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SAMPLE PAPER TEST 12 FOR BOARD EXAM 2024
(ANSWERS)

SUBJECT: MATHEMATICS — **MAX. MARKS : 80**
CLASS : X — **DURATION : 3 HRS**

General Instruction:
1. This Question Paper has 5 Sections A-E.
2. **Section A** has 20 MCQs carrying 1 mark each.
3. **Section B** has 5 questions carrying 02 marks each.
4. **Section C** has 6 questions carrying 03 marks each.
5. **Section D** has 4 questions carrying 05 marks each.
6. **Section E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

SECTION – A
Questions 1 to 20 carry 1 mark each.

1. The next term of the A.P.: $\sqrt{6}$, $\sqrt{24}$, $\sqrt{54}$ is:
(a) $\sqrt{60}$ (b) $\sqrt{96}$ (c) $\sqrt{72}$ (d) $\sqrt{216}$
Ans: (b) $\sqrt{96}$
First term, $a_1 = \sqrt{6}$
Second term, $a_2 = \sqrt{24} = 2\sqrt{6}$
Common difference $= 2\sqrt{6} - \sqrt{6} = \sqrt{6}(2 - 1) = \sqrt{6}$
Next term of A.P. is = Third term + common difference
$= \sqrt{54} + 6 = 3\sqrt{6} + 6 = 4\sqrt{6} = \sqrt{96}$

2. If the quadratic equation $ax^2 + bx + c = 0$ has two real and equal roots, then 'c' is equal to
(a) $-b/2a$ (b) $b/2a$ (c) $-b^2/4a$ (d) $b^2/4a$
Ans: (d) $b^2/4a$
For equation having real and equal roots
$D = b^2 - 4ac = 0$
$\Rightarrow b^2 - 4ac = 0 \Rightarrow b^2 = 4ac \Rightarrow c = b^2/4a$

3. The distance of the point (–6, 8) from x-axis is
(a) 6 units (b) – 6 units (c) 8 units (d) 10 units
Ans: (c) 8 units
The perpendicular distance of point (– 6, 8) from x-axis is y-coordinate i.e. 8 units.

4. The ratio of HCF to LCM of the least composite number and the least prime number is:
(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3
Ans: (a) 1 : 2
Least composite number is 4 and the least prime number is 2.
HCF (4, 2) : LCM(4, 2) = 2 : 4 = 1 : 2

5. The circumferences of two circles are in the ratio 4 : 5. What is the ratio of their radii ?
(a) 16 : 25 (b) 25 : 16 (c) 2 : 5 (d) 4 : 5
Ans: (d) 4 : 5
Circumference of circle = $2\pi r$

$\therefore \frac{2\pi r_1}{2\pi r_2} = \frac{4}{5} \Rightarrow \frac{r_1}{r_2} = \frac{4}{5}$

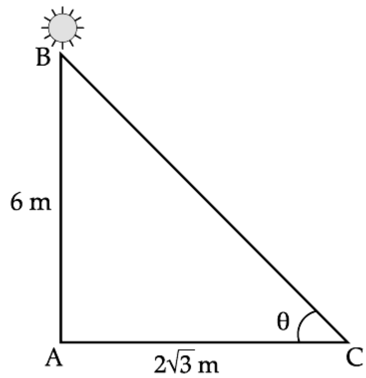
Hence, Ratio of their radii = 4 : 5.

6. The empirical relation between the mode, median and mean of a distribution is:
(a) Mode = 3 Median – 2 Mean (b) Mode = 3 Mean – 2 Median
(c) Mode = 2 Median – 3 Mean (d) Mode = 2 Mean – 3 Median
Ans: (a) Mode = 3 Median – 2 Mean
Empirical formula, Mode = 3 Median – 2 Mean

7. The point of intersection of the line represented by 3x – y = 3 and y-axis is given by
(a) (0, –3) (b) (0, 3) (c) (2, 0) (d) (–2, 0)
Ans: (a) (0, –3)
3x – y = 3 (Given)
At the y-axis, value of x = 0
Substitute value of 'x' in given equations we have,
3 × 0 – y = 3
⇒ – y = 3 ⇒ y = – 3
Hence, the line 3x – y = 3 cuts y axis at point (0, – 3).

8. If a and b are the zeroes of the polynomial $x^2 - 1$, then the value of (ab) is
(a) 2 (b) 1 (c) – 1 (d) 0
Ans: (c) – 1
$(x^2 - 1) = (x + 1)(x - 1)$
⇒ x + 1 = 0 & x – 1 = 0
⇒ x = – 1, x = 1
Thus, a = – 1 and b = 1
∴ ab = (– 1)(1) = – 1

9. If a pole 6 m high casts a shadow 2√3 m long on the ground, then sun's elevation is:
(a) 60° (b) 45° (c) 30° (d) 90°
Ans: (a) 60°



$$\tan\theta = \frac{AB}{AC} = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60^0 \Rightarrow \theta = 60^0$$

10. $\frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$, in simplified form is:
(a) $\tan^2\theta$ (b) $\sec^2\theta$ (c) 1 (d) – 1
Ans: (d) – 1

$$\frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta} = \cot^2\theta - \operatorname{cosec}^2\theta = -1 \quad [\because \operatorname{cosec}^2\theta - \cot^2\theta = 1]$$

11. A bag contains 5 pink, 8 blue and 7 yellow balls. One ball is drawn at random from the bag. What is the probability of getting neither a blue nor a pink ball ?

