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SAMPLE PAPER TEST 13 FOR BOARD EXAM 2024
(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**
CLASS : X **DURATION : 3 HRS**

General Instruction:

1. This Question Paper has 5 Sections A-E.
2. **Section A** has 20 MCQs carrying 1 mark each.
3. **Section B** has 5 questions carrying 02 marks each.
4. **Section C** has 6 questions carrying 03 marks each.
5. **Section D** has 4 questions carrying 05 marks each.
6. **Section E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

SECTION – A
Questions 1 to 20 carry 1 mark each.

1. The distance of the point (–1, 7) from x-axis is:
(a) – 1 (b) 7 (c) 6 (d) 50
Ans: (b) 7
The distance of (–1, 7) from x-axis is y-coordinate i.e. 7 units.

2. What is the area of a semi-circle of diameter 'd' ?
(a) $\frac{1}{16}\pi d^2$ (b) $\frac{1}{4}\pi d^2$ (c) $\frac{1}{8}\pi d^2$ (d) $\frac{1}{2}\pi d^2$
Ans: (c) $\frac{1}{8}\pi d^2$
Given, diameter of semi-circle = d
∴ radius of semi-circle = d/2

Therefore, area of semi-circle = $\dfrac{\pi\left(\frac{d}{2}\right)^2}{2} = \dfrac{\pi d^2}{8}$

3. If a, b are zeroes of the polynomial $x^2 - 1$, then value of (a + b) is:
(a) 2 (b) 1 (c) – 1 (d) 0
Ans: (d) 0
Given polynomial: $x^2 - 1 = (x - 1)(x + 1)$
For zeroes, (x – 1)(x + 1) = 0
∴ x = 1 and x = – 1
Let a = 1 and b = – 1
Sum of a + b = 1 + (– 1) = 0

4. sec θ when expressed in terms of cot θ, is equal to:
(a) $\dfrac{1+\cot^2\theta}{\cot\theta}$ (b) $\sqrt{1+\cot^2\theta}$ (c) $\dfrac{\sqrt{1+\cot^2\theta}}{\cot\theta}$ (d) $\dfrac{\sqrt{1-\cot^2\theta}}{\cot\theta}$

Ans: $\dfrac{\sqrt{1+\cot^2\theta}}{\cot\theta}$

5. If $p^2 = \dfrac{32}{50}$, then p is a/an

(a) whole number (b) integer (c) rational number (d) irrational number

Ans: (c) rational number

$p^2 = \dfrac{32}{50} \Rightarrow p^2 = \dfrac{16}{25} \Rightarrow p = \dfrac{4}{5}$

Since p is in form of p/q where q $\neq$ 0.

$\therefore$ p is a rational number.

6. The pair of linear equations 2x = 5y + 6 and 15y = 6x – 18 represents two lines which are:

(a) intersecting (b) parallel (c) coincident (d) either intersecting or parallel

Ans: (c) coincident

Given equations can be rewrite as:

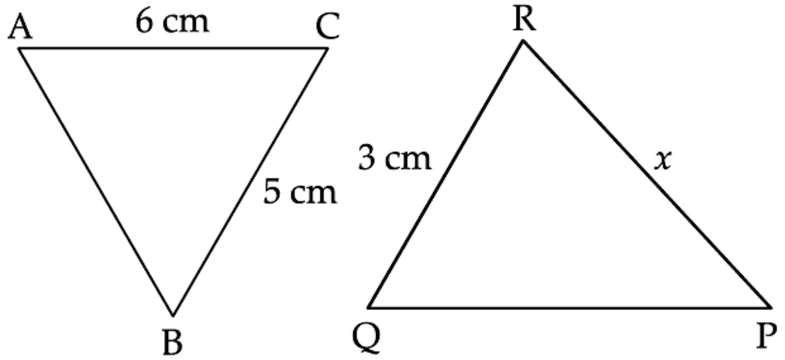
2x – 5y – 6 = 0

6x – 15y – 18 = 0

$\dfrac{a_1}{a_2} = \dfrac{2}{6} = \dfrac{1}{3}$, $\dfrac{b_1}{b_2} = \dfrac{-5}{-15} = \dfrac{1}{3}$ and $\dfrac{c_1}{c_2} = \dfrac{-6}{-18} = \dfrac{1}{3}$

Therefore, the pair of equations has infinitely many solutions. Graphically pair of linear equations represent coincident.

7. In the given figure, ΔABC ~ ΔQPR. If AC = 6 cm, BC = 5 cm, QR = 3 cm and PR = x; then the value of x is:



(a) 3.6 cm (b) 2.5 cm (c) 10 cm (d) 3.2 cm

Ans: (b) 2.5 cm

Given, ΔABC ~ ΔQPR

$\therefore \dfrac{AB}{QP} = \dfrac{BC}{PR} = \dfrac{AC}{QR} \Rightarrow \dfrac{AB}{QP} = \dfrac{5}{x} = \dfrac{6}{3} \Rightarrow x = \dfrac{5\times 3}{6} = \dfrac{5}{2} = 2.5cm$

