	14	y	4

Ans:

Class	**Frequency**	**Cumulative Frequency**
65-85	4	4
85-105	x	4 + x
105-125	13	17+ x
125-145	20	37+ x
145-165	14	51+ x
165-185	y	51 + x + y
185-205	4	55 + x + y

∴ Median class is "125–145." cf = 17 + x , l = 125 , f = 20 h = 20, N = 68

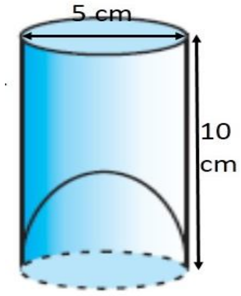
Median = $l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h$

$\Rightarrow 137 = 125 + \left(\frac{34 - 17 - x}{20} \right) \times 20$

$\Rightarrow 137 - 125 = 17 - x$

$\Rightarrow x = 17 - 12 = 5 \Rightarrow 55 + 5 + y = 68 \Rightarrow y = 8$

34. A juice seller serves his customers using a glass as shown in figure. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass has a hemispherical portion raised which reduces the capacity of the glass. If the height of the glass is 10 cm, find the apparent capacity of the glass and its actual capacity. [$\pi = 3.14$]



Ans:

Apparent capacity of glass = $\pi r^2 h$

$= 3.14 \times \left(\frac{5}{2}\right)^2 \times 10 \text{ cm}^3 = 196.25 \text{ cm}^3$

Actual capacity of glass = apparent capacity – volume of hemispherical part

$= 196.25 \text{ cm}^3 - \frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^2 \text{ cm}^3 = 196.25 \text{ cm}^3 - 32.70 \text{ cm}^3 = 163.55 \text{ cm}^3$

OR

A rectangular sheet of paper 30 cm × 18 cm can be transformed into the curved surface of a right circular cylinder in two ways either by rolling the paper along its length or by rolling it along its breadth. Find the ratio of the volumes of the two cylinders thus formed.

Ans:

Rolling along length, circumference of base = 30 cm

$2\pi r = 30$ cm

$\Rightarrow r = \frac{30}{2\pi}$ cm

Volume = $\pi \times \frac{30}{2\pi} \times \frac{30}{2\pi} \times 18 = \frac{225 \times 18}{\pi}$ cm^3

Rolling along its width, circumference of base = 18 cm

$\Rightarrow 2\pi R = 18$

$\Rightarrow R = \frac{18}{2\pi} = \frac{9}{\pi}$ cm

Volume = $\pi \times \frac{9}{\pi} \times \frac{9}{\pi} \times 30 = \frac{81 \times 30}{\pi}$ cm^3

Ratio of volumes = $\frac{225 \times 18}{\pi} : \frac{81 \times 30}{\pi} = 5 : 3$

35. A person on tour has Rs.360 for his expenses. If he extends his tour for 4 days, he has to cut down his daily expenses by Rs.3. Find the original duration of the tour.

Ans: Let days be the original duration of the tour.

Total expenditure on tour ₹ 360

Expenditure per day ₹ 360/x

Duration of the extended tour (x + 4) days

Expenditure per day according to the new schedule ₹ 360/(x + 4)

Given that daily expenses are cut down by ₹ 3

As per the given condition, $\frac{360}{x} - \frac{360}{x+4} = 3 \Rightarrow 360\left(\frac{1}{x} - \frac{1}{x+4}\right) = 3$

$\Rightarrow \left(\frac{1}{x} - \frac{1}{x+4}\right) = \frac{3}{360} = \frac{1}{120} \Rightarrow \frac{x+4-x}{x(x+4)} = \frac{1}{120} \Rightarrow \frac{4}{x(x+4)} = \frac{1}{120}$

$\Rightarrow x(x + 4) = 480 \Rightarrow x^2 + 4x = 480 \Rightarrow x^2 + 4x - 480 = 0$

$\Rightarrow x^2 + 24x - 20x - 480 = 0 \Rightarrow x(x + 24) - 20(x + 24) = 0$

$\Rightarrow x - 20 = 0$ or $x + 24 = 0 \Rightarrow x = 20$ or $x = -24$

Since the number of days cannot be negative. So, x = 20

Therefore, the original duration of the tour was 20 days

OR

Rs.6500 were divided equally among a certain number of persons. Had there been 15 more persons, each would have got Rs.30 less. Find the original number of persons.