(a) 1/4 (b) 2/5 (c) 7/20 (d) 13/20

Ans: (c) 7/20

Number of balls which is neither a blue nor a Pink = 7

∴ P(Getting a ball which is neither blue or pink) = 7/20

12. If $\Delta PQR \sim \Delta ABC$; PQ = 6 cm, AB = 8 cm and the perimeter of ΔABC is 36 cm, then the perimeter of ΔPQR is

(a) 20.25 cm (b) 27 cm (c) 48 cm (d) 64 cm

Ans: (b) 27 cm

$\Delta PQR \sim \Delta ABC$

PQ = 6 cm, AB = 8 cm

Perimeter of ΔABC = 36 cm

We know that, Ratio of perimeter of two similar triangles is same as the ratio of their corresponding sides.

$$\therefore \frac{Perimeter\ of\ \Delta ABC}{Perimeter\ of\ \Delta PQR} = \frac{8}{6} \Rightarrow \frac{36}{x} = \frac{8}{6} \Rightarrow x = \frac{36 \times 6}{8} = 27cm$$

13. The distance of the point (–6, 8) from origin is:

(a) 6 (b) – 6 (c) 8 (d) 10

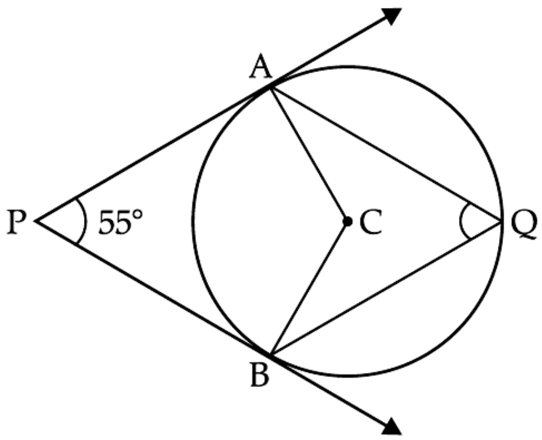
Ans: (d) 10

Distance between (– 6, 8) and (0, 0) is

$a^2 = (-6 - 0)^2 + (8 - 0)^2$

$\Rightarrow a^2 = 36 + 64 = 100 \Rightarrow a = 10$

14. In the given figure, PA and PB are tangents from external point P to a circle with centre C and Q is any point on the circle. Then the measure of ∠AQB is



(a) 62½° (b) 125° (c) 55° (d) 90°

Ans: (a) 62½°

∠PAC = 90° (Tangent is perpendicular to the radius through point of contact)

∠PBA = 90°

∠APB = 55° (Given)

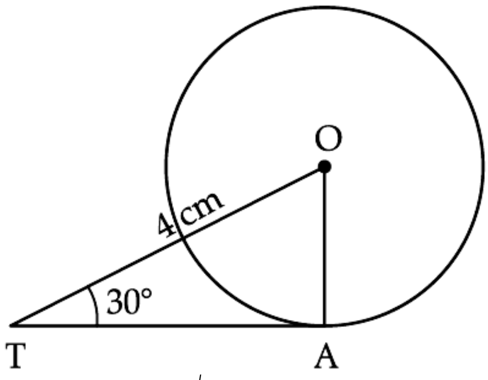
So, ∠APB + ∠PAC + ∠PBA + ∠ACB = 360° (Sum of all angles of quadrilaterals is 360°)

∠ACB = 360° – 235° = 125°

∠ACB = 2∠AQB

∴ ∠AQB = 125/2 ° = 62½° (∵ Angle subtended by an arc at centre is double the angle subtended by it at any other point of contact.)

15. In the given figure, TA is a tangent to the circle with centre O such that OT = 4 cm, ∠OTA = 30°, then length of TA is:



(a) $2\sqrt{3}$ cm (b) 2 cm (c) $2\sqrt{2}$ cm (d) 3 cm

Ans: (a) $2\sqrt{3}$ cm

Here, $\angle OAT = 90°$ (angle between tangent and radius)

In ΔOAT, $\cos 30° = TA/OT$

$\Rightarrow \sqrt{3}/2 = TA / 4$

$\Rightarrow TA = 4\sqrt{3}/2 = 2\sqrt{3}$ cm

16. If a, b are the zeroes of the polynomial $p(x) = 4x^2 - 3x - 7$, then $\left(\frac{1}{a}+\frac{1}{b}\right)$ is equal to:

(a) 7/3 (b) −7/3 (c) 3/7 (d) −3/7

Ans: (d) −3/7

For zeroes of polynomial, put p(x) = 0

$4x^2 - 3x - 7 = 0$

$\Rightarrow 4x^2 - 7x + 4x - 7 = 0$

$\Rightarrow x(4x - 7) + 1(4x - 7) = 0$

$\Rightarrow (4x - 7)(x + 1) = 0$

$\Rightarrow x = 7/4$ and $x = -1$

Let a = 7/4 and b = – 1

$\therefore \frac{1}{a}+\frac{1}{b}=\frac{4}{7}+\frac{1}{-1}=\frac{4}{7}-1=\frac{-3}{7}$