8. The roots of the equation $x^2 + 3x - 10 = 0$ are:

(a) 2, – 5 (b) – 2, 5 (c) 2, 5 (d) –2, – 5

Ans: (a) 2, – 5

$x^2 + 3x - 10 = 0 \Rightarrow x^2 + 5x - 2x - 10 = 0$

$\Rightarrow x(x + 5) - 2(x + 5) = 0 \Rightarrow x = 2$ and $x = -5$

9. The number of quadratic polynomials having zeroes – 5 and – 3 is

(a) 1 (b) 2 (c) 3 (d) more than 3

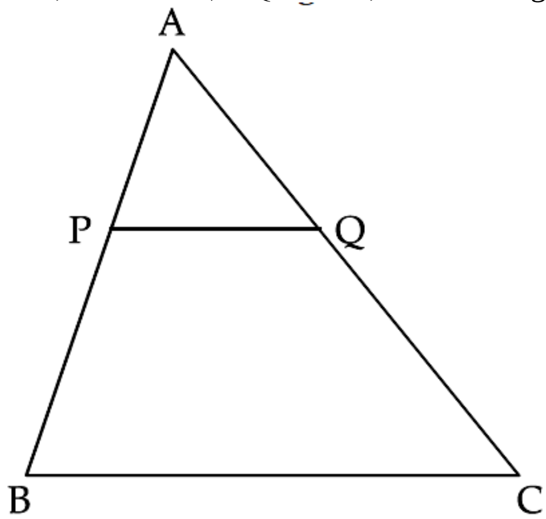
Ans: (d) more than 3

Let the zeroes of polynomial be a = – 5 and b = – 3

The general form of polynomial with a and b as the zeroes is given by

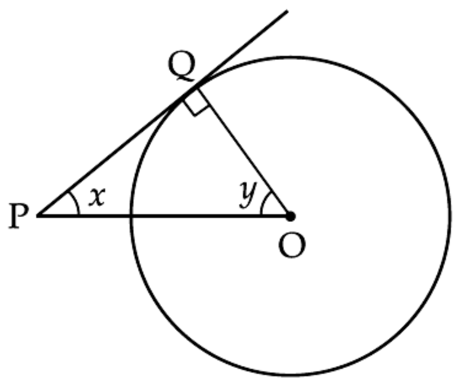
$k[x^2 – (a + b) x + ab]$ where k is any real number
$k[x^2 – (– 8)x + (– 15)]$ ($\because$ a + b = (– 5) + (– 3) = – 8 & ab = – 5 × – 3 = 15)
$\Rightarrow k(x^2 + 8x + 15)$
Here k can have any value
Hence, more than 3 polynomials can have the zeroes – 5 and – 3.

10. In ΔABC, PQ || BC. If PB = 6 cm, AP = 4 cm, AQ = 8 cm, find the length of AC.



(a) 12 cm (b) 20 cm (c) 6 cm (d) 14 cm
Ans: (b) 20 cm
As PQ || BC by using basic proportionality theorem,
AP/PB = AQ/QC
⇒ 4/6 = 8/QC
⇒ QC = 8 × 6 / 4
⇒ QC = 12 cm
Now, AC = AQ + QC = 8 + 12 = 20 cm

11. In the given figure, PQ is a tangent to the circle with centre O. If ∠OPQ = x, ∠POQ = y, then x + y is:



(a) 45° (b) 90° (c) 60° (d) 180°
Ans: (b) 90°
Here, ∠OQP = 90° (angle between radius and tangent)
Now, in ΔOQP, ∠OQP + ∠QOP + ∠OPQ = 180°
⇒ 90° + y + x = 180°
⇒ x + y = 90°

12. Two dice are thrown together. The probability of getting the difference of numbers on their upper faces equals to 3 is:
(a) 1/9 (b) 2/9 (c) 1/6 (d) 1/12
Ans: (c) 1/6

Total number of possible outcomes = 36
Favourable outcomes to get difference of number on the dice as 3 are:
(1, 4), (2, 5), (3, 6), (4, 1), (5, 2), (6, 3)
Required Probability = 6/36 = 1/6

13. A card is drawn at random from a well-shuffled pack of 52 cards. The probability that the card drawn is not an ace is:
(a) 1/13 (b) 9/13 (c) 4/13 (d) 12/13
Ans: (d) 12/13
No. of ace cards in a pack of 52 cards = 4
∴ No. of non-ace cards in a pack of 52 cards = 48
Required probability = 48/52 = 12/13

14. If θ is an acute angle of a right angled triangle, then which of the following equation is not true?
(a) $\sin\theta \cot\theta = \cos\theta$ (b) $\cos\theta \tan\theta = \sin\theta$
(c) $\text{cosec}^2\theta - \cot^2\theta = 1$ (d) $\tan^2\theta - \sec^2\theta = 1$
Ans: (d) $\tan^2\theta - \sec^2\theta = 1$
For the given acute angle (θ), $1 + \tan^2\theta = \sec^2\theta$
So, $\sec^2\theta - \tan^2\theta = 1$ but in option (d) is incorrect
Hence, option (d) is false.

15. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and – 3, then
(a) a = – 7, b = – 1 (b) a = 5, b = – 1 (c) a = 2, b = – 6 (d) a = 0, b = – 6
Ans: (d) a = 0, b = – 6
Zeroes of Quadratic Polynomial $x^2 + (a + 1)x + b$ are 2 and – 3
∴ $\alpha = 2$ and $\beta = -3$
Then, Sum of zeroes $(\alpha + \beta) = (2 + (-3)) = -1$
Product of zeroes $(\alpha\beta) = 2 \times -3 = -6$
∴ Quadratic Polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$
$\Rightarrow x^2 + 1x - 6 = 0$ ...(ii)
From Equation (i) and (ii), $a + 1 = 1 \Rightarrow a = 0$ and $b = -6$

16. If the sum of the first n terms of an A.P. be $3n^2 + n$ and its common difference is 6, then its first term is
(a) 2 (b) 3 (c) 1 (d) 4
Ans: (d) 4
$S_n = 3n^2 + n$ and d = 6
Substituting n = 1, we get
$S_1 = 3(1)^2 + 1 = 3 + 1 = 4$
$\Rightarrow$ a = 4
Thus, first term = 4.