Ans: Let the original number of persons be x

Total money which was divided = Rs. 6500

Each person share = Rs. 6500/x

According to the question, $\frac{6500}{x} - \frac{6500}{x+15} = 30 \Rightarrow \frac{6500x + 97500 - 6500x}{x(x+15)} = 30$

$\Rightarrow \frac{97500}{x(x+15)} = 30 \Rightarrow \frac{3250}{x(x+15)} = 1 \Rightarrow x^2 + 15x - 3250 = 0$

$\Rightarrow x^2 + 65x - 50x - 3250 = 0 \Rightarrow x(x + 65) - 50(x + 65) = 0$

$\Rightarrow (x + 65)(x - 50) = 0 \Rightarrow x = -65, 50$

Since the number of persons cannot be negative, hence the original numbers of person is 50

SECTION-E (Case Study Based Questions)
Questions 36 to 38 carry 4 marks each

36. Case Study – 1:

In the month of April to June 2022, the exports of passenger cars from India increased by 26% in the corresponding quarter of 2021–22, as per a report. A car manufacturing company planned to produce 1800 cars in 4th year and 2600 cars in 8th year. Assuming that the production increases uniformly by a fixed number every year.

Based on the above information answer the following questions.
(i) Find the production in the 1st year. (1)
(ii) Find the production in the 12th year. (1)
(iii) Find the total production in first 10 years. (2)

OR

(iii) In how many years will the total production reach 31200 cars? (2)

Ans: (i) Since the production increases uniformly by a fixed number every year, the number of Cars manufactured in 1st, 2nd, 3rd, . . .,years will form an AP.
So, a + 3d = 1800 & a + 7d = 2600
So d = 200 & a = 1200
(ii) $a_{12} = a + 11d \Rightarrow a_{30} = 1200 + 11 \times 200$
$\Rightarrow a_{12} = 3400$
(iii) $S_n = \frac{n}{2}[2a+(n-1)d] \Rightarrow S_{10} = \frac{10}{2}[2\times1200+(10-1)\times200]$
$\Rightarrow S_{10} = 5[2400+1800] = 5\times4200 = 21000$

OR

$S_n = \frac{n}{2}[2a+(n-1)d] = 31200$

$\Rightarrow \frac{n}{2}[2\times1200+(n-1)\times200] = 31200$

$\Rightarrow \frac{n}{2}\times200[12+(n-1)] = 31200$

$\Rightarrow n[12 + (n - 1)] = 312$
$\Rightarrow n^2 + 11n - 312 = 0$
$\Rightarrow n^2 + 24n - 13n - 312 = 0$
$\Rightarrow (n+24)(n - 13) = 0$
$\Rightarrow n = 13$ or -24.
As n can't be negative. So n = 13

37. Case Study – 2:

In order to conduct sports day activities in your school, lines have been drawn with chalk powder at a distance of 1 m each in a rectangular shaped ground ABCD. 100 flower pots have been placed at the distance of 1 m from each other along AD, as shown in the following figure. Niharika runs ($\frac{1}{4}$)th distance AD on the 2nd line and posts a green Flag. Preet runs ($\frac{1}{5}$) th distance AD on the eighth line and posts are red flags. Taking A as the origin AB along x-axis and AD along y-axis, answer the following questions:

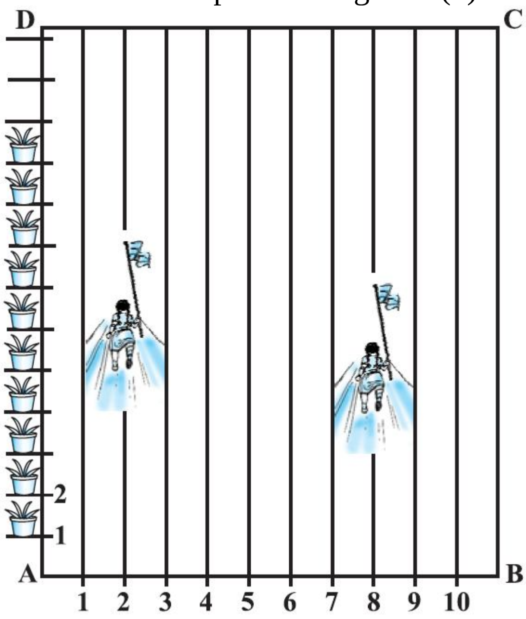
(i) Find the coordinates of the green flag. (1)

(ii) Find the distance between the two flags. (1)

(iii) If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag? (2)

OR

(iii) If Joy has to post a flag at one fourth distance from the green flag, in the line segment joining the green and red flags, then where should he post his flag? (2)