17. A card is drawn at random from a well shuffled deck of 52 playing cards. The probability of getting a face card is

(a) 1/2 (b) 3/13 (c) 4/13 (d) 1/13

Ans: (b) 3/13

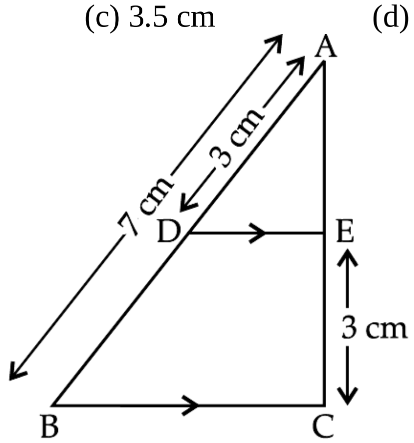
Total Number of Cards = 52

Total Number of Face Cards = 12

∴ P(Probability of getting a Face card) = 12/52 = 3/13

18. In the given figure, DE||BC. If AD = 3 cm, AB = 7 cm and EC = 3 cm, then the length of AE is

(a) 2 cm (b) 2.25 cm (c) 3.5 cm (d) 4 cm



Ans: (b) 2.25 cm

In ΔABC, $DE \parallel BC$
Then, $AD/DB = AE/EC$ (By Basic Proportionality theorem)
$\Rightarrow 3/4 = AE/3$ ($\because DB = AB - AD = 7 - 3 = 4$ cm)
$\therefore AE = 9/4 = 2.25$ cm.

Direction : In the question number 19 & 20 , A statement of Assertion (A) is followed by a statement of Reason(R) . Choose the correct option
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. Assertion (A): a, b, c are in A.P. if only if $2b = a + c$.
Reason (R): The sum of first n odd natural numbers is n^2.
Ans: (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

20. Assertion (A): The probability that a leap year has 53 Sunday is 2/7.
Reason (R): The probability that a non-leap year has 53 Sunday is 5/7.
Ans: (c) Assertion (A) is true but Reason (R) is false.
Assertion: A week has 7 days and total days are 366
Number of Sundays is a leap year = 52 Sundays + 2 days
Therefore, probability of leap year with 53 Sundays = 2/7
Reason: There are 52 Sundays in a non-leap year.
But one left over days apart from those 52 weeks can be either a Monday. Tuesday, Wednesday, Thursday, Friday, Saturday or Sunday.
$\therefore$ Required probability = 1/7

SECTION-B
Questions 21 to 25 carry 2M each

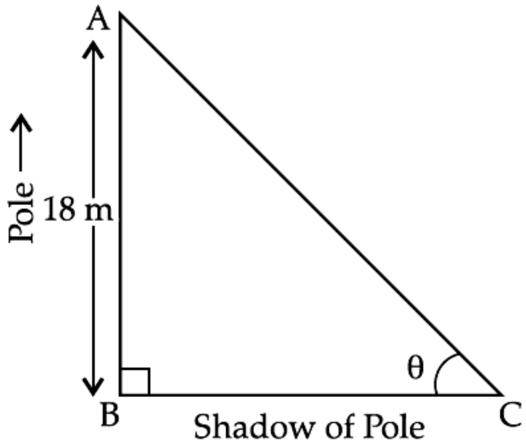
21. Show that 6^n cannot end with digit 0 for any natural number 'n'.
Ans: If 6^n ends with 0 then it must have 5 as a factor.
But, $6^n = (2 \times 3)^n = 2^n \times 3^n$
This shows that 2 and 3 are the only Prime Factors of 6^n.
According to Fundamental theorem of arithmetic prime factorization of each number is Unique.
So, 5 is not a factor of 6^n
Hence, 6^n can never end with the digit 0.

22. Find the sum and product of the roots of the quadratic equation $2x^2 - 9x + 4 = 0$.
OR
Find the discriminant of the quadratic equation $4x^2 - 5 = 0$ and hence comment on the nature of roots of the equation.
Ans: Given quadratic equation is $2x^2 - 9x + 4 = 0$
Sum of roots $= -(-9)/2 = 9/2$
Product of roots $= 4/2 = 2$
[For quadratic equation $ax^2 + bx + c = 0$, sum of roots $= -b/a$ and product of roots $= c/a$]
OR
Given quadratic equation is $4x^2 - 5 = 0$
$\because$ discriminant, $D = b^2 - 4ac$
$\Rightarrow D = 0 - 4(4)(-5) \Rightarrow D = 80$
Thus, discriminant D = 80

Since, D > 0, then roots are real and distinct.

23. If a fair coin is tossed twice, find the probability of getting 'atmost one head'.
Ans: When a coin is tossed two times.
Total possible outcomes are {TT, HH, TH, HT} = 4
No. of favourable outcomes = {HH, HT, TH} = 3
Required probability = 3/4

24. Find the length of the shadow on the ground of a pole of height 18 m when angle of elevation θ of the sun is such that tanθ = 6/7.
Ans: In right ΔABC,
tanθ = AB/BC = P/B ...(i)



But tanθ = 6/7 (Given)
Substitute value of tanθ and height of pole AB = 18 m is equation (i)
⇒ 6/7 = 18/BC
⇒ BC = 18 × 7 / 6 = 21 m
Hence, the length of the shadow = 21 m.