17. The volume of a right circular cone whose area of the base is 156 cm² and the vertical height is 8 cm, is:
(a) 2496 cm³ (b) 1248 cm³ (c) 1664 cm³ (d) 416 cm³
Ans: (d) 416 cm³
Volume of cone = $1/3 \times \pi r^2 h$
$= 1/3 \times 156 \times 8$ (∵ Area of base $= \pi r^2 = 156$ cm²)
= 416 cm³

18. 3 chairs and 1 table cost Rs. 900; whereas 5 chairs and 3 tables cost Rs. 2,100. If the cost of 1 chair is Rs. x and the cost of 1 table is Rs. y, then the situation can be represented algebraically as
(a) $3x + y = 900, 3x + 5y = 2100$ (b) $x + 3y = 900, 3x + 5y = 2100$
(c) $3x + y = 900, 5x + 3y = 2100$ (d) $x + 3y = 900, 5x + 3y = 2100$

Ans: (c) $3x + y = 900$, $5x + 3y = 2100$
cost of one chair = x
cost of one table = y
$\therefore$ $3x + y$ = Rs. 900
$5x + 3y$ = Rs. 2100

Direction : In the question number 19 & 20 , A statement of Assertion (A) is followed by a statement of Reason(R) . Choose the correct option
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. Assertion (A): For $0 < \theta \leq 90°$, $\operatorname{cosec}\theta - \cot\theta$ and $\operatorname{cosec}\theta + \cot\theta$ are reciprocal of each other.
Reason (R): $\operatorname{cosec}^2\theta - \cot^2\theta = 1$
Ans: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
According to Trigonometry Identity, $\operatorname{cosec}2\theta - \cot2\theta = 1$
$\therefore (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$
$\Rightarrow (\operatorname{cosec}\theta - \cot\theta) = 1/(\operatorname{cosec}\theta + \cot\theta)$
or $(\operatorname{cosec}\theta + \cot\theta) = 1/(\operatorname{cosec}\theta - \cot\theta)$
$\therefore$ Assertion is True.
Reason : It is a Trigonometric Identity which is used in Assertion
$\therefore$ Reason is also true and correct. Explanation of Assertion.

20. Assertion (A): If $5 + \sqrt{7}$ is a root of a quadratic equation with rational coefficients, then its other root is $5 - \sqrt{7}$.
Reason (R): Surd roots of quadratic equation with rational coefficients occur in conjugate pairs.
Ans: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
In Quadratic Equation with rational coefficient, irrational roots occur in conjugate pairs.
$\therefore$ If one root = $5 + \sqrt{7}$ then second root = $5 - \sqrt{7}$
Hence, Assertion is True and Reason is also true and correct explanation.

SECTION-B
Questions 21 to 25 carry 2M each

21. Two number are in the ratio 2 : 3 and their LCM is 180. What is the HCF of these numbers?
Ans: We know that, LCM × HCF = a × b (a, b are two numbers) ...(i)
Let numbers = 2x and 3x
$\therefore$ LCM = $2 \times 3 \times x = 6x$
$\Rightarrow 6x = 180 \Rightarrow x = 30$
Numbers are: $2 \times 30 = 60$ and $3 \times 30 = 90$
From eq (i), $180 \times \text{HCF} = 60 \times 90$
HCF = $60 \times 90 / 180 = 30$
Hence, HCF = 30

OR

Find the HCF and LCM of 72 and 120.
Ans: By Prime Factorisation, we get
Factors of 72 = $2^3 \times 3^2$
$120 = 2^3 \times 3^1 \times 5^1$

HCF (72, 120) = Product of common terms with lowest power
$= 2^3 \times 3^1 = 8 \times 3 = 24$
LCM (72, 120) = Product of Prime Factors with highest power.
$= 2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 360$
Thus, HCF and LCM of 72 and 120 are 24 and 360 respectively.

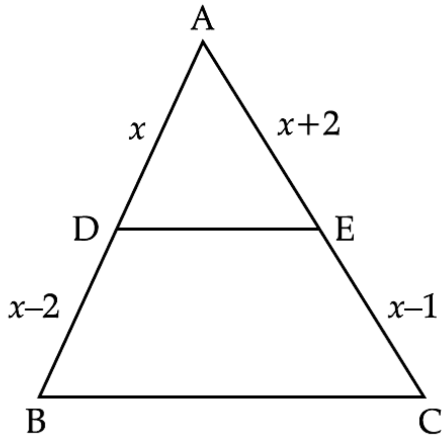
22. Evaluate: $\dfrac{5\cos^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\sin^2 30^0 + \cos^2 30^0}$

Ans:

We have, $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{5}{4} + \frac{16}{3} - 1 = \frac{15 + 64 - 12}{12} = \frac{67}{12}$$

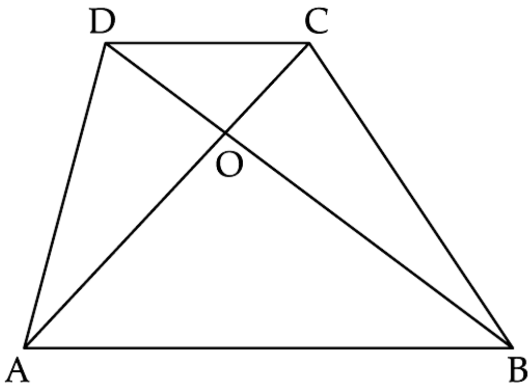
23. In the given figure, ABC is a triangle in which DE||BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, then find the value of x.