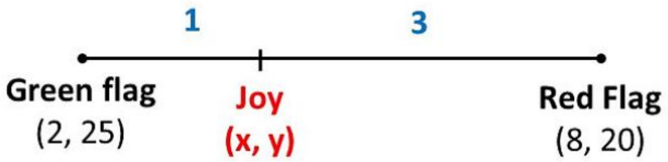


Ans: (i) Position of the green flag is $\left(2, \frac{1}{4} \times 100\right) = (2, 25)$

(ii) Distance between the two flags = $\sqrt{(36+25)} = \sqrt{61}\,cm$

(iii) Position of the blue flag = $\left(\frac{2+8}{2}, \frac{25+20}{2}\right) = (5, 22.5)$

OR



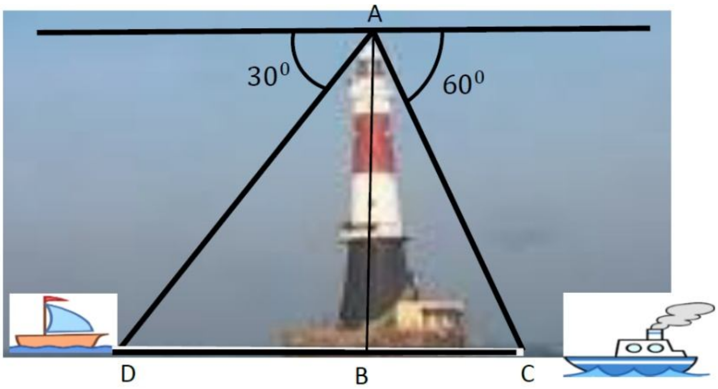
$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{1 \times 8 + 3 \times 2}{1 + 3} = \frac{8+6}{4} = \frac{14}{4} = \mathbf{3.5}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1 \times 20 + 3 \times 25}{1 + 3} = \frac{20+75}{4} = \frac{95}{4} = \mathbf{23.75}$$

Required point is (3.5, 23.75)

38. Case Study – 3:

A lighthouse is a tall tower with light near the top. These are often built on islands, coasts or on cliffs. Lighthouses on water surface act as a navigational aid to the mariners and send warning to boats and ships for dangers. Initially wood, coal would be used as illuminators. Gradually it was replaced by candles, lanterns, electric lights. Nowadays they are run by machines and remote monitoring. Prongs Reef lighthouse of Mumbai was constructed in 1874-75. It is approximately 40 meters high and its beam can be seen at a distance of 30 kilometres. A ship and a boat are coming towards the lighthouse from opposite directions. Angles of depression of flash light from the lighthouse to the boat and the ship are 30^0 and 60^0 respectively.



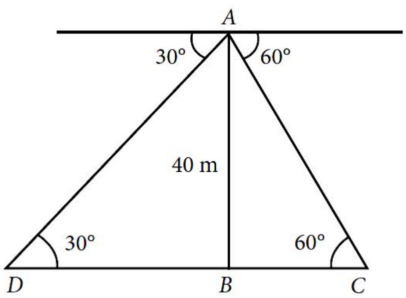
(i) Which of the two, boat or the ship is nearer to the light house. Find its distance from the lighthouse? (2)
(ii) Find the time taken by the boat to reach the light house if it is moving at the rate of 2 km per hour. (2)

OR

(ii) The ratio of the height of a light house and the length of its shadow on the ground is √3 : 1 . What is the angle of elevation of the sun?

Ans:

(i) Here, height of lighthouse (AB) = 40 m (Given)



In ΔACB, $\tan 60° = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{40}{BC} \Rightarrow BC = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3}$ m

Also, in ΔADB, $\tan 30° = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{DB} \Rightarrow DB = 40\sqrt{3}$ m

Thus, ship is nearer to the light house.

(ii) Boat moving at the speed of 2 km/hr *i.e.*, $\frac{2000}{60}$ m/min.

$\therefore$ Time taken to cover the distance $= \frac{\text{Distance } DB}{\text{Speed}} = \frac{60}{2000} \times 40\sqrt{3} = 2.078$ minutes

(iii) Let height of light house be AB and its shadow be BC.

In ΔABC, $\tan\theta = \frac{AB}{AC}$

But $\frac{AB}{AC} = \frac{\sqrt{3}}{1} = \sqrt{3} \Rightarrow \tan\theta = \sqrt{3} = \tan 60^0 \Rightarrow \theta = 60^0$