OR

If A and B are acute angles such that sin (A – B) = 0 and 2 cos (A + B) – 1 = 0, then find angles A and B.
Ans: Given sin (A – B) = 0 and 2 cos (A + B) – 1 = 0
sin (A – B) = 0
and 2 cos (A + B) – 1 = 0
⇒ sin (A – B) = sin 0°
and cos (A + B) = 1/2
⇒ A – B = 0° ...(i)
and cos (A + B) = cos 60°
and A + B = 60° ...(ii)
On solving eqs (i) and (ii), we get
A = 30° and B = 30°

25. If one zero of the polynomial p(x) = $6x^2 + 37x - (k - 2)$ is reciprocal of the other, then find the value of k.
Ans: Let the zeroes of polynomials are α and 1/α.
product of zeroes = −(k − 2)/6
⇒ α × 1/α = −(k − 2)/6
⇒ 6 = – (k – 2) ⇒ k = 2 – 6 ⇒ k = – 4
Therefore, value of k is – 4.

SECTION-C
Questions 26 to 31 carry 3 marks each

26. Find the HCF and LCM of 26, 65 and 117, using prime factorisation.

Ans: By Prime Factorization

Factors of 26 = 2 × 13

Factors of 65 = 5 × 13

Factors of 117 = 3^2 × 13

HCF of (26, 65, 117) = Product of common terms with lowest power = 13

LCM of (26, 65, 117) = Product of Prime Factors with highest Power

= 2 × 5 × 3^2 × 13 = 1170

OR

Prove that √2 is an irrational number.

Ans: Let $\sqrt{2}$ is a rational number then we have $\sqrt{2} = \frac{p}{q}$, where p and q are co-primes.

$\Rightarrow p = \sqrt{2}\ q$

Squaring both sides, we get $p^2 = 2q^2$

$\Rightarrow$ p^2 is divisible by 2 $\Rightarrow$ p is also divisible by 2

So, assume p = 2m where m is any integer.

Squaring both sides, we get $p^2 = 4m^2$

But $p^2 = 2q^2$

Therefore, $2q^2 = 4m^2 \Rightarrow q^2 = 2m^2$

$\Rightarrow$ q^2 is divisible by 2 $\Rightarrow$ q is also divisible by 2

From above we conclude that p and q have one common factor i.e. 2 which contradicts that p and q are co-primes.

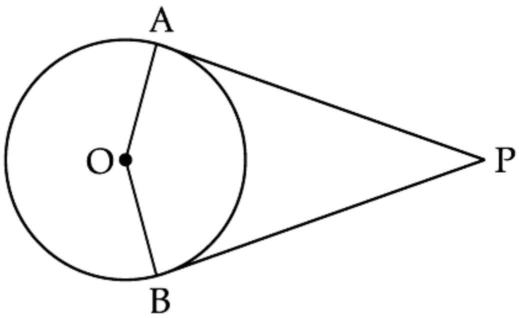
Therefore, our assumption is wrong.

Hence, $\sqrt{2}$ is an irrational number.

27. Prove that the angle between the two tangents drawn from an external to circle is supplementary to the angle subtended by the line joining the points of contact at the centre.

Ans: Given: PA and PB are the tangent drawn from a point P to a circle with centre O Also, the line segments OA and OB are drawn.

To prove: ∠APB + ∠AOB = 180°



Proof: We know that the tangents to a circle is perpendicular to the radius through the points of contact.

∴ PA ⊥ OA ⇒ ∠OAP = 90°

and PB ⊥ OB ⇒ ∠OBP = 90°

Therefore, ∠OAP + ∠OBP = 180°

Hence ∠APB + ∠AOB = 180° [Sum of the all the angles of a quadrilateral is 360°]

28. Prove that: $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

Ans: $RHS = \frac{\sin^2 A}{1-\cos A} = \frac{1-\cos^2 A}{1-\cos A} = \frac{(1+\cos A)(1-\cos A)}{1-\cos A}$

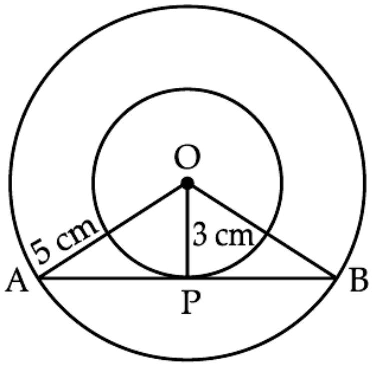
$$= 1 + \cos A = 1 + \frac{1}{\sec A} = \frac{\sec A + 1}{\sec A} = LHS$$

29. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Ans: Let the two concentric circles with centres O. Let AB be the chord of the larger circle which touches the smaller circle at point P.

Therefore, AB is tangent to the smaller circle to the point P.

$\therefore$ OP $\perp$ AB



In ΔOPA, $AO^2 = OP^2 + AP^2$

$\Rightarrow (5)^2 = (3)^2 + AP^2$

$\Rightarrow AP^2 = 25 - 9 \Rightarrow AP = 4$ cm

Now, in ΔOPB, OP $\perp$ AB

$\Rightarrow$ AP = PB (Perpendicular form the centre of the circle bisects the chord)

Thus, AB = 2AP = 2 × 4 = 8 cm

Hence, length of the chord of the larger circle is 8 cm.