Ans: According to Basic Proportionality theorem.
$\dfrac{AD}{DB} = \dfrac{AE}{EC} \Rightarrow \dfrac{x}{x-2} = \dfrac{x+2}{x-1}$
$\Rightarrow x(x - 1) = (x - 2)(x + 2)$
$\Rightarrow x^2 - x = x^2 - 4$
$\Rightarrow -x = -4$
$\Rightarrow x = 4.$

OR

Diagonals AC and BD of trapezium ABCD with AB||DC intersect each other at point O. Show that OA/OC = OB/OD.



Ans: Given: ABCD is a trapezium, AB || DC.
Diagonals AC and BD intersect at O.

To Prove: $\frac{OA}{OC} = \frac{OB}{OD}$

Construction: Draw OE || AB, through O, meeting AD at E.
Proof: In ΔADC, EO || DC (∵ EO || AB || DC)

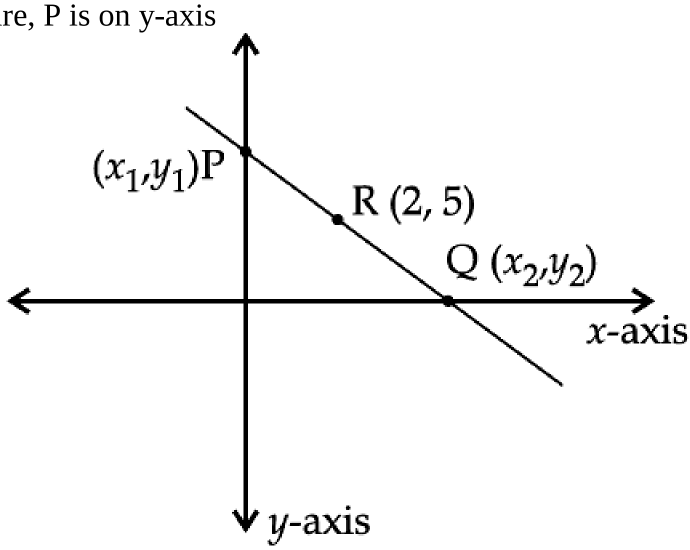
$\frac{AE}{ED} = \frac{OA}{OC}$ (By Thale's Theorem (i))

In ΔDAB, EO || AB (By constructions)

$\frac{AE}{ED} = \frac{OB}{OD}$ (By Thale's Theorem) ...(ii)

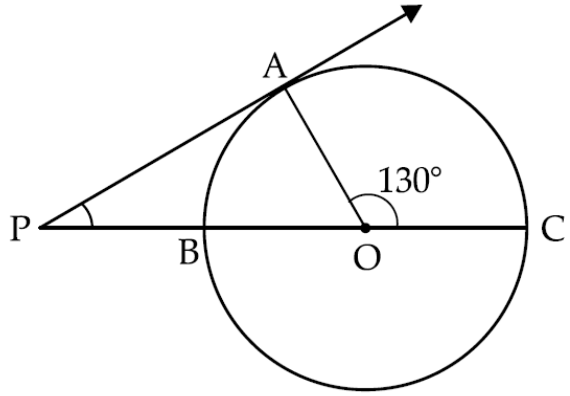
From (i) and (ii), we get $\frac{OA}{OC} = \frac{OB}{OD}$

24. A line intersects y-axis and x-axis at point P and Q, respectively. If R(2, 5) is the mid-point of line segment PQ, then find the coordinates of P and Q.
Ans: According to figure, P is on y-axis



∴ Coordinates of P are $(0, y_1)$
Q is on x-axis
∴ Co-ordinates of Q are $(x_2, 0)$
According to mid-point Formula
$2 = 0 + x_2 / 2$ and $5 = y_1 + 0 /2$
$\Rightarrow 4 = x_2$; $10 = y_1$
Thus coordinates of P are (0, 10)
And, coordinates of Q are (4, 0).

25. In the given figure, PA is a tangent to the circle drawn from the external point P and PBC is the secant to the circle with BC as diameter. If ∠AOC = 130°, then find the measure of ∠APB, where O is the centre of the circle.



Ans: We know that the tangent at a point to a circle is perpendicular to the radius passing through the point of contact.

∴ ∠OAP = 90°

Now, ∠AOC + ∠AOB = 180° (Linear pair)

⇒ ∠AOP = 50°

In ΔPAO, ∠APO + ∠PAO + ∠AOP = 180° (Sum of all angles of a triangle is 180°)

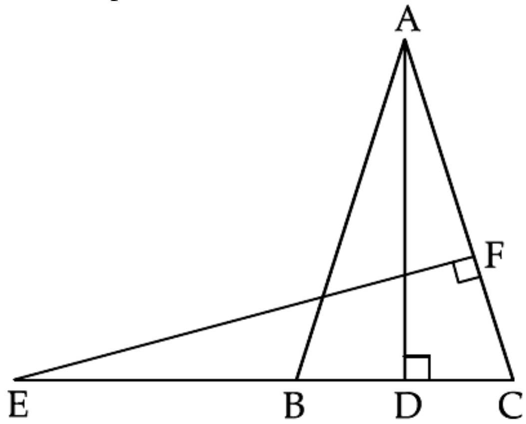
⇒ ∠APO = 180° – (∠PAO + ∠AOP)

= 180° – (90° + 50°) = 40°.