30. The sum of two numbers is 15. If the sum of their reciprocals is 3/10, find the two numbers.

Ans: Let First Number = x

Other Number = 15 – x

So, $\frac{1}{x} + \frac{1}{15-x} = \frac{3}{10} \Rightarrow \frac{15-x+x}{x(15-x)} = \frac{3}{10}$

$\Rightarrow 15 \times 10 = 3x(15 - x)$

$\Rightarrow 150 = 45x - 3x^2$

$\Rightarrow 3x^2 - 45x + 150 = 0$

$\Rightarrow x^2 - 15x + 50 = 0$

$\Rightarrow x^2 - 10x - 5x + 50 = 0$

$\Rightarrow x(x - 10) - 5(x - 10) = 0$

$\Rightarrow (x - 10)(x - 5) = 0$

$\Rightarrow x = 10, x = 5$

If First Number (x) = 10

Other Number (15 – x) = 5

If First Number (x) = 5

Other Number (15 – x) = 10

31. How many terms are there in an A.P. whose first and fifth terms are –14 and 2, respectively and the last term is 62.

OR

Which term of the A.P.: 65, 61, 57, 53, ... is the first negative term?

Ans: Given, first term (a) = – 14, fifth term (a_5) = 2

and last term (a_n) = 62 1

Let common difference be d.

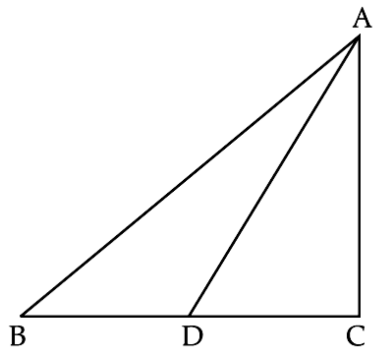
$\therefore a_5 = a + 4d \Rightarrow 2 = -14 + 4d \Rightarrow d = 4$ ...(i) 1
$a_n = a + (n-1)d$
$\Rightarrow 62 = -14 + (n-1)4$ [From eq (i)]
$\Rightarrow n - 1 = 19 \Rightarrow n = 20$
Thus, number of terms in A.P. are 20

OR

Given A.P. is 65, 61 57, 53, ...
Here, first term, a = 65
common difference, d = – 4
Let the nth term of the given A.P. be the first negative term.
$\therefore a_n < 0 \Rightarrow a + (n-1)d < 0$
$\Rightarrow 65 + (n-1)(-4) < 0 \Rightarrow 69 - 4n < 0 \Rightarrow -4n < -69$
$\Rightarrow n > 69/4 \Rightarrow n > 17.25$
Since, 18 is the natural number just greater than 17.25
So, n = 18
Hence, 18th term is first negative term.

SECTION-D
Questions 32 to 35 carry 5M each

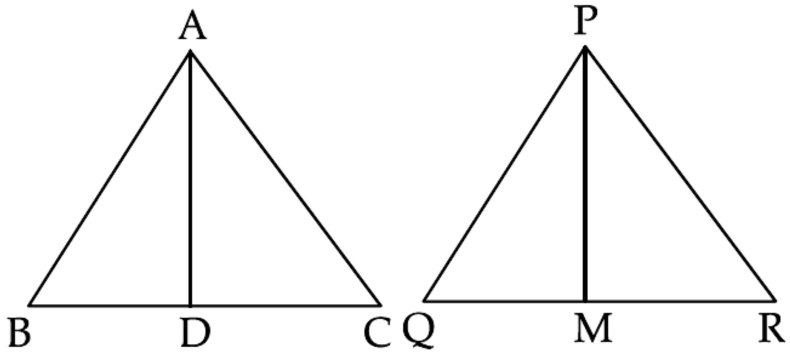
32. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$, prove that $CA^2 = CB.\ CD$
Ans: Given: D is the point on the side BC of ΔABC such that $\angle ADC = \angle BAC$
To prove: $CA^2 = CB.CD$



Proof: In ΔADC and ΔBAC,
$\angle ADC = \angle BAC$ (Given)
$\angle ACD = \angle BCA$ (common angle)
$\therefore \Delta ADC \sim \Delta BAC$ (By AA similarly criterion)
We know that, the corresponding sides of similar triangles are in proportion.
$\therefore CA/CB = CD/CA$
$\Rightarrow CA^2 = CB.CD$

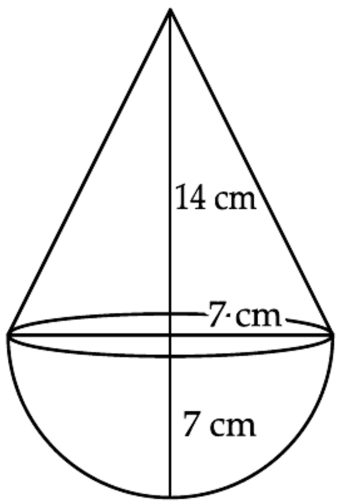
OR

If AD and PM are medians of triangles ABC and PQR respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.
Ans: Given, $\Delta ABC \sim \Delta PQR$
We know that the corresponding sides of similar triangles are in proportion.
$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ ...(i)



Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R ...(ii)
Since AD and PM are medians, they will divide opposite sides.
∴ BD = BC/2 and QM = QR/2 ...(iii)
From eqs. (i) and (ii), we get $\frac{AB}{PQ} = \frac{BD}{QM}$ ...(iv)
In ΔABD and ΔPQM,
∠B = ∠Q [using eq. (ii)]
$\frac{AB}{PQ} = \frac{BD}{QM}$
∴ ΔABD ~ ΔPQM (By SAS similarity criterion)
Thus, $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$
Hence, $\frac{AB}{PQ} = \frac{AD}{PM}$

33. A solid is in the shape of a right-circular cone surmounted on a hemisphere, the radius of each of them being 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid.
Ans: Here, Radius = 7 cm
Height = 2 × Radius = 14 cm



Volume of cone = $\frac{1}{3}\pi r^2 h$
Volume of hemisphere = $\frac{2}{3}\pi r^3$
Volume of solid = Volume of cone + Volume of hemisphere
$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 (h + 2r)$
$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 (14 + 2 \times 7) = \frac{154}{3} \times 28 = \frac{4312}{3} = 1437.33 \text{ cm}^3$

34. 250 apples of a box were weighted and the distribution of masses of the apples is given in the following table:

Mass (in grams)	80 – 100	100 – 120	120 – 140	140 – 160	160 – 180
Number of apples	20	60	70	x	60

(i) Find the value of x and the mean mass of the apples.
(ii) Find the modal mass of the apples.
Ans: (i) Total Number of apples = 250
⇒ 210 + x = 250
⇒ x = 40

Mass (in grams)	**Number of apples**	**Class mark 'x'**	**fx**
80 – 100	20	90	1800
100 – 120	60	110	6600
120 – 140	70	130	9100
140 – 160	$x = 40$	150	6000
160 – 180	60	170	10200
Total	$210 + x$		33700

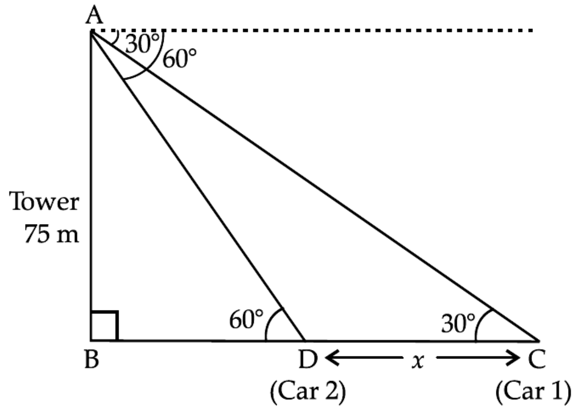
$$Mean, \bar{x} = \frac{\sum fx}{\sum f} = \frac{33700}{250} = 134.8$$

(ii) Here modal class is 120 – 140 as it has maximum frequency.
$l = 120$, h = 20, $f_1 = 70$, $f_0 = 60$, $f_2 = 40$

$$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h\right) = 120 + \left(\frac{70-60}{140-60-40} \times 20\right)$$

$$= 120 + \left(\frac{10}{40} \times 20\right) = 120 + 5 = 125$$

35. A straight highway leads to the foot of a tower. A man standing on the top of the 75 m high tower observes two cars at angles of depression of 30° and 60°, which are approaching the foot of the tower. If one car is exactly behind the other on the same side of the tower, find the distance between the two cars. (Use √3 = 1.73)
Ans: Let AB be the tower C is the position of first car and D is the position of second car.
CD is the distance between two cars.



In right ΔABC,

$$\tan 30° = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BD + x}$$

$$\therefore\ BD + x = 75\sqrt{3} \qquad \text{...(i)}$$

In right ΔABD, $\tan 60° = \frac{AB}{BD}$

$$\Rightarrow \sqrt{3} = \frac{75}{BD} \Rightarrow BD = \frac{75}{\sqrt{3}} \quad ...(ii)$$

From eqs. (i) and (ii), we get $\frac{75}{\sqrt{3}} + x = 75\sqrt{3}$

$$\Rightarrow x = 75\sqrt{3} - \frac{75}{\sqrt{3}} \Rightarrow x = 75\sqrt{3} - \frac{75\sqrt{3}}{3}$$

$$\Rightarrow x = 75\sqrt{3}\left(1 - \frac{1}{3}\right) \Rightarrow x = 75\sqrt{3} \times \frac{2}{3}$$

$$\Rightarrow x = \frac{150}{\sqrt{3}} \Rightarrow x = \frac{150}{1.73} \Rightarrow x = 86.705 \Rightarrow x = 86.71 \text{ m}$$

OR

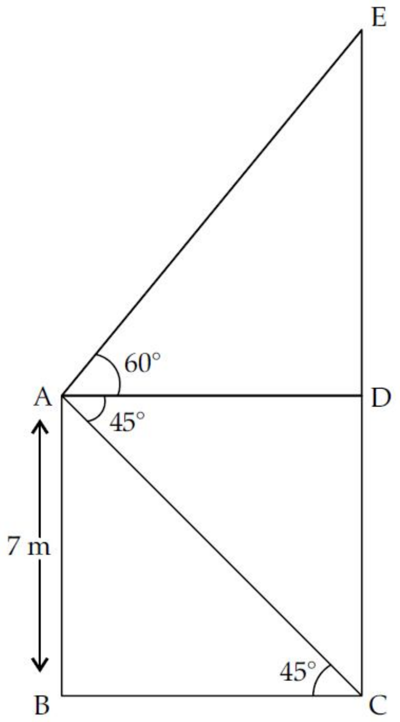
From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower.

Ans: Let AB be the building of height 7 m and EC be the height of the tower.

A is the point from where elevation of tower is 60° and the angle of depression of its foot is 45°.

EC = DE + CD

Also, CD = AB = 7 m and BC = AD



In right ΔABC, tan 45° = AB/BC

⇒ 1 = 7/BC ⇒ BC = 7

Since, BC = AD

So, AD = 7 m

In right ΔADE, tan 60° = DE/AD

⇒ √3 = DE/7 ⇒ DE = 7√3 cm

Hence, EC = CD + ED

= 7 + 7√3 = 7(1 + √3) = 7(1 + 1.732)

= 7 × 2.732 = 19.124 m = 19 m

Thus, height of the tower in approximately 19 m.

SECTION-E (Case Study Based Questions)
Questions 36 to 38 carry 4M each

36. A coaching institute of Mathematics conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, there are 20 poor and 5 rich children, whereas in batch II, there are 5 poor and 25 rich children. The total monthly collection of fees from batch I is Rs. 9000 and from batch II is Rs. 26,000. Assume that each poor child pays Rs. x per month and each rich child pays Rs. y per month.

Based on the above information, answer the following questions:
(i) Represent the information given above in terms of x and y.
(ii) Find the monthly fee paid by a poor child.

OR

Find the difference in the monthly fee paid by a poor child and a rich child.
(iii) If there are 10 poor and 20 rich children in batch II, what is the total monthly collection of fees from batch II?

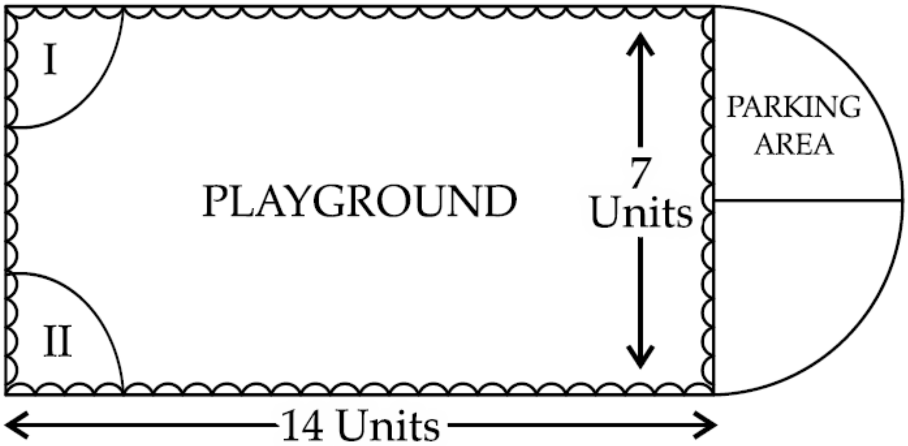
Ans: Monthly fees paid by each poor children = ` x
Monthly fees paid by each rich children = ` y
(i) For batch I
$20x + 5y = 9000$ ...(i)
For batch II
$5x + 25y = 26000$ ...(ii)
(ii) Multiply equation (i) by 5 we get
$100x + 25y = 45000$ (iii)
Subtract (ii) from (iii), we get
$95x = 19000$
$\Rightarrow x = 19000/95 = 200$
Thus, monthly fee paid by Poor Child = ` 200

OR

Substitute value of x in equation (i)
$20 \times 200 + 5y = 9000$
$\Rightarrow 5y = 9000 – 4000$
$\Rightarrow y = 5000/5 = 1000$
Monthly fee paid by Rich child = Rs. 1000
Difference in monthly fee paid by poor child and a rich child = 1000 – 200
= Rs. 800
(iii) Poor children = 10
Rich children = 20
Total monthly collection of fees from batch II
$= 10 \times 200 + 200 \times 800$

= 2000 + 16000
= Rs. 18000

37. Governing council of a local public development authority of Dehradun decided to build an adventurous playground on the top of a hill, which will have adequate space for parking. After survey, it was decided to build rectangular playground, with a semi-circular are allotted for parking at one end of the playground. The length and breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants of radius 2 units on one side for special seats.



Based on the above information, answer the following questions:
(i) What is the total perimeter of the parking area?
(ii) (a) What is the total area of parking and the two quadrants?