SECTION-C

Questions 26 to 31 carry 3 marks each

26. In the given figure, E is a point on the side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, them prove that ΔABD ~ ΔECF.



Ans: Given: AB = AC

∴ ∠B = ∠C ...(i) (Angles opposite to equal sides are equal)

AD ⊥ BC

∴ ∠ADB = 90° ...(ii)

EF ⊥ AC

∴ ∠EFC = 90° ...(iii) 1

In ΔABD and ΔECF,

∠B = ∠C (From (i))

∠ADB = ∠EFC = 90° (From (ii) & (iii))

∴ ΔABD ~ ΔECF (By AA Criterion)

27. Find the value of 'p' for which the quadratic equation px(x – 2) + 6 = 0 has two equal real roots.

Ans: For equal roots, discriminant = 0
i.e., $b^2 - 4ac = 0$ …(i)
Given equation is $px(x - 2) + 6 = 0$
i.e., $px^2 - 2px + 6 = 0$
here, $a = p$, $b = -2p$ and $c = 6$ (On comparing with $ax^2 + bx + c = 0$)
From eq, (i) $(-2p)^2 - 4(p)(6) = 0$
$\Rightarrow 4p^2 - 24p = 0 \Rightarrow 4p^2 = 24p \Rightarrow p^2 = 6p$
$\Rightarrow p^2 - 6p = 0 \Rightarrow p(p - 6) = 0 \Rightarrow p = 0$ or $p = 6$
$\Rightarrow p = 6$ (Since, If $p = 0$, then given equation is not quadratic equation)

OR

If α and β are roots of the quadratic equation $x^2 - 7x + 10 = 0$, find the quadratic equation whose roots are α^2 and β^2.
Ans: For Given Quadratic Equation
$x^2 - 7x + 10 = 0$
$\Rightarrow x^2 - 5x - 2x + 10 = 0$
$\Rightarrow x(x - 5) - 2(x - 5) = 0$
$\Rightarrow (x - 5)(x - 2) = 0 \Rightarrow x = 5$ and $x = 2$
$\therefore \alpha = 5$ and $\beta = 2$
Thus $\alpha^2 = 25$ and $\beta^2 = 4$
Quadratic Equation whose roots are α^2 and β^2
$\Rightarrow x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0 \Rightarrow x^2 - (25 + 4)x + 25 \times 4 = 0$
$\Rightarrow x^2 - 29x + 100 = 0$

28. Prove that: $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$

OR

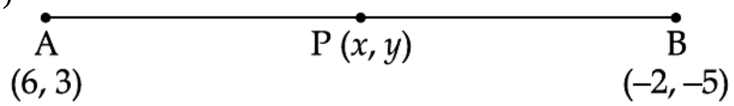
Prove that sec A (1 – sin A) (sec A + tan A) = 1.
Ans:

$$\text{L.H.S.} = \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$$
$$= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)} = \frac{\sin A[1 - 2\sin^2 A]}{\cos A[2(1 - \sin^2 A) - 1]}$$
$$= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(1 - 2\sin^2 A)} = \tan A = \text{R.H.S}$$

OR

$$\text{L.H.S} = \sec A(1 - \sin A)(\sec A + \tan A)$$
$$= \left(\sec A - \frac{\sin A}{\cos A}\right)(\sec A + \tan A) \quad \left[\because \sec A = \frac{1}{\cos A}\right]$$
$$= (\sec A - \tan A)(\sec A + \tan A) = \sec^2 A - \tan^2 A = 1 = \text{R.H.S}$$

29. Find the ratio in which the line segment joining the points A(6, 3) and B(–2, –5) is divided by x-axis.
Ans: We know that on x-axis, $y = 0$.
$\therefore$ point will be P(x, 0)



and let the ratio be k : 1

then $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$ = (x, 0)

$\Rightarrow \left(\frac{-2k+6}{k+1}, \frac{-5k+3}{k+1}\right)$ = (x, 0)

$\therefore \frac{-5k+3}{k+1} = 0 \Rightarrow -5k + 3 = 0 \Rightarrow k = \frac{3}{5}$

30. Prove that √5 is and irrational number.

Ans: Let $\sqrt{5}$ is a rational number then we have $\sqrt{5} = \frac{p}{q}$, where p and q are co-primes.

$\Rightarrow p = \sqrt{5}q$

Squaring both sides, we get $p^2 = 5q^2$

$\Rightarrow$ p^2 is divisible by 5 $\Rightarrow$ p is also divisible by 5

So, assume p = 5m where m is any integer.

Squaring both sides, we get $p^2 = 25m^2$

But $p^2 = 5q^2$

Therefore, $5q^2 = 25m^2 \Rightarrow q^2 = 5m^2$

$\Rightarrow$ q^2 is divisible by 5 $\Rightarrow$ q is also divisible by 5

From above we conclude that p and q have one common factor i.e. 5 which contradicts that p and q are co-primes.

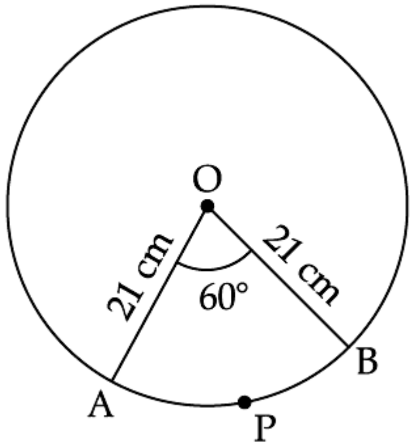
Therefore, our assumption is wrong.

Hence, $\sqrt{5}$ is an irrational number.

31. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the area of the sector formed by the arc. Also, find the length of the arc.

Ans: (i) Area of sector APB = $\frac{\theta}{360^0}\pi r^2 = \frac{60^0}{360^0} \times \frac{22}{7} \times 21 \times 21$

$= \frac{1}{6} \times 22 \times 3 \times 21 = 11 \times 21 = 231 cm^2$



(ii) Length of the arc APB = $\frac{\theta}{360^0} \times 2\pi r = \frac{60^0}{360^0} \times 2 \times \frac{22}{7} \times 21$

$= \frac{1}{6} \times 2 \times 22 \times 3 = 22 cm$

SECTION-D

Questions 32 to 35 carry 5M each

32. The ratio of the 11th term to the 18th term of an A.P. is 2 : 3. Find the ratio of the 5th term to the 21st term. Also, find the ratio of the sum of first 5 terms to the sum of first 21 terms.

Ans: Let First term = a
Common difference = d

$\frac{a_{11}}{a_{18}} = \frac{2}{3} \Rightarrow \frac{a+10d}{a+17d} = \frac{2}{3}$ ($\because$ a_{11} = a + 10d and a_{18} = a + 17d)

⇒ 3a + 30d = 2a + 34d
⇒ a = 4d ...(i)

Ratio of 5th term to 21st term is $\frac{S_5}{S_{21}} = \frac{a+4d}{a+20d}$

Substitute value of a = 4d from (i) we get

$\frac{S_5}{S_{21}} = \frac{4d+4d}{4d+20d} = \frac{8d}{24d} = \frac{1}{3}$

Ratio of S_5 to S_{21} is $\frac{S_5}{S_{21}} = \frac{\frac{5}{2}(2a+4d)}{\frac{21}{2}(2a+20d)} = \frac{5(2a+4d)}{21(2a+20d)}$ $\left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$

Substitute a = 4d in equation (ii)

$\frac{S_5}{S_{21}} = \frac{5(8d+4d)}{21(8d+20d)} = \frac{5(12d)}{21(28d)} = \frac{60d}{588d} \Rightarrow \frac{S_5}{S_{21}} = \frac{5}{49}$

Hence $a_5 : a_{21}$ = 1 : 3
$S_5 : S_{21}$ = 5 : 49

OR

If the sum of first 6 terms of an A.P. is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.

Ans: We know that $S_n = \frac{n}{2}[2a+(n-1)d]$

Now, S_6 = 36 (Given)

$\Rightarrow \frac{6}{2}[2a+(6-1)d] = 36$

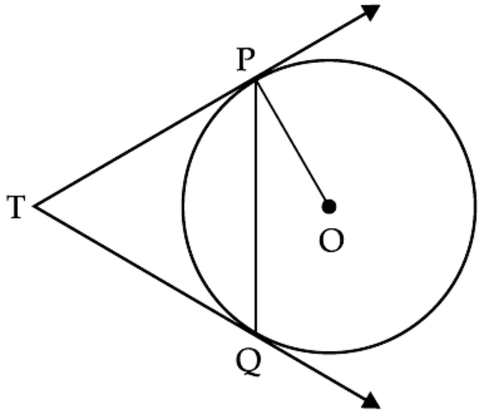
⇒ 2a + 5d = 12 ...(i)
Also, S_{16} = 256 (Given)

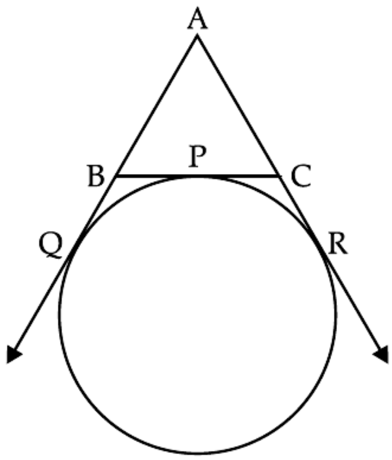
$\Rightarrow \frac{16}{2}[2a+(16-1)d] = 256$

⇒ 2a + 15d = 32 ...(ii)
Subtract (i) from (ii), we get
10d = 20 ⇒ d = 2
Substitute d = 2 in equation (i)
2a + 5 × 2 = 12
⇒ 2a = 12 – 10 ⇒ 2a = 2 ⇒ a = 1
Thus, the sum of first 10 terms of AP

$S_{10} = \frac{10}{2}[2\times1+(10-1)\times2]$ = 5(2 + 18) = 5 × 20 = 100

33. Two tangents TP and TQ are drawn to a circle with centre O from an external point T (see below left figure). Prove that ∠PTQ = 2 ∠OPQ.





Ans: Given: TP = TQ (Two tangents from external point T are equal)
To Prove: ∠PTQ = 2∠OPQ
Proof: Let ∠PTQ = x
TP = TQ (Given)
∴ ∠TPQ = ∠TQP
In ΔTPQ, ∠TPQ + ∠TPQ + ∠PTQ = 180°
⇒ 2∠TPQ + x = 180°
⇒ ∠TPQ = (180°− x)/2
∠TPQ = 90°− (x/2) ...(i)
Now, OP is radius
∴ ∠OPT = 90° (Tangent at any point of a circle is perpendicular to the radius through point of contact.)
⇒ ∠OPQ + ∠QPT = 90°
⇒ ∠OPQ = 90° – ∠QPT
⇒ ∠OPQ = 90° − [90°− (x/2)] (From (i))
⇒ ∠OPQ = 90° − 90°+ (x/2)
⇒ ∠OPQ = x/2
⇒ 2∠OPQ = x
⇒ ∠PTQ = 2∠OPQ (∵ ∠PTQ = x)

OR

A circle touches the side BC of a ΔABC at a point P and touches AB and AC when produced at Q and R respectively (see above right sided figure). Show that AQ = $\frac{1}{2}$(Perimeter of ΔABC).

Ans: Lengths of tangents drawn from an external point to a circle are equal.
∴ AQ = AR ...(i) (Tangents from A)
BP = BQ ...(ii) (Tangents from B)
CP = CR ...(iii) (Tangents from C) 2
Perimeter of ΔABC = AB + BC + AC
= AB + BP + PC + AC
= AB + BQ + CR + AC [Using (ii) and (iii)]
= AQ + AR
= 2AQ [From (i)]
∴ AQ = $\frac{1}{2}$ × Perimeter of ΔABC.

34. The monthly expenditure on milk in 200 families of a Housing Society is given below:

Monthly Expenditure (in Rs.)	1000 – 1500	1500 – 2000	2000 – 2500	2500 – 3000	3000 – 3500	3500 – 4000	4000 – 4500	4500 – 5000

Number of families	24	40	33	x	30	22	16	7

Find the value of x and also, find the median and mean expenditure on milk.

Ans: We have, 24 + 40 + 33 + x + 30 + 22 + 16 + 7 = 200 [∵ Total no. of families = 200]

⇒ x + 172 = 200 ⇒ x = 28

Expenditure (in ₹)	No. of families (f_i)	Cumulative frequency ($c.f.$)	x_i	$u_i = \frac{x - 2750}{h}$	$f_i u_i$
1000 – 1500	24	24	1250	– 3	– 72
1500 – 2000	40	64	1750	– 2	– 80
2000 – 2500	33	97	2250	– 1	– 33
2500 – 3000	28	125	2750	0	0
3000 – 3500	30	155	3250	1	30
3500 – 4000	22	177	3750	2	44
4000 – 4500	16	193	4250	3	48
4500 – 5000	7	200	4750	4	28
Total	200				– 35

For mean,

From table, $\Sigma f_i = 200$, $\Sigma f_i u_i = -35$, $h = 500$, $a = 2750$

$$Mean, \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h = 2750 + \left(\frac{-35}{200} \times 500\right) = 2750 - 87.5 = 2662.5$$

So, the mean monthly expenditure was ` 2662.50.

For median,

From table, $\Sigma f_i = N = 200$, then N/2 = 200/2 = 100,

which lies in interval 2500 – 3000.

Median class : 2500 – 3000

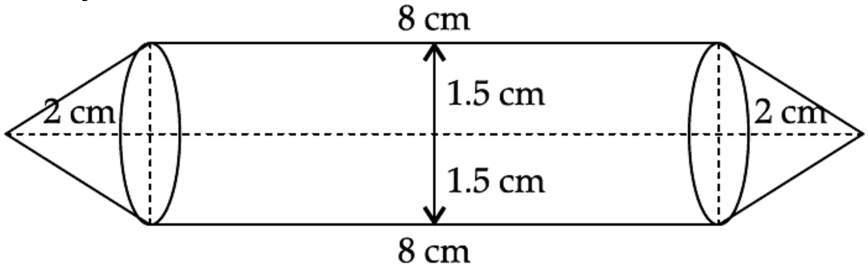
So, $l = 2500$, $f = 28$, $cf = 97$ and $h = 500$

$$Median = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 2500 + \left(\frac{100 - 97}{28} \times 500\right)$$

$$= 2500 + \left(\frac{3}{28} \times 500\right) = 2500 + 53.57 = 2553.57$$

35. A student was asked to make a model shaped like a cylinder with two cones attached to its ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its total length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model.

Ans: Here, Height of cylinder = 12 – 4 = 8 cm



Radius of cone/cylinder = 3/2 = 1.5 cm

Height of cone = 2 cm

Volume of cylinder = $\pi r^2 h$
$= \pi(1.5)^2 \times 8 = 18\pi$ cm^3
Volume of cone = $1/3\ \pi r^2 h$
$= 1/3\ \pi(1.5)^2 \times 2$
$= 1.5\pi$ cm^3
Total volume = Volume of cylinder + (Volume of cone) × 2
$= 18\pi + 1.5\pi \times 2 = 18\pi + 3\pi$
$= 21\pi = 21 \times 22/7 = 66$ cm^3.

SECTION-E (Case Study Based Questions)
Questions 36 to 38 carry 4M each

36. Two schools 'P' and 'Q' decided to award prizes to their students for two games of Hockey Rs. x per student and Cricket Rs. y per student. School 'P' decided to award a total of Rs. 9,500 for the two games to 5 and 4 students respectively; while school 'Q' decided to award Rs. 7,370 for the two games to 4 and 3 students respectively.

Based on the above information, answer the following questions:
(i) Represent the following information algebraically (in terms of x and y).
(ii) (a) What is the prize amount for hockey?
(b) Prize amount on which game is more and by how much?

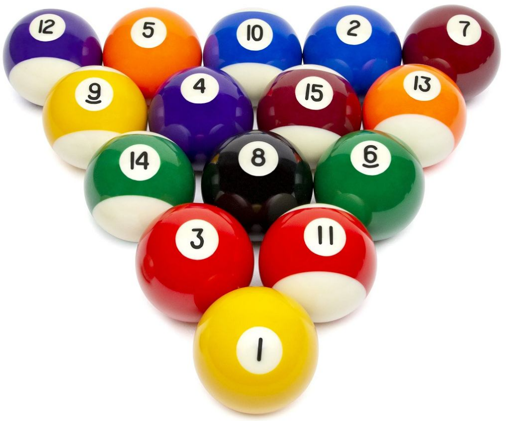
OR

(iii) What will be the total prize amount if there are 2 students each from two games ?
Ans: (i) Given Rs. x and Rs. y are the prize money per student for Hockey and Cricket, respectively.
$\therefore 5x + 4y = 9500$ ...(i)
and $4x + 3y = 7370$ ...(ii)
(ii) (a) On multiplying eq (i) by 4 and eq (ii) by 5 then subtracting, we get $y = 1150$
On substituting value of y in equation (i), we get
$5x + 4(1150) = 9500$
$\Rightarrow 5x + 4600 = 9500 \Rightarrow 5x = 4900 \Rightarrow x = 980$
Thus, prize money for Hockey is Rs. 980.

OR

(b) From part (a),
Prize money for Hockey = Rs. 980
Prize money for Cricket = Rs. 1150
Difference between prize money = Rs. (1150 – 980) = Rs. 170
Thus, prize money is Rs. 170 more for cricket in comparison to Hockey.
(iii) Total prize money = 2 (Prize money for Hockey + Prize money for Cricket)
= 2(980 + 1150) = 2 × 2130 = Rs. 4260

37. "Eight Ball" is a game played on a pool table with 15 balls numbered 1 to 15 and a "cue ball" that is solid and white. Of the 15 numbered balls, eight are solid (non-white) coloured and numbered 1 to 8 and seven are striped balls numbered 9 to 15.



The 15 numbered pool balls (no cue ball) are placed in a large bowl and mixed, then one ball is drawn out at random.

Based on the above information, answer the following question:

(i) What is the probability that the drawn ball bears number 8?

(ii) What is probability that the drawn ball bears an even number?

OR

What is the probability that the drawn ball bears a number, which is a multiple of 3?

(iii) What is the probability that the drawn ball is a solid coloured and bears an even number?

Ans: (i) Total number of balls = 15

Number of Ball bears number 8 = 1

∴ P(Getting ball bears number 8) = 1/15

(ii) Number of balls having even numbers = 7

∴ P(Getting even number balls) = 7/15

OR

Number of balls bearing a number, which is multiple of 3 = 5

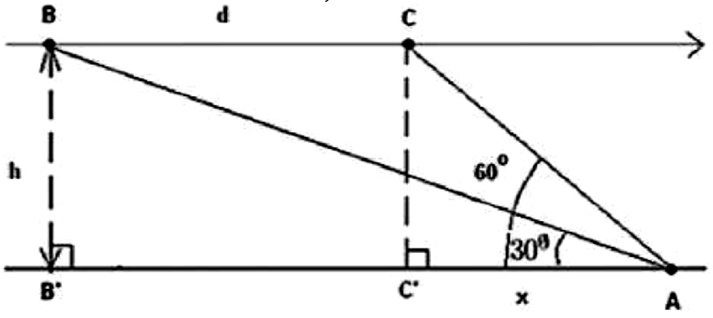
∴ P(Getting balls having multiple of 3) = 5/15 = 1/3

(iii) Solid coloured balls = 8

Number of solid coloured balls having an even number = 4.

∴ P(Getting Solid Coloured even number Ball) = 4/15

38. Aditya is a pilot in Air India. During the Covid-19 pandemic, many Indian passengers were stuck at Dubai Airport. The government of India sent special aircraft to take them. Mr. Vinod was leading this operation. He is flying from Dubai to New Delhi with these passengers. His airplane is approaching point A along a straight line and at a constant altitude h. At 10:00 am, the angle of elevation of the airplane is 30° and at 10:01 am, it is 60°.



(i) What is the distance d is covered by the airplane from 10:00 am to 10:01 am if the speed of the airplane is constant and equal to 600 miles/hour?

(ii) What is the altitude h of the airplane? (round answer to 2 decimal places)

OR

Find the distance between passenger and airplane when the angle of elevation is 60°.

(iii) Find the distance between passenger and airplane when the angle of elevation is 30°.
Ans: (i) Time covered 10.00 am to 10.01 am = 1 minute = 1/60 hour
Given: Speed = 600 miles/hour
Thus, distance d= 600 x (1/60) = 10 miles

(ii) Now, $\tan 30^0 = \frac{BB'}{B'A} = \frac{h}{10+x}$ and $\tan 60^0 = \frac{CC'}{C'A} = \frac{BB'}{C'A} = \frac{h}{x}$

$$x = \frac{h}{\tan 60^0} = \frac{h}{\sqrt{3}}$$

$$\therefore \tan 30^0 = \frac{h}{10 + \frac{h}{\sqrt{3}}} = \frac{\sqrt{3}h}{10\sqrt{3}+h}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}h}{10\sqrt{3}+h} \Rightarrow 3h = 10\sqrt{3} + h \Rightarrow 2h = 10\sqrt{3}$$

$$\Rightarrow h = 5\sqrt{3} = 8.66 \ \ miles$$

Thus, the altitude 'h' of the airplane is 8.66 miles.

OR

(ii) The distance between passenger and airplane when the angle of elevation is 60°.

In ΔACC', $\sin 60^0 = \frac{CC'}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{AC} \Rightarrow AC = 10 \ \ miles$

(iii) The distance between passenger and airplane when the angle of elevation is 30º.

In ΔABB', $\sin 30^0 = \frac{BB'}{AB} \Rightarrow \frac{1}{2} = \frac{8.66}{AB} \Rightarrow AB = 17.32 \ \ miles$