OR

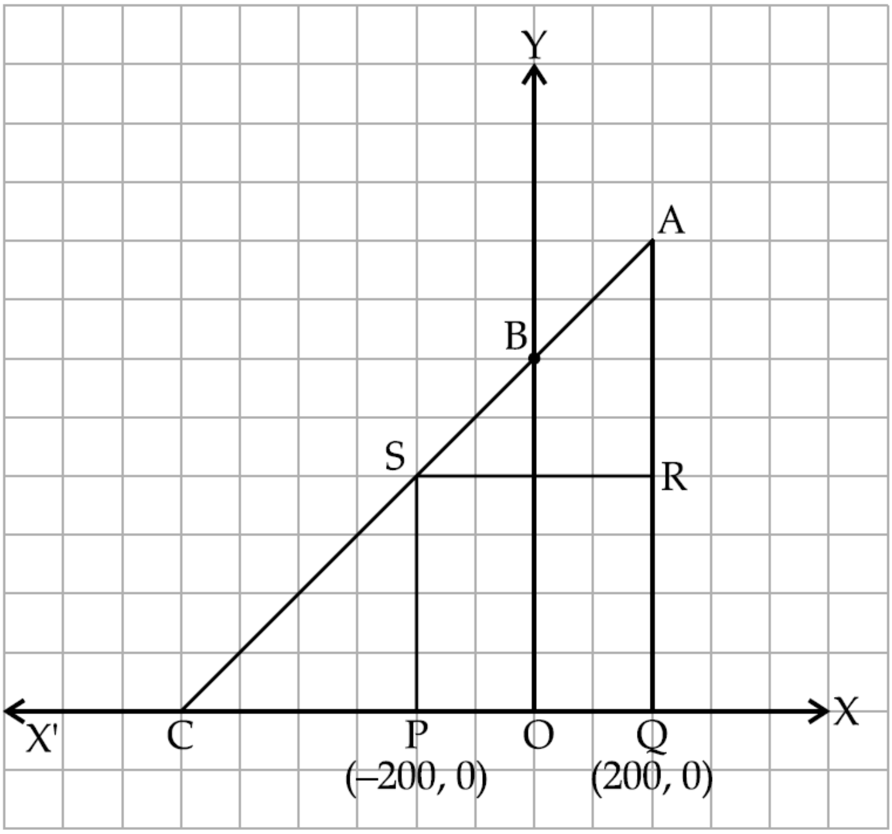
(b) What is the ratio of area of playground to the area of parking area?
(iii) Find the cost of fencing the playground and parking area at the rate of Rs. 2 per unit.
Ans: (i) Radius of semi-circle (r) = 7/2 = 3.5 units
Circumference of semi-circle = πr = 22/7 × 3.5 = 11 units
∴ Perimeter of parking area
= circumference of semi-circle + diameter of semi-circle
= 11 + 7 = 18 units
(ii) (a) Area of parking = $\frac{\pi r^2}{2} = \frac{22}{7} \times \frac{1}{2} \times (3.5)^2$
= 11 × 0.5 × 3.5
= 19.25 unit²
Area of quadrants = 2 × area of one quadrant
$2 \times \frac{\pi r_1^2}{4} = 2 \times \frac{22}{7} \times \frac{1}{4} \times (2)^2 \quad [\because r_1 = 2 \ \ units]$
= 6.285 unit²
Thus, total area = 19.25 + 6.285 = 25.535 unit²

OR

(b) Area of playground = length × breadth
= 14 × 7 = 98 unit²
Area of parking = 19.25 unit² [from part (ii) a]
∴ Ratio of playground : Ratio of parking area
= 98 : 19.25
= 9800/1925
= 56/11
Thus, required ratio is 56 : 11.
(iii) We know that, Perimeter of parking area = 18 units
Also, Perimeter of playground = 2(*l* + *b*)

= 2(14 + 7) = 2 × 21 = 42 units
Thus, total perimeter of parking area and playground
= 18 + 42 – 7 = 53 units
Hence, total cost = Rs. 2 × 53 = Rs. 106

38. Jagdhish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field from growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.



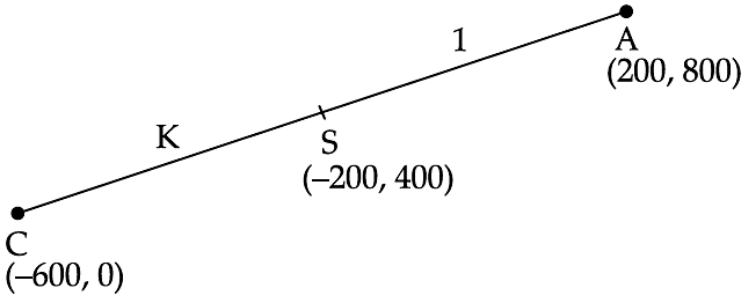
Based on the above information, answer the following questions:
(i) Taking O as origin, coordinates of P are (–200, 0) and of Q are (200, 0). PQRS being a square, what are the coordinates of R and S?
(ii) (a) What is the area of square PQRS ?

OR

(b) What is the length of diagonal PR in square PQRS?
(iii) If S divides CA in the ratio K : 1, what is the value of K, where point A is (200, 800) ?
Ans: (i) Coordinates of R = (200, 400)
Coordinates of S = (– 200, 400)
(ii) Since, side of square PQRS = 400
Thus, area of square PQRS = $(\text{side})^2$
= $(400)^2$ = 160000 unit^2

OR

We know that, diagonal of square = $\sqrt{2}$ × side
∴ Diagonal PR of square PQRS = $\sqrt{2}$ × 400 = 400 $\sqrt{2}$ units
(iii)



Using section formula, $-200 = \frac{200K + 1(-600)}{K+1}$

⇒ – 200 K – 200 = 200 K – 600

⇒ – 400 K = – 400 ⇒ K = 1

[Note: Here, S is the mid-point of CA, hence S divides CA in ratio 1 : 